Questions

Q1.

(a) Show that the transformation $x = e^t$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^2 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + 3y = e^{2t} \qquad \text{(II)}$$

(6)

(b) Find the general solution of the differential equation (II), expressing *y* as a function of *t*.

(6)

(c) Hence find the general solution of the differential equation (I).

(1)

(Total for question = 13 marks)

Q2.

(a) Show that the substitution $y = vx$ transforms the differential equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + (2 - x^2)y = 2x^3 \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2v}{dx^2} - v = 2 \qquad \text{(II)}$$

(5)

(b) By solving differential equation (II), find the general solution of differential equation (I) in the form $y = f(x)$.

(6)

Given that $y = e$ and $\frac{dy}{dx} = e$ at $x = 1$

(c) find the particular solution of differential equation (I).

(4)

(Total for question = 15 marks)

Q3.

(a) Show that the transformation $x = e^u$ transforms the differential equation

$$x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 8y = 4\ln x \qquad x > 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{d^2y}{du^2} + 2\frac{dy}{du} - 8y = 4u \qquad \text{(II)}$$

(6)

(b) Determine the general solution of differential equation (II), expressing y as a function of u.

(7)

(c) Hence obtain the general solution of differential equation (I).

(1)

(Total for question = 14 marks)

Q4.

$$y = 3e^{-x}\cos 3x + Ae^{-x}\sin 3x$$

is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 40e^{-x}\sin 3x$$

where A is a constant.

(a) Find the value of A.

(5)

(b) Hence find the general solution of this differential equation.

(4)

(c) Find the particular solution of this differential equation for which both $y = 3$ and $\frac{dy}{dx} = 3$ at $x = 0$

(4)

(Total for question = 13 marks)

Q5.

(a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2\sin x \qquad \text{(I)}$$

(8)

Given that $y = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$

(b) find the particular solution of differential equation (I).

(5)

(Total for question = 13 marks)

Q6.

(a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3x^2 + 2x + 1$$

(9)

(b) Find the particular solution of this differential equation for which $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$

(5)

(Total for question = 14 marks)

<u>Examiner's Report</u>

Q1.

This question on solving a second order differential equation by first changing the variable also produced a range of mark profiles.
Part (a) required students to carry out a given transformation. Many excellent proofs were seen, usually from students who took care with their presentation and notation. Many were able to differentiate $x = e^t$ to obtain an appropriate expression but the second differentiation, which required both chain and product rules, had more mixed results. It was clear that some students were ill-prepared to handle the differentiation of expressions where a change of variable was involved. Those that obtained a correct first and second derivative usually proceeded correctly by substitution. A few who had obtained expressions for the derivatives of y with respect to t substituted into equation (II) to prove equation (I). Some students were clearly working backwards from the given answer and were unlikely to construct a convincing proof.
Part (b) required the solution of the transformed differential equation and the method was largely well known. It was rare to see an incorrect auxiliary equation or errors in solving it. Students should be aware that in questions on this topic which are not proofs, it is perfectly acceptable to write the auxiliary equation straight down. A correct complementary function usually followed although it was sometimes given in terms of x and not changed to t at any point. Use of a correct form of particular integral was also common although peqt was occasionally used needlessly instead of $k\mathrm{e}^2t$. Differentiation of $y = k\mathrm{e}^{2t}$ occasionally led to the incorrect $y' = 2kt\mathrm{e}^{2t}$. Slips in determining the correct value for k were fairly rare. The general solution was occasionally given as "GS =" rather than "y =".
Those who had correctly answered part (b) generally had little difficulty stating the general solution to differential equation (I) in part (c). Although a polynomial in x was anticipated, answers with unsimplified logarithmic powers of e were accepted.

Q2.

This question on a second order differential equation saw an encouraging number of completely correct solutions. Scoring was good in part (a). Most knew to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ from $y = vx$ and then substitute to obtain the given answer. Those who worked with other derivatives generally did not make much progress although a small number who obtained $\frac{\mathrm{d}v}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2v}{\mathrm{d}x^2}$ were able to use these to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.
A very small number of candidates substituted into (II) to derive (I).

A few errors were seen in forming the auxiliary equation in part (b) with $m^2 - m = 0$ rather than $m^2 - 1 = 0$ occasionally and the correct equation was sometimes solved incorrectly to give $m = \pm\,\mathrm{i}$ rather than ± 1. The correct form of complementary function was usually obtained although a few gave the form for equal roots. The mark for a correct particular integral of -2 was widely scored although a small number needlessly started with forms such as $\lambda + \mu x$ or $\lambda + \mu x + vx^2$. Most added their CF and PI but some were not working with v here and gave this expression as their final answer. However, the correct solution in the form $y = \mathrm{f}(x)$ was widely seen.

In part (c), the method was again clear to most candidates although the product rule was not always used to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$. Most obtained two equations in A and B but the algebra required to solve them simultaneously often led to error. Candidates persisting with their "v =" form were penalised further in this part.

Q3.

This question on a second order differential equation saw a good number of completely correct solutions although part (a) predictably proved a challenge to many.

In part (a), those who identified that the best strategy was to directly replace the derivatives in *y* with respect to *x* with those in *y* with respect to *u* were much more likely to be successful. The first two marks for differentiating $x = e^u$ (or $u = \ln x$) and for use of the chain rule for a first derivative were widely scored. Obtaining a second derivative was more challenging – most knew to apply the product rule but failing to use the chain rule when differentiating was a common error. Those who obtained the correct $\frac{d^2y}{dx^2}$ almost always were able to substitute into equation (I) to obtain the given result. Candidates who decided to find the derivatives of *y* with respect to *u* were often left confused as to how to proceed. Some extra work was needed to go on from there to show that equation (I) ⇒ (II). Indeed, some substituted into equation (II) and then offered the given answer. The acceptable option of showing (II) ⇒ (I) was not common.

As expected, a significant number of candidates were completely ill-prepared for this part of the question and many very poor quality attempts at "fudging" the answer were evident. Misconceptions included thinking that the chain rule for first derivatives also worked when first derivatives were replaced with second derivatives. However, on the whole, a commendable amount of succinct proofs was seen.

Scoring was very good in part (b). It was rare to see an incorrect auxiliary equation although many unnecessarily showed its derivation from $y = e^{mx}$. The quadratic was almost always solved correctly and was usually followed by the correct form of complementary function. Most knew the method from here and looked for a particular integral, but a common pitfall was to choose the form of the particular integral to be $y = \lambda u$. Many chose $\lambda + \mu u + \nu u^2$ instead of $\lambda + \mu u$ but were generally not much more prone to error. There were a few basic mistakes (usually sign errors) when obtaining the constants. Most added their CF and PI but some were working with mixed variables here. The part (c) mark for the correct solution in the form $y = f(x)$ was widely seen although again variables were occasionally confused. A small number gave away a mark they could easily have scored by carelessly omitting the replacement of the *u* in $-\frac{1}{2}u$ with $\ln x$. It was also unfortunate to see students losing achievable marks by giving answers starting with "CF + PI =" or "GS =" rather than "*y* =".

Q4.

Varied responses were seen to this second order differential equation question and some attempts were abandoned early on. Part (a) required the determination of a constant in a non-standard particular integral. A significant number of students began by writing the auxiliary equation and this often led to subsequent confusion between complementary functions, particular integrals, general and particular solutions. The differentiation required proved demanding, although students who simplified their expressions as they proceeded were more likely to be successful. Four of the five marks in part (a) were for method and were commonly scored. Some students introduced their own incorrect particular integral. Those who chose to use $\lambda e^{-x}\cos x + \mu e^{-x}\sin x$ could access all the marks but gave themselves additional simultaneous equations to solve.

Most produced the correct auxiliary equation in part (b) although occasional incorrect solutions were seen. The correct form of complementary function usually followed although the e^x or constants were sometimes missing. Those who chose the alternative exponential form often ran into difficulties differentiating in part (c). The subsequent follow through mark for combining their complementary function and particular integral was widely scored.

Three of the four marks in part (c) were for method and were fairly accessible. The final mark for a fully correct solution was only scored by the most confident and organised students.

Q5.

This question was very well answered by the vast majority of students, with many fully correct solutions obtained in (a). Common errors were mainly down to students being unable to solve their set of linear equations correctly to get their constants for the particular integral. Students would be best advised to double check that the values they obtain do satisfy their original equations. It was relatively rare to see a student who did not know how to find the particular integral, although some poor responses were seen where only one trigonometric function was used. This showed a poor level of preparation by such students in what was a routine question. Some minor errors were also observed when finding the complementary function, with some students simply forgetting the form of the solution when two distinct roots are obtained from the auxiliary equation. In part (b) the most common errors were either in writing down the wrong set of linear equations, or solving them incorrectly, however such students nearly always benefitted from the follow through marks available to them after these errors.

Q6.

The presentation of work in this question was often of a high standard and solutions were clear and easy to follow.

The method of obtaining the complementary function was well understood with only a relatively few errors. Similarly, the correct quadratic form of the particular integral was well known and the method of obtaining the coefficients of that quadratic by substitution was well executed.

Students knew how to take the general solution and apply the boundary conditions in order to obtain the remaining two constants.

There were only few students who lost marks by not writing their answer in the correct form.

Generally the only errors were errors in solving simultaneous equations and students are advised, if they have time, to check their answers.

Mark Scheme

Q1.

Question Number	Scheme	Notes	Marks
	$x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+3y=x^2$		
(a)	$x=e^t \Rightarrow \frac{dx}{dy}=e^t\frac{dt}{dy} \Rightarrow \frac{dy}{dx}=e^{-t}\frac{dy}{dt}$	M1: Attempt first derivative using the chain rule to obtain $\frac{dx}{dy}=e^t\frac{dt}{dy}$	M1A1
		A1: $\frac{dy}{dx}=e^{-t}\frac{dy}{dt}$ oe	
	$\frac{dy}{dx}=x^{-1}\frac{dy}{dt} \Rightarrow \frac{d^2y}{dx^2}=-x^{-2}\frac{dy}{dt}+x^{-1}\frac{d^2y}{dt^2}.\frac{dt}{dx}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only	dM1A1
		A1: Correct second derivative oe	
	$x^2\left(\frac{1}{x^2}\frac{d^2y}{dt^2}-\frac{1}{x^2}\frac{dy}{dt}\right)-3x\left(\frac{1}{x}\frac{dy}{dt}\right)+3y=\left(e^t\right)^2$	Substitutes their $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ in terms of t into the differential equation	M1
	$\frac{d^2y}{dt^2}-4\frac{dy}{dt}+3y=e^{2t}$	cso	A1
			(6)
	Alternative		
	$x=e^t \Rightarrow \frac{dy}{dt}=e^t\frac{dy}{dx}=x\frac{dy}{dx}$	M1: Attempt first derivative using $\frac{dy}{dt}=\frac{dx}{dt}\times\frac{dy}{dx}$	M1A1
		A1: $\frac{dy}{dt}=x\frac{dy}{dx}$ oe	
	$\frac{d^2y}{dt^2}=\frac{dx}{dt}\frac{dy}{dx}+x\frac{d^2y}{dx^2}.\frac{dx}{dt}=x\frac{dy}{dx}+x^2\frac{d^2y}{dx^2}$	dM1: Attempt product rule and chain rule. Dependent on the first method mark and must be a fully correct method with sign errors only	dM1A1
		A1: Correct second derivative oe	
	$\frac{d^2y}{dt^2}-x\frac{dy}{dx}-3x\frac{dy}{dx}+3y=e^{2t}$ $=\frac{d^2y}{dt^2}-\frac{dy}{dt}-3\frac{dy}{dt}+3y=e^{2t}$	Substitutes their $\frac{d^2y}{dx^2}$ and $x\frac{dy}{dx}$ in terms of t into the differential equation	M1
	$\frac{d^2y}{dt^2}-4\frac{dy}{dt}+3y=e^{2t}$	Cso	A1
			(6)

(b)	$m^2-4m+3=0 \Rightarrow m=1,\ 3$	Solves (according to the General Guidance) the correct quadratic (so should be $m=\pm1, \pm3$)	M1
	$(y=)Ae^{3t}+Be^{t}$	Correct CF in terms of t not x. (May be seen later in their GS)	A1
	$y=ke^{2t}, y'=2ke^{2t}, y''=4ke^{2t}$	Correct form for PI and differentiates twice to obtain multiples of e^{2t} each time but do not allow if they are clearly integrating.	M1
	$4ke^{2t}-8ke^{2t}+3ke^{2t}=e^{2t} \Rightarrow k=...$	Substitutes their $y,\ y',\ y''$ that are of the form αe^{2t} into the differential equation and sets $= e^{2t}$ and proceeds to find their k	M1
	$(y)=-e^{2t}$	Correct PI or $k = -1$	A1
	$y=Ae^{3t}+Be^{t}-e^{2t}$	Correct ft GS **in terms of** t (their CF + their PI with non-zero PI). **Must be** $y = ...$	B1ft
			(6)
(c)	$(y=)Ax^3+Bx-x^2$	Allow equivalent expressions in terms of x e.g. $(y=)Ae^{3\ln x}+Be^{\ln x}-e^{2\ln x}$. **Note that** $y = ...$ **is not needed here.**	B1
			(1)
			Total 13

Q2.

Question Number	Scheme	Notes	Marks
	$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+(2-x^2)y=2x^3$		
(a)	$y=vx \Rightarrow \frac{dy}{dx}=\frac{dv}{dx}x+v$	Correct expression	B1
	$\frac{d^2y}{dx^2}=\frac{d^2v}{dx^2}x+2\frac{dv}{dx}$	$\frac{d^2y}{dx^2}=\alpha x\frac{d^2v}{dx^2}+\beta\frac{dv}{dx}$	M1
		$\frac{d^2y}{dx^2}=\frac{d^2v}{dx^2}x+2\frac{dv}{dx}$	A1
	$x^2\left(\frac{d^2v}{dx^2}x+2\frac{dv}{dx}\right)-2x\left(\frac{dv}{dx}x+v\right)+\left(2-x^2\right)vx=2x^3$ Substitutes their $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ into the given equation to give an equation in x and v only		M1
	$\frac{d^2v}{dx^2}-v=2$ *	Correct proof with no errors with at least one more line of working (usually $x^3\frac{d^2v}{dx^2}-x^3v=2x^3$)	A1
			(5)
(b)	$m^2-1=0 \Rightarrow m=\pm 1$	Attempts to solve "m"$^2-1=0$	M1
	$(v=)Ae^x+Be^{-x}$	Correct CF ($v=\ldots$ not required)	A1
	PI is -2	Correct PI	B1
	$v=CF+PI=\ldots$	Adds their CF and their non-zero PI to find v in terms of x. **Must be $v=\ldots$ here unless this is implied by subsequent work.**	M1
	$(y=)x(CF+PI)$	Multiplies their v by x to find y in terms of x. (Can be awarded if no PI is found or their PI = 0)	M1
	$y=Axe^x+Bxe^{-x}-2x$	Correct expression. Must be $y=\ldots$	A1
			(6)
(c)	$\frac{dy}{dx}=Axe^x+Ae^x-Bxe^{-x}+Be^{-x}-2$	Differentiate their GS wrt x using the Product Rule	M1
	$x=1, y=e \Rightarrow e=Ae+Be^{-1}-2$ and $x=1, \frac{dy}{dx}=e$ $\Rightarrow e=Ae+Ae-Be^{-1}+Be^{-1}-2$	Substitutes the given values in for y and $\frac{dy}{dx}$ to obtain 2 equations	M1
	$A=\frac{e+2}{2e},\ B=\frac{e(e+2)}{2}$	Both values correct or exact equivalent	A1
	$y=\left(\frac{e+2}{2e}\right)xe^x+\left(\frac{e(e+2)}{2}\right)xe^{-x}-2x$	Correct expression (or equivalent). Must be $y=\ldots$	A1
			(4)
			Total 15

Q3.

Question Number	Scheme	Marks
(a)	$x=\mathrm{e}^u \quad \dfrac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u$ or $\dfrac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}$ or $\dfrac{\mathrm{d}x}{\mathrm{d}u}=x$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}u}\times\dfrac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-u}\dfrac{\mathrm{d}y}{\mathrm{d}u}$	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\mathrm{e}^{-u}\dfrac{\mathrm{d}u}{\mathrm{d}x}\dfrac{\mathrm{d}y}{\mathrm{d}u}+\mathrm{e}^{-u}\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\dfrac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^{-2u}\left(-\dfrac{\mathrm{d}y}{\mathrm{d}u}+\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)$	M1A1
	$x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+3x\dfrac{\mathrm{d}y}{\mathrm{d}x}-8y=4\ln x$	
	$\mathrm{e}^{2u}\times\mathrm{e}^{-2u}\left(-\dfrac{\mathrm{d}y}{\mathrm{d}u}+\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}\right)+3\mathrm{e}^u\times\mathrm{e}^{-u}\dfrac{\mathrm{d}y}{\mathrm{d}u}-8y=4\ln\left(\mathrm{e}^u\right)$	dM1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}+2\dfrac{\mathrm{d}y}{\mathrm{d}u}-8y=4u \quad *$	A1*cso (6)

B1	$\dfrac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u$ oe as shown seen explicitly or used
M1	Obtaining $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ using chain rule here or seen later
M1	Obtaining $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ using product rule (penalise lack of chain rule by the A mark)
A1	Correct expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ any equivalent form
dM1	Substituting in the equation to eliminate x (u and y **only**). Depends on the 2nd M mark
A1*cso	Obtaining the **given** result from completely correct work

	ALTERNATIVE 1	
	$x=\mathrm{e}^u \quad \dfrac{\mathrm{d}x}{\mathrm{d}u}=\mathrm{e}^u=x$	B1
	$\dfrac{\mathrm{d}y}{\mathrm{d}u}=\dfrac{\mathrm{d}y}{\mathrm{d}x}\times\dfrac{\mathrm{d}x}{\mathrm{d}u}=x\dfrac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}=1\dfrac{\mathrm{d}x}{\mathrm{d}u}\times\dfrac{\mathrm{d}y}{\mathrm{d}x}+x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\times\dfrac{\mathrm{d}x}{\mathrm{d}u}=x\dfrac{\mathrm{d}y}{\mathrm{d}x}+x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$	M1A1
	$x^2\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}-\dfrac{\mathrm{d}y}{\mathrm{d}u}$	
	$\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}-\dfrac{\mathrm{d}y}{\mathrm{d}u}\right)+3x\times\dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}u}-8y=4\ln\left(\mathrm{e}^u\right)$	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}u^2}+2\dfrac{\mathrm{d}y}{\mathrm{d}u}-8y=4u \quad *$	dM1A1*cso (6)

Question Number	Scheme	Marks
B1	$\dfrac{\mathrm{d}x}{\mathrm{d}u} = \mathrm{e}^u$ oe as shown seen explicitly or used	
M1	Obtaining $\dfrac{\mathrm{d}y}{\mathrm{d}u}$ using chain rule here or seen later	
M1	Obtaining $\dfrac{\mathrm{d}^2 y}{\mathrm{d}u^2}$ using product rule (penalise lack of chain rule by the A mark)	
A1	Correct expression for $\dfrac{\mathrm{d}^2 y}{\mathrm{d}u^2}$ any equivalent form	
dM1 **A1***cso	Substituting in the equation to eliminate x (u and y **only**). Depends on the 2nd M mark Obtaining the **given** result from completely correct work	
	ALTERNATIVE 2: $u = \ln x \quad \dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{x}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}u} \times \dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}u}$ $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}u} + \dfrac{1}{x}\dfrac{\mathrm{d}^2 y}{\mathrm{d}u^2} \times \dfrac{\mathrm{d}u}{\mathrm{d}x} = -\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}u} + \dfrac{1}{x^2}\dfrac{\mathrm{d}^2 y}{\mathrm{d}u^2}$ $x^2\left(-\dfrac{1}{x^2}\dfrac{\mathrm{d}y}{\mathrm{d}u} + \dfrac{1}{x^2}\dfrac{\mathrm{d}^2 y}{\mathrm{d}u^2}\right) + 3x \times \dfrac{1}{x}\dfrac{\mathrm{d}y}{\mathrm{d}u} - 8y = 4u$ $\dfrac{\mathrm{d}^2 y}{\mathrm{d}u^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}u} - 8y = 4u \qquad *$	B1 M1 M1A1 M1A1*cso
	Notes as for main scheme	

There are also **other solutions** which will appear, either starting from equation II and obtaining equation I, or mixing letters x, y and u until the final stage.
Mark as follows:

B1 as shown in schemes above
M1 obtaining a first derivative with chain rule
M1 obtaining a second derivative with product rule
A1 correct second derivative with 2 or 3 variables present
dM1 Either substitute in equation I or substitute in equation II according to method chosen **and** obtain an equation with only y and u (following sub in eqn I) or with only x and y (following sub in eqn II)
A1cso Obtaining the required result from completely correct work

Question Number	Scheme	Marks
(b)	$m^2+2m-8=0$	
	$(m+4)(m-2)=0, \quad m=-4, 2$	M1A1
	CF $=Ae^{-4u}+Be^{2u}$	A1
	PI: try $y=au+b$ (or $y=cu^2+au+b$ different derivatives, $c=0$)	
	$\frac{dy}{du}=a \quad \frac{d^2y}{du^2}=0$	M1
	$0+2a-8(au+b)=4u$	
	$a=-\frac{1}{2} \quad b=-\frac{1}{8}$	dM1A1
	$\therefore y=Ae^{-4u}+Be^{2u}-\frac{1}{2}u-\frac{1}{8}$	B1ft (7)
(c)	$y=Ax^{-4}+Bx^2-\frac{1}{2}\ln x-\frac{1}{8}$	B1 (1)
		[14]

(b) M1	Writing down the correct aux equation and solving to $m=\dots$ (usual rules)
A1	Correct solution $(m=-4, 2)$
A1	Correct CF – can use any (single) variable
M1	Using an appropriate PI and finding $\frac{dy}{du}$ **and** $\frac{d^2y}{du^2}$ Use of $y=\lambda u$ scores M0
dM1	Substitute in the equation to obtain values for the unknowns. Depends on the second M1
A1	Correct unknowns two or three (with $c=0$)
B1ft	A complete solution, follow through their CF and a non-zero PI. Must have $y=$ a function of u Allow recovery of incorrect variables.
(c) B1	Reverse the substitution to obtain a correct expression for y in terms of x No ft here x^{-4} or $e^{-4\ln x}$ and x^2 or $e^{2\ln x}$ allowed. Must start $y=\dots$

Q4.

Question Number	Scheme	Notes	Marks
(a)	$y = 3e^{-x}\cos 3x + Ae^{-x}\sin 3x$ $\Rightarrow \frac{dy}{dx} = -3e^{-x}\cos 3x - 9e^{-x}\sin 3x - Ae^{-x}\sin 3x + 3Ae^{-x}\cos 3x$ $\left(= (-3+3A)e^{-x}\cos 3x + (-9-A)e^{-x}\sin 3x\right)$ Attempts to differentiate the **given expression** by using the product rule on $3e^{-x}\cos 3x$ to give $\alpha e^{-x}\cos 3x + \beta e^{-x}\sin 3x$ or by using the product rule on $Ae^{-x}\sin 3x$ to give $\alpha Ae^{-x}\cos 3x + \beta Ae^{-x}\sin 3x$		M1
	$\frac{d^2y}{dx^2} = (-24-6A)e^{-x}\cos 3x + (18-8A)e^{-x}\sin 3x$ (Terms may be uncollected) Uses the product rule again on an expression of the form $e^{-x}\sin 3x$ or $e^{-x}\cos 3x$ to give $\alpha e^{-x}\cos 3x + \beta e^{-x}\sin 3x$. **Dependent on the first method mark.**		dM1
	$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = (12-12A)e^{-x}\cos 3x + (36+4A)e^{-x}\sin 3x$ Substitute their results into the differential equation. (May be implied)		M1
	$12-12A=0$ or $36+4A=40 \Rightarrow A = \ldots$	Compares coefficients of $e^{-x}\sin 3x$ or $e^{-x}\cos 3x$ and attempts to find A. **Dependent on the previous method mark.**	dM1
	$\Rightarrow A = 1$	cao	A1
			(5)

Question Number	Scheme	Notes	Marks
(b) **Marks for (b) can score anywhere in their answer.**	$m^2 - 2m + 10 = 0 \Rightarrow m = 1 \pm 3i$	M1: Forms and attempts to solve the Auxiliary Equation. See General Principles. A1: Correct solution for the AE	M1 A1
	$(y=)e^{x}(C\cos 3x + D\sin 3x)$ or $(y=)Ce^{(1+3i)x} + De^{(1-3i)x}$	Correct form for CF using their complex roots from the AE	M1
	$y = e^{x}(C\cos 3x + D\sin 3x) + 3e^{-x}\cos 3x + e^{-x}\sin 3x$ GS = their CF + their PI (**Allow ft on their CF and PI**) Must start $y = \ldots$ and depends on at least one the M's being scored and must have been using a PI of the form given.		A1ft
			(4)
(c)	$x=0, y=3 \Rightarrow 3 = C + 3 (\Rightarrow C = 0)$	Attempts to substitute $x = 0$ and $y = 3$ into their answer to (b)	M1
	$\frac{dy}{dx} = (C+3D)e^{x}\cos 3x + (-3C+D)e^{x}\sin 3x - 10e^{-x}\sin 3x$ Attempt to differentiate their GS with or without their C		M1
	$3 = C + 3D$	Attempt to substitute $x = 0$ and $\frac{dy}{dx} = 3$ into their $\frac{dy}{dx}$	M1
	$y = e^{x}\sin 3x + 3e^{-x}\cos 3x + e^{-x}\sin 3x$	Correct answer. **Must start** $y = \ldots$	A1cao
			(4)
			Total 13

Q5.

Question Number	Scheme	Notes	Marks
(a)	$\frac{d^2y}{dx^2}-2\frac{dy}{dx}-3y=2\sin x$		
	AE: $m^2-2m-3=0$		
	$m^2-2m-3=0 \Rightarrow m=...(-1,3)$	Forms Auxiliary Equation and attempts to solve (usual rules)	M1
	$(y=)Ae^{3x}+Be^{-x}$	Cao	A1
	PI: $(y=)p\sin x+q\cos x$	Correct form for PI	B1
	$(y'=)p\cos x-q\sin x$ $(y''=)-p\sin x-q\cos x$		
	$-p\sin x-q\cos x-2(p\cos x-q\sin x)-3p\sin x-3q\cos x=2\sin x$ Differentiates twice and substitutes		M1
	$2q-4p=2,\ 4q+2p=0$	Correct equations	A1
	$p=-\frac{2}{5},\ q=\frac{1}{5}$	A1A1 both correct A1A0 one correct	A1A1
	$y=\frac{1}{5}\cos x-\frac{2}{5}\sin x$		
	$y=Ae^{3x}+Be^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Follow through their p and q and their CF	B1ft
			(8)
(b)	$y'=3Ae^{3x}-Be^{-x}-\frac{1}{5}\sin x-\frac{2}{5}\cos x$	Differentiates their GS	M1
	$0=A+B+\frac{1}{5},\ 1=3A-B-\frac{2}{5}$	M1: Uses the given conditions to give two equations in A and B A1: Correct equations	M1A1
	$A=\frac{3}{10},\ B=-\frac{1}{2}$	Solves for A and B Both correct	A1
	$y=\frac{3}{10}e^{3x}-\frac{1}{2}e^{-x}+\frac{1}{5}\cos x-\frac{2}{5}\sin x$	Sub their values of A and B in their GS	A1ft
			(5)
			Total 13

Q6.

Question Number	Scheme	Notes	Marks
	$\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=3x^2+2x+1$		
(a)	$m^2+3m+2=0 \Rightarrow m=-1,-2$	Correct roots (may be implied by their CF)	B1
	$y=Ae^{-2x}+Be^{-x}$	M1: CF of the correct form A1: Correct CF	M1A1
	$y=ax^2+bx+c$	Correct form for PI	B1
	$\frac{dy}{dx}=2ax+b,\ \frac{d^2y}{dx^2}=2a \Rightarrow 2a+3(2ax+b)+2(ax^2+bx+c)=3x^2+2x+1$		M1
	M1: Differentiates twice and substitutes into the lhs of the given differential equation and puts equal to $3x^2+2x+1$ or substitutes into the lhs of the given differential equation and compares coefficients with $3x^2+2x+1$. For the substitution, at least one of y, y' or y'' must be correctly placed.		
	$a=\frac{3}{2}$		A1
	$6a+2b=2 \Rightarrow b=-\frac{7}{2} \Rightarrow c=\frac{17}{4}$	M1: Solves to obtain one of b or c A1: Correct b and c	M1A1
	$y=Ae^{-2x}+Be^{-x}+\frac{3}{2}x^2-\frac{7}{2}x+\frac{17}{4}$	Correct ft (their CF + their PI) but must be $y=\ldots$	B1ft
			(9)
(b)	$0=A+B+\frac{17}{4}$	Substitutes $x=0$ and $y=0$ into their GS	M1
	$\frac{dy}{dx}=-2Ae^{-2x}-Be^{-x}+3x-\frac{7}{2} \Rightarrow 0=-2A-B-\frac{7}{2}$ Attempts to differentiate and substitutes $x=0$ and $y'=0$		M1
	$0=A+B+\frac{17}{4},\ 0=-2A-B-\frac{7}{2} \Rightarrow A=..,B=..$	Solves simultaneously to obtain values for A and B	M1
	$A=\frac{3}{4},\quad B=-5$	Correct values	A1
	$y=\frac{3}{4}e^{-2x}-5e^{-x}+\frac{3}{2}x^2-\frac{7}{2}x+\frac{17}{4}$	Correct ft (their CF + their PI) but must be $y=\ldots$	B1ft
			(5)
			Total 14