Edexcel International A Level (IAL) Further Maths: Further Pure 1

Your notes

Matrix Algebra

Contents

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Introduction to Matrices

What is a matrix?

- A **matrix** is a rectangular grid (array) of **elements** (numbers or letters) arranged in **rows** and **columns**
  - The plural of matrix is **matrices**
- The **order** (**dimensions**) of a matrix is its **number of rows × number of columns**
  - a 2 × 1 matrix is
    - this is also called a **column matrix** or a **column vector**
  - a 2 × 2 matrix is
    - this is called a **square** matrix
- A **bold capital** letter is often used to represent a matrix
  - $\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 0 & 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$
- **2D coordinates** can be written as a column matrix
  - The point $(3, 5)$ is
- You can use **subscript** notation to refer to **elements** in a matrix
  - Matrix $\mathbf{A}$ is $\mathbf{A} = (a_{i,j})$ where and $j = 1, 2, 3, \ldots, n$
    - $a_{i,j}$ refers to the element in row , column
    - The **order** of $\mathbf{A}$ is $m \times n$ (rows × columns)

$$\boldsymbol{A} = (a_{i,j}) = \begin{array}{c} \overbrace{\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix}}^{\text{Number of columns, } n\,=\,3} \end{array} \left.\vphantom{\begin{pmatrix} a \\ a \end{pmatrix}}\right\} \text{Number of rows, } m = 2$$

What type of matrices do I need to know?

- A **column matrix** (or column vector) is a matrix with a **single column**
  - Order $m \times 1$
- A **row matrix** is a matrix with a **single row**
  - Order $1 \times n$

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- A **square matrix** is one in which the number of rows is **equal** to the number of columns
  - Order $n \times n$
- Two matrices are **equal** when they are of the **same order** and their **corresponding elements** are **equal**
  - $a_{i,j} = b_{i,j}$ for all elements
- A **zero matrix**, $\mathbf{0}$, is a matrix in which all the elements are zero
  - For example, the 2 × 2 zero matrix is $\mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
- An **identity matrix**, $\mathbf{I}$, is a **square** matrix in which all elements along the **leading diagonal** (top-left to bottom right) are 1
  - The rest of the elements are zero
  - For example, the 2 × 2 identity matrix is $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
  - The notation $\mathbf{I}_n$ can be used to specify the $n \times n$ identity matrix

Basic Operations with Matrices

How do I multiply a matrix by a scalar?

- To multiply any matrix by a **scalar** (a number), **multiply each element** by that scalar
  - If $\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 0 & 4 \end{pmatrix}$ then $2\mathbf{A} = 2\begin{pmatrix} 5 & 2 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 2 \times 5 & 2 \times 2 \\ 2 \times 0 & 2 \times 4 \end{pmatrix} = \begin{pmatrix} 10 & 4 \\ 0 & 8 \end{pmatrix}$
- Multiplying by a **negative** scalar changes the **sign** of each element in the matrix
- **Lower case** letters often refer to scalar multiples
  - $k\mathbf{A}$ is the matrix $\mathbf{A}$ multiplied by the scalar

How do I add and subtract matrices?

- Two matrices of the **same order** can be added (or subtracted) by adding (or subtracting) **corresponding elements**
  - The answer is a matrix of the **same order**
  - For example, $\begin{pmatrix} 1 & 2 \\ -3 & 0 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 3 & -1 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1+3 & 2+0 \\ -3+3 & 0-1 \\ 2+4 & 1+1 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 0 & -1 \\ 6 & 2 \end{pmatrix}$

What properties of matrix addition do I need to know?

- $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

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- Matrix **addition** is **commutative**
    - You can swap the order
- Matrix **subtraction** is **not** commutative
    - $\mathbf{A} - \mathbf{B} \neq \mathbf{B} - \mathbf{A}$

- $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$
    - **Subtraction** is the same as **adding** a **negative**
- $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$
    - Matrix **addition** is **associative**
        - To add three matrices, you can start with the first two, or the last two
    - Matrix **subtraction** is **not** associative
        - $\mathbf{A} - (\mathbf{B} - \mathbf{C}) \neq (\mathbf{A} - \mathbf{B}) - \mathbf{C}$
        - Try expanding the brackets to see
- $\mathbf{A} + \mathbf{0} = \mathbf{A}$
    - Adding the **zero** matrix has no effect
- $\mathbf{0} - \mathbf{A} = -\mathbf{A}$

Worked Example

Consider the matrices $\mathbf{A} = \begin{pmatrix} -4 & 2 \\ 7 & 3 \\ 1 & -5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 6 \\ 5 & -9 \\ -2 & -3 \end{pmatrix}$.

(a) Find $\mathbf{A} + \mathbf{B}$.

The matrices have the same order (dimensions)
Corresponding elements can be added

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} -4+2 & 2+6 \\ 7+5 & 3+(-9) \\ 1+(-2) & -5+(-3) \end{pmatrix}$$

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} -2 & 8 \\ 12 & -6 \\ -1 & -8 \end{pmatrix}$$

(b) Find $-10\mathbf{B}$.

Multiply each element by -10

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$$-10\mathbf{B} = -10\begin{pmatrix} 2 & 6 \\ 5 & -9 \\ -2 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} -10\times 2 & -10\times 6 \\ -10\times 5 & -10\times(-9) \\ -10\times(-2) & -10\times(-3) \end{pmatrix}$$

$$-10\mathbf{B} = \begin{pmatrix} -20 & -60 \\ -50 & 90 \\ 20 & 30 \end{pmatrix}$$

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Multiplying Matrices

How do I multiply a 2x2 matrix by a 2x1 matrix?

- Multiply the corresponding elements in the **row** of the **first** matrix with the corresponding elements in the **column** of the **second** matrix, writing their **sum** in the answer matrix
- The answer will be a **2 × 1 matrix**
  -
-

How do I multiply a 2x2 matrix by another 2x2 matrix?

- Multiply the corresponding elements in the **row** of the **first** matrix with the corresponding elements in the **column** of the **second** matrix, writing their **sum** in the answer matrix
- The answer will be a **2 × 2 matrix**
  -
-
- This process becomes more natural the more times you do it!

How do I square a matrix?

- **Do not** square each individual element inside the matrix
- Write out a **matrix multiplication**
  - If then
- It is possible to have **negative** elements in a squared matrix

How do I multiply matrices of any dimensions?

- To multiply a matrix by another matrix:
  - The **number of columns** in the **first** matrix must be **equal** to the **number of rows** in the **second** matrix
  - For example, the **first** matrix is and the **second** matrix is
  - The **order** of the **resultant** matrix will be
- Multiply corresponding elements in the **row** of the **first** matrix with the corresponding elements in the **column** of the **second** matrix
  - Place their **sums** in the resultant matrix

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- For example, if $\mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} g & h \\ i & j \\ k & l \end{pmatrix}$
- then $\mathbf{AB} = \begin{pmatrix} (ag+bi+ck) & (ah+bj+cl) \\ (dg+ei+fk) & (dh+ej+fl) \end{pmatrix}$
- whereas $\mathbf{BA} = \begin{pmatrix} (ga+hd) & (gb+he) & (gc+hf) \\ (ia+jd) & (ib+je) & (ic+jf) \\ (ka+ld) & (kb+le) & (kc+lf) \end{pmatrix}$

- It is possible for **AB** to exist but **BA** not to exist
  - For **square matrices** of the same order, **AB** and **BA** will both exist

What does commutative mean?

- **Commutative** means "swapping the order doesn't change the result"
  - 5 × 4 = 4 × 5 and 3 + 2 = 2 + 3
    - Multiplication and addition of **numbers** are commutative
  - 4 ÷ 2 ≠ 2 ÷ 4 and 5 - 3 ≠ 3 - 5
    - Division and subtraction of **numbers** are not commutative
- However, **matrix multiplication** is **not commutative**
  - In general **AB** ≠ **BA**
- For example, but

What does associative mean?

- **Associative** means "it doesn't matter which order you group operations into"
  - To do 5 + 4 + 3, either (5 + 4) + 3 or 5 + (4 + 3) works
  - To do 8 × 9 × 10, either (8 × 9) × 10 or 8 × (9 × 10) works
    - Multiplication and addition of **numbers** are associative
  - (8 ÷ 4) ÷ 2 ≠ 8 ÷ (4 ÷ 2) and (5 - 4) - 3 ≠ 5 - (4 - 3)
    - Division and subtraction of **numbers** are not associative
- **Matrix multiplication** is **associative**
  - **(AB)C** ≡ **A(BC)**
- To multiply **three** matrices together, it is fine to either:
  - start by multiplying the first two together,
  - or start by multiplying the second two together
    - Just don't switch the order

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- **A(BC)** is not the same as **(BC)A**

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Worked Example

If , and $\mathbf{R}=\begin{pmatrix}10 & -1 & 4\\ 8 & 0 & 5\end{pmatrix}$, find the following:

(a)

Write out $\mathbf{PQ}$ in full

$$\begin{pmatrix}3 & 1\\ -2 & 0\end{pmatrix}\times\begin{pmatrix}5 & -5\\ 4 & 2\end{pmatrix}$$

Multiply the matrices

$$\begin{pmatrix}(3\times 5\ +\ 1\times 4) & (3\times -5\ +\ 1\times 2)\\ (-2\times 5\ +\ 0\times 4) & (-2\times -5\ +\ 0\times 2)\end{pmatrix}=\begin{pmatrix}(15\ +\ 4) & (-15\ +\ 2)\\ (-10\ +\ 0) & (10\ +\ 0)\end{pmatrix}$$

Simplify

$$\mathbf{PQ}=\begin{pmatrix}19 & -13\\ -10 & 10\end{pmatrix}$$

(b) $\mathbf{PR}$

Write out $\mathbf{PR}$ in full

$$\begin{pmatrix}3 & 1\\ -2 & 0\end{pmatrix}\times\begin{pmatrix}10 & -1 & 4\\ 8 & 0 & 5\end{pmatrix}$$

The order (dimensions) agree, as $\mathbf{P}$ has 2 columns and $\mathbf{R}$ has 2 rows
Multiply the matrices

$$\begin{pmatrix}(3\times 10\ +\ 1\times 8) & (3\times -1\ +\ 1\times 0) & (3\times 4\ +\ 1\times 5)\\ (-2\times 10\ +\ 0\times 8) & (-2\times -1\ +\ 0\times 0) & (-2\times 4\ +\ 0\times 5)\end{pmatrix}$$

Simplify

$$\mathbf{PR}=\begin{pmatrix}38 & -3 & 17\\ -20 & 2 & -8\end{pmatrix}$$

(c)

Write out $\mathbf{Q}^2$ as $\mathbf{Q}\times\mathbf{Q}$

$$\begin{pmatrix}5 & -5\\ 4 & 2\end{pmatrix}\times\begin{pmatrix}5 & -5\\ 4 & 2\end{pmatrix}$$

Multiply the matrices

$$\begin{pmatrix}(5\times 5\ +\ -5\times 4) & (5\times -5\ +\ -5\times 2)\\ (4\times 5\ +\ 2\times 4) & (4\times -5\ +\ 2\times 2)\end{pmatrix}=\begin{pmatrix}(25\ +\ -20) & (-25\ +\ -10)\\ (20\ +\ 8) & (-20\ +\ 4)\end{pmatrix}$$

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Simplify

$$Q^2 = \begin{pmatrix} 5 & -35 \\ 28 & -16 \end{pmatrix}$$

(d) Explain why the matrix $\mathbf{RP}$ does not exist.

An $m \times n$ matrix can only be multiplied by an $n \times p$ matrix

The order (dimensions) do not agree, as $\mathbf{R}$ has 3 columns but $\mathbf{P}$ has 2 rows

The Identity Matrix

What is the Identity Matrix?

- The **identity matrix**, $\mathbf{I}$, is a **square** matrix with:
  - **Ones** along the **leading diagonal** (from top-left to bottom-right)
  - and **zeros** everywhere else
    - The **2 × 2** identity matrix is
    - The **3 × 3** identity matrix is $\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
  - The notation $\mathbf{I}_n$ can be used to specify the $n \times n$ identity matrix
- **Multiplying** any **square** matrix by the same-sized **identity** matrix leaves it **unchanged**
  - Both and
  - and
    - This result can be proved by multiplying together the matrices

Examiner Tips and Tricks

The identity matrix is an important matrix which you should know or recognise as $\mathbf{I}$ in a question.

Worked Example

If show that .

Write out $\mathbf{A}^2$ as $\mathbf{A} \times \mathbf{A}$

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$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

Multiply the matrices

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} (0 \times 0 + 2 \times 2) & (0 \times 2 + 2 \times 0) \\ (2 \times 0 + 0 \times 2) & (2 \times 2 + 0 \times 0) \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

Write in terms of the identity matrix, $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ by factorising out 4

$$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} = 4\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 4\mathbf{I}$$

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Determinant of a 2x2 Matrix

What is the determinant?

- The **determinant** is a **numerical value** (positive or negative) calculated from elements of a **square** matrix
  - It is used to find the **inverse** of a matrix
- For a 2 × 2 matrix, $\mathbf{A}$, the determinant, $\det(\mathbf{A})$ or $|\mathbf{A}|$, is given by

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \det \mathbf{A} = |\mathbf{A}| = ad - bc$$

What properties of determinants do I need to know?

- If $\det(\mathbf{A}) = 0$ then $\mathbf{A}$ is called a **singular matrix**
- If $\det(\mathbf{A}) \neq 0$ then $\mathbf{A}$ is called a **non-singular matrix**
- The determinant of the **identity matrix** is 1
  - $\det(\mathbf{I}) = 1$
- The determinant of the **zero matrix** is 0
  - $\det(\mathbf{0}) = 0$
- You do **not** need to know the following rules, but they can be **good checks** in an exam:
  - $\det(\mathbf{AB}) = \det(\mathbf{A}) \times \det(\mathbf{B})$
  - $\det(\mathbf{A}^{-1}) = \dfrac{1}{\det(\mathbf{A})}$

Worked Example

Consider the matrix $\mathbf{A} = \begin{pmatrix} 3 & -6 \\ p & 7 \end{pmatrix}$, where p is a constant.

Given that $\det \mathbf{A} = -3$, find the value of .

Use that if $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $\det \mathbf{A} = |\mathbf{A}| = ad - bc$

Work out the determinant algebraically

$$\begin{aligned} \det(\mathbf{A}) &= 3 \times 7 - (-6) \times p \\ &= 21 + 6p \end{aligned}$$

Set this expression equal to -3 and solve for p

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$$21 + 6p = -3$$
$$6p = -24$$
$$p = -4$$

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Inverse of a 2x2 Matrix

What is the inverse of a matrix?

- The **inverse** of a **square** matrix $\mathbf{A}$ is the matrix $\mathbf{A}^{-1}$ which, when **multiplied** together (in either order), gives the **identity** matrix
  - $\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$
  - $\mathbf{A}^{-1}$ has the same dimensions (order) as $\mathbf{A}$

How do I find the inverse of a 2x2 matrix?

- To find the inverse of a 2 × 2 matrix:
  - **Switch** the two entries on **leading diagonal** (top-left to bottom right)
  - **Change** the **signs** of the **other** two entries
  - **Divide** by the **determinant**

$$\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

- Note that you **cannot divide** by zero
  - If $\det \mathbf{A} = 0$, then $\mathbf{A}$ is not invertible ($\mathbf{A}^{-1}$ does **not exist**)
    - $\mathbf{A}$ is **singular**
  - If $\det \mathbf{A} \neq 0$, then $\mathbf{A}$ is **invertible**

How do I find the inverse of a product of matrices?

- The inverse of a **product** of matrices is the product of the inverses of the matrices in **reverse order**
  - $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

Examiner Tips and Tricks

There are two ways to check whether your inverse matrix is correct:

- use a calculator (they can find inverses),
- or calculate $\mathbf{AA}^{-1}$ to see if you get the identity $\mathbf{I}$.

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Worked Example

Let $\mathbf{P} = \begin{pmatrix} 1 & -2 \\ 4 & k \end{pmatrix}$ where $k \neq -8$.

(a) Find $\mathbf{P}^{-1}$, giving your answer in terms of k.

Use that $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \mathbf{A}^{-1} = \frac{1}{\det \mathbf{A}}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ where $\det \mathbf{A} = ad - bc$

First find $\det(\mathbf{P})$

$$\begin{aligned} \det(\mathbf{P}) &= 1 \times k - 4 \times (-2) \\ &= k + 8 \end{aligned}$$

Now divide $\mathbf{P}$ by $\det(\mathbf{P})$, swap a and d, and change signs of b and c

$$\mathbf{P}^{-1} = \frac{1}{k+8}\begin{pmatrix} k & 2 \\ -4 & 1 \end{pmatrix}$$

You can also write this as $\mathbf{P}^{-1} = \begin{pmatrix} \frac{k}{k+8} & \frac{2}{k+8} \\ -\frac{4}{k+8} & \frac{1}{k+8} \end{pmatrix}$

(b) Verify that $\mathbf{P}^{-1}\mathbf{P} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix.

Verify means check that it is true

Substitute $\mathbf{P}^{-1}$ and $\mathbf{P}$ into the left-hand side and multiply out the matrices

$$\begin{aligned} \mathbf{P}^{-1}\mathbf{P} &= \frac{1}{k+8}\begin{pmatrix} k & 2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 4 & k \end{pmatrix} \\ &= \frac{1}{k+8}\begin{pmatrix} k \times 1 + 2 \times 4 & k \times (-2) + 2 \times k \\ -4 \times 1 + 1 \times 4 & -4 \times (-2) + 1 \times k \end{pmatrix} \\ &= \frac{1}{k+8}\begin{pmatrix} k+8 & 0 \\ 0 & 8+k \end{pmatrix} \end{aligned}$$

The $8 + k$ is the same as $k + 8$

Since $k \neq -8$ in the question, cancel the $(k+8)$'s

$$\mathbf{P}^{-1}\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The right-hand side is the 2×2 identity matrix

$$\mathbf{P}^{-1}\mathbf{P} = \mathbf{I}$$

Even though $\mathbf{P}\mathbf{P}^{-1} = \mathbf{I}$ is also true, this question only asks for the order shown

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Proving Matrix Relationships

What is a matrix relationship?

- A matrix **relationship** is an equation that connects different matrices
  - For example, if $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are matrices then possible relationships are
    - $\mathbf{AB} = \mathbf{C}$
    - $\mathbf{A} + 2\mathbf{B} = \mathbf{C}$
- Remember that in general matrix multiplication is **not commutative**
  - $\mathbf{AB} \neq \mathbf{BA}$
  - $\mathbf{ABC} \neq \mathbf{ACB} \neq \mathbf{BAC} \neq \dots$
  - $\mathbf{CBC}$ does not simplify to $\mathbf{C}^2\mathbf{B}$
    - Since $\mathbf{C}(\mathbf{BC}) \neq \mathbf{C}(\mathbf{CB})$

What matrix relationships do I need to know?

- If $\mathbf{A}$ and $\mathbf{B}$ are matrices and $\mathbf{I}$ is the **identity matrix**, you need to know the following:
  - $\mathbf{AA}^{-1} = \mathbf{I}$
  - $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$
  - $\mathbf{IA} = \mathbf{AI} = \mathbf{A}$
  - $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
    - The **order reverses**

How do I prove matrix relationships?

- You need to carefully **pre-multiply** (on the left) or **post-multiply** (on the right) **both sides** by matrices or their inverses
  - To make $\mathbf{B}$ the subject of $\mathbf{AB} = \mathbf{C}$
    - $\mathbf{A}^{-1}\mathbf{AB} = \mathbf{A}^{-1}\mathbf{C}$ (pre-multiply by $\mathbf{A}^{-1}$)
    - $\mathbf{IB} = \mathbf{A}^{-1}\mathbf{C}$ (form the identity)
    - $\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$ (simplify)
  - To make $\mathbf{A}$ the subject of $\mathbf{AB} = \mathbf{C}$
    - $\mathbf{ABB}^{-1} = \mathbf{CB}^{-1}$ (post-multiply by $\mathbf{B}^{-1}$)
    - $\mathbf{AI} = \mathbf{CB}^{-1}$ (form the identity)

- $\mathbf{A} = \mathbf{CB}^{-1}$ (simplify)

How do I prove that $(AB)^{-1} = B^{-1}A^{-1}$?

- Start by **multiplying AB** by its **inverse** to form the **identity**
  - $\mathbf{AB}(\mathbf{AB})^{-1} = \mathbf{I}$
- Then make $(\mathbf{AB})^{-1}$ the **subject**
  - Pre-multiply by $\mathbf{A}^{-1}$ and simplify
    - $\mathbf{A}^{-1}\mathbf{AB}(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{I}$
    - $\mathbf{IB}(\mathbf{AB})^{-1}=\mathbf{A}^{-1}$
    - $\mathbf{B}(\mathbf{AB})^{-1}=\mathbf{A}^{-1}$
  - Then pre-multiply by $\mathbf{B}^{-1}$ and simplify
    - $\mathbf{B}^{-1}\mathbf{B}(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$
    - $\mathbf{I}(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$
    - $(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$

Examiner Tips and Tricks

- Learn the formula $(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$ as you are not given it in the Formulae Booklet.
- Show lots of steps when doing matrix algebra.
  - Examiners want to see pre- or post-multiplying clearly.

Worked Example

Let $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ be three matrices such that $\mathbf{PQR}^{-1} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix.

Prove that $\mathbf{Q} = \mathbf{P}^{-1}\mathbf{R}$.

You need to use matrix algebra to make $\mathbf{Q}$ the subject
One possible way is to remove $\mathbf{P}$ from the left by pre-multiplying both sides by $\mathbf{P}^{-1}$

$$\mathbf{P}^{-1}\mathbf{PQR}^{-1} = \mathbf{P}^{-1}\mathbf{I}$$

The $\mathbf{P}^{-1}\mathbf{P}$ forms the identity matrix, $\mathbf{I}$, on the left
The $\mathbf{P}^{-1}\mathbf{I}$ simplifies to just $\mathbf{P}^{-1}$ on the right

$$\mathbf{IQR}^{-1} = \mathbf{P}^{-1}$$

The $\mathbf{IQR}^{-1}$ on the left simplifies to just $\mathbf{QR}^{-1}$

$$\mathbf{QR}^{-1} = \mathbf{P}^{-1}$$

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Now post-multiply both sides by $\mathbf{R}$

$$\mathbf{Q}\mathbf{R}^{-1}\mathbf{R} = \mathbf{P}^{-1}\mathbf{R}$$

The $\mathbf{R}^{-1}\mathbf{R}$ on the left forms the identity matrix

$$\mathbf{Q}\mathbf{I} = \mathbf{P}^{-1}\mathbf{R}$$

The $\mathbf{QI}$ on the left simplifies to $\mathbf{Q}$

$$\mathbf{Q} = \mathbf{P}^{-1}\mathbf{R}$$

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