2020 May

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8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n} \frac{2r^2 - 1}{r^2(r+1)^2} = \frac{n^2}{(n+1)^2}$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 12^n + 2 \times 5^{n-1}$$

is divisible by 7

(6)

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Question 8 continued

2021 Jan

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9. (i) A sequence of numbers $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = \frac{1}{3}(2u_n - 1) \qquad u_1 = 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3\left(\frac{2}{3}\right)^n - 1$$

(6)

(ii) $\mathrm{f}(n) = 2^{n+2} + 3^{2n+1}$

Prove by induction that, for $n \in \mathbb{Z}^+$, $\mathrm{f}(n)$ is a multiple of 7

(6)

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Question 9 continued

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7. (a) Prove by induction that for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^2 = \frac{n}{6}(n+1)(2n+1)$$

(5)

(b) Hence show that

$$\sum_{r=1}^{n} (r^2 + 2) = \frac{n}{6}(an^2 + bn + c)$$

where a, b and c are integers to be found.

(4)

(c) Using your answers to part (b), find the value of

$$\sum_{r=10}^{25} (r^2 + 2)$$

(2)

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Question 7 continued

2021 June

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8. Prove by induction that $4^{n+2} + 5^{2n+1}$ is divisible by 21 for all positive integers n.

(6)

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Question 8 continued

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9. (i) A sequence of numbers is defined by

$$u_1 = 0 \qquad u_2 = -6$$

$$u_{n+2} = 5u_{n+1} - 6u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3 \times 2^n - 2 \times 3^n$$

(5)

(ii) Prove by induction that, for all positive integers n,

$$f(n) = 3^{3n-2} + 2^{4n-1}$$

is divisible by 11

(5)

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Question 9 continued

2022 Jan

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9. (a) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2 (n+1)^2$$

(5)

(b) Using the standard summation formulae, show that

$$\sum_{r=1}^{n} r(r+1)(r-1) = \frac{1}{4} n(n+A)(n+B)(n+C)$$

where A, B and C are constants to be determined.

(4)

(c) Determine the value of n for which

$$3\sum_{r=1}^{n} r(r+1)(r-1) = 17\sum_{r=n}^{2n} r^2$$

(5)

Leave blank

Question 9 continued

2022 June

9. (i) A sequence of numbers is defined by

$$u_1 = 3$$

$$u_{n+1} = 2u_n - 2^{n+1} \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{N}$

$$u_n = 5 \times 2^{n-1} - n \times 2^n$$

(5)

(ii) Prove by induction that, for $n \in \mathbb{N}$

$$f(n) = 5^{n+2} - 4n - 9$$

is divisible by 16

(5)

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Question 9 continued

2023 Jan

9. Prove by induction that for all positive integers n

$$\sum_{r=1}^{n} \log(2r-1) = \log\left(\frac{(2n)!}{2^n n!}\right)$$

(6)

Question 9 continued

2023 June

9. Prove, by induction, that for $n \in \mathbb{Z}, n \geqslant 2$

$$4^n + 6n - 10$$

is divisible by 18

(5)

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Question 9 continued

2024 Jan

10. (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}^n = 3^{n-1}\begin{pmatrix} 2n+3 & -n \\ 4n & 3-2n \end{pmatrix}$$

(5)

(ii) Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 8^{2n+1} + 6^{2n-1}$$

is divisible by 7

(5)

Question 10 continued

2024 June

6. (a) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & (2^n - 1)r \\ 0 & 2^n \end{pmatrix}$$

where r is a constant.

(4)

$$\mathbf{M} = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} \qquad \mathbf{N} = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}^4$$

The transformation represented by matrix **M** followed by the transformation represented by matrix **N** is represented by the matrix **B**

(b) (i) Determine **N** in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c and d are integers.

(ii) Determine **B**

(3)

Hexagon S is transformed onto hexagon S' by matrix **B**

(c) Given that the area of S' is 720 square units, determine the area of S

(2)

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Question 6 continued

2024 June

8. Prove by induction that for $n \in \mathbb{Z}^+$

$$f(n) = 7^{n-1} + 8^{2n+1}$$

is divisible by 57

(6)

Question 8 continued

(Total for Question 8 is 6 marks)

2025 Jan

8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & 9n \\ -n & 3n+1 \end{pmatrix}$$

(5)

(ii) A sequence of numbers is defined by

$$u_1 = 1 \qquad u_2 = 4$$

$$u_{n+2} = 6u_{n+1} - 9u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3^{n-2}(n+2)$$

(5)

Question 8 continued

2025 June

8: (a) Using the standard summation formulae show that

$$\sum_{r=1}^{2n}\left(2r^2-1\right)=\frac{4}{3}n(n+1)(an+b)$$

where a and b are integers to be determined.

(5)

(b) Prove by induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} r(3r-2)^2=\frac{n^2(n+1)(9n-7)}{4}$$

(5)

Using the results from parts (a) and (b) and showing all stages of your working,

(c) determine the value of n for which

$$8\sum_{r=1}^{n} r(3r-2)^2=15\sum_{r=1}^{2n}\left(2r^2-1\right)$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

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Question 8 continued