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4. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3} n(4n^2 - 1)$$

for all positive integers n.

(5)

(b) Hence find the exact value of the sum of the squares of the odd numbers between 200 and 500

(4)

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Question 4 continued

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5. (a) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n} (r+1)(r+5) = \frac{n}{6}(n+7)(2n+7)$$

for all positive integers n.

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n} (r+1)(r+5) = \frac{7n}{6}(n+1)(an+b)$$

where a and b are integers to be determined.

(2)

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Question 5 continued

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7. (a) Prove by induction that for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

(5)

(b) Hence show that

$$\sum_{r=1}^{n} (r^2 + 2) = \frac{n}{6}(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

(c) Using your answers to part (b), find the value of

$$\sum_{r=10}^{25} (r^2 + 2)$$

(2)

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Question 7 continued

2021 Oct

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5. (a) Use the standard results for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n,

$$\sum_{r=1}^{n} r(r-1)(r-3) = \frac{1}{12} n(n+1)(n-1)(3n-10)$$

(5)

(b) Hence show that

$$\sum_{r=n+1}^{2n+1} r(r-1)(r-3) = \frac{1}{12} n(n+1)(an^2 + bn + c)$$

where a, b and c are integers to be determined.

(3)

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Question 5 continued

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9. (a) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2 (n+1)^2$$

(5)

(b) Using the standard summation formulae, show that

$$\sum_{r=1}^{n} r(r+1)(r-1) = \frac{1}{4} n(n+A)(n+B)(n+C)$$

where A, B and C are constants to be determined.

(4)

(c) Determine the value of n for which

$$3\sum_{r=1}^{n} r(r+1)(r-1) = 17\sum_{r=n}^{2n} r^2$$

(5)

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Question 9 continued

2022 June

8. (a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that for all positive integers n

$$\sum_{r=0}^{n} (r+1)(r+2) = \frac{1}{3}(n+1)(n+2)(n+3)$$

(5)

(b) Hence determine the value of

$$10 \times 11 + 11 \times 12 + 12 \times 13 + ... + 100 \times 101$$

(3)

Question 8 continued

2023 Jan

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that for all positive integers n

$$\sum_{r=1}^{n} (7r - 5)^2 = \frac{n}{6}(7n + 1)(An + B)$$

where A and B are integers to be determined.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

2023 June

1. Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n

$$\sum_{r=1}^{n} r^2 (r + 2) = \frac{1}{12} n(n + 1)(an^2 + bn + c)$$

where a, b and c are integers to be determined.

(4)

Question 1 continued

(Total for Question 1 is 4 marks)

2024 Jan

8. (a) Use the standard results for summations to show that, for all positive integers n,

$$\sum_{r=1}^{n} r\left(2r^2 - 3r - 1\right) = \frac{1}{2}n(n+1)^2(n-2)$$

(4)

(b) Hence show that, for all positive integers n,

$$\sum_{r=n}^{2n} r\left(2r^2 - 3r - 1\right) = \frac{1}{2}n(n-1)(an+b)(cn+d)$$

where a, b, c and d are integers to be determined.

(4)

Question 8 continued

2024 June

7. **In this question use the standard results for summations.**

(a) Show that for all positive integers n

$$\sum_{r=1}^{n}(12r^2 + 2r - 3) = An^3 + Bn^2$$

where A and B are integers to be determined.

(4)

(b) Hence determine the value of n for which

$$\sum_{r=1}^{2n} r^3 - \sum_{r=1}^{n}(12r^2 + 2r - 3) = 270$$

(4)

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Question 7 continued

(Total for Question 7 is 8 marks)

2025 Jan

5. (a) Use the standard results for summations to show that for all positive integers n

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n+a)(n+b)(n+c)$$

where a, b and c are integers to be determined.

(5)

(b) Hence determine the value of

$$20 \times 21 \times 25 + 21 \times 22 \times 26 + ... + 40 \times 41 \times 45$$

(2)

Question 5 continued

(Total for Question 5 is 7 marks)

2025 June

8: (a) Using the standard summation formulae show that

$$\sum_{r=1}^{2n}\left(2r^2-1\right)=\frac{4}{3}n(n+1)(an+b)$$

where a and b are integers to be determined.

(5)

(b) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r(3r-2)^2=\frac{n^2(n+1)(9n-7)}{4}$$

(5)

Using the results from parts (a) and (b) and showing all stages of your working,

(c) determine the value of n for which

$$8\sum_{r=1}^{n} r(3r-2)^2=15\sum_{r=1}^{2n}\left(2r^2-1\right)$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

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Question 8 continued