2020 May

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**2.** The quadratic equation

$$5x^2 - 2x + 3 = 0$$

has roots $\alpha$ and $\beta$.

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

**(1)**

(b) determine, giving each answer as a simplified fraction, the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

**(4)**

(c) determine a quadratic equation that has roots

$$(\alpha + \beta^2) \text{ and } (\beta + \alpha^2)$$

giving your answer in the form $px^2 + qx + r = 0$ where $p$, $q$ and $r$ are integers.

**(4)**

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**Question 2 continued**

2021 Jan

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**4.** The equation $2x^2 + 5x + 7 = 0$ has roots $\alpha$ and $\beta$

Without solving the equation

(a) determine the exact value of $\alpha^3 + \beta^3$

**(3)**

(b) form a quadratic equation, with integer coefficients, which has roots

$$\frac{\alpha^2}{\beta} \text{ and } \frac{\beta^2}{\alpha}$$

**(5)**

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**Question 4 continued**

Leave blank

**3.** The quadratic equation

$$2x^2 - 5x + 7 = 0$$

has roots $\alpha$ and $\beta$

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

**(1)**

(b) determine, giving each answer as a simplified fraction, the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

**(4)**

(c) find a quadratic equation that has roots

$$\frac{1}{\alpha^2 + \beta} \text{ and } \frac{1}{\beta^2 + \alpha}$$

giving your answer in the form $px^2 + qx + r = 0$ where $p$, $q$ and $r$ are integers to be determined.

**(4)**

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**Question 3 continued**

Leave blank

**6.** The quadratic equation

$$Ax^2 + 5x - 12 = 0$$

where $A$ is a constant, has roots $\alpha$ and $\beta$

(a) Write down an expression in terms of $A$ for

(i) $\alpha + \beta$

(ii) $\alpha\beta$

**(2)**

The equation

$$4x^2 - 5x + B = 0$$

where $B$ is a constant, has roots $\alpha - \frac{3}{\beta}$ and $\beta - \frac{3}{\alpha}$

(b) Determine the value of $A$

**(3)**

(c) Determine the value of $B$

**(3)**

Leave blank

**Question 6 continued**

2022 June

**5.** The quadratic equation

$$2x^2 - 3x + 5 = 0$$

has roots $\alpha$ and $\beta$

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

**(1)**

(b) determine the value of

(i) $\alpha^2 + \beta^2$

(ii) $\alpha^3 + \beta^3$

**(4)**

(c) find a quadratic equation which has roots

$$(\alpha^3 - \beta) \text{ and } (\beta^3 - \alpha)$$

giving your answer in the form $px^2 + qx + r = 0$ where $p$, $q$ and $r$ are integers to be determined.

**(5)**

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**Question 5 continued**

2023 Jan

**5.** The quadratic equation

$$4x^2 + 3x + k = 0$$

where $k$ is an integer, has roots $\alpha$ and $\beta$

(a) Write down, in terms of $k$ where appropriate, the value of $\alpha + \beta$ and the value of $\alpha\beta$

**(2)**

(b) Determine, in simplest form in terms of $k$, the value of $\dfrac{\alpha}{\beta^2} + \dfrac{\beta}{\alpha^2}$

**(4)**

(c) Determine a quadratic equation which has roots

$$\frac{\alpha}{\beta^2} \text{ and } \frac{\beta}{\alpha^2}$$

giving your answer in the form $px^2 + qx + r = 0$ where $p$, $q$ and $r$ are integer values in terms of $k$

**(3)**

**Question 5 continued**

2023 June

**5.** $f(x) = x^2 - 6x + 3$

The equation $f(x) = 0$ has roots $\alpha$ and $\beta$

Without solving the equation,

(a) determine the value of

$$(\alpha^2 + 1)(\beta^2 + 1)$$

**(4)**

(b) find a quadratic equation which has roots

$$\frac{\alpha}{(\alpha^2 + 1)} \text{ and } \frac{\beta}{(\beta^2 + 1)}$$

giving your answer in the form $px^2 + qx + r = 0$ where $p$, $q$ and $r$ are integers to be determined.

**(6)**

**Question 5 continued**

**5.** The quadratic equation

$$2x^2 - 3x + 7 = 0$$

has roots $\alpha$ and $\beta$

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

**(1)**

(b) determine the value of $\alpha^2 + \beta^2$

**(2)**

(c) find a quadratic equation which has roots

$$\left(\alpha - \frac{1}{\beta^2}\right) \text{ and } \left(\beta - \frac{1}{\alpha^2}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where $p$, $q$ and $r$ are integers to be determined.

**(6)**

**Question 5 continued**

2024 June

**5.** The equation $5x^2 - 4x + 2 = 0$ has roots $\frac{1}{p}$ and $\frac{1}{q}$

(a) Without solving the equation,

(i) show that $pq = \frac{5}{2}$

(ii) determine the value of $p + q$

**(4)**

(b) Hence, without finding the values of $p$ and $q$, determine a quadratic equation with roots

$$\frac{p}{p^2 + 1} \text{ and } \frac{q}{q^2 + 1}$$

giving your answer in the form $ax^2 + bx + c = 0$ where $a$, $b$ and $c$ are integers.

**(5)**

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**Question 5 continued**

2025 Jan

**3.** The quadratic equation

$$3x^2 - 2x + 5 = 0$$

has roots $\alpha$ and $\beta$

Without solving the equation,

(a) write down the value of $(\alpha + \beta)$ and the value of $\alpha\beta$

**(1)**

(b) determine the value of $\alpha^2 + \beta^2$

**(2)**

(c) determine a quadratic equation that has roots

$$\left(\alpha + \frac{1}{\alpha}\right) \text{ and } \left(\beta + \frac{1}{\beta}\right)$$

giving your answer in the form $px^2 + qx + r = 0$ where $p$, $q$ and $r$ are integers.

**(4)**

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**Question 3 continued**

**(Total for Question 3 is 7 marks)**

2025 June

**6:** $f(x) = 3x^2 + kx - 5$

where $k$ is a constant.

The equation $f(x) = 0$ has roots $\alpha$ and $\beta$.

(a) State the value of $\alpha\beta$.

**(1)**

Given that $\alpha + \beta = 9\alpha\beta$

(b) determine the value of $k$.

**(2)**

(c) By first expanding $(\alpha + \beta)^3$ prove that

$$\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$$

**(2)**

Without solving the equation $f(x) = 0$

(d) find a quadratic equation with integer coefficients that has roots

$$(\alpha^2 + \beta) \text{ and } (\alpha + \beta^2)$$

**(6)**

**Question 6 continued**