2020 May

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1. $$f(x) = x^3 - \frac{10\sqrt{x} - 4x}{x^2} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.4, 1.5]

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 1.4$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to calculate a second approximation to α, giving your answer to 3 decimal places.

(2)

Leave blank

Question 1 continued

Q1

(Total 7 marks)

2021 Jan

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1. (a) Show that the equation $4x - 2\sin x - 1 = 0$, where x is in radians, has a root α in the interval [0.2, 0.6]

(2)

(b) Starting with the interval [0.2, 0.6], use interval bisection twice to find an interval of width 0.1 in which α lies.

(3)

Leave blank

Question 1 continued

Q1

(Total 5 marks)

2021 June

Leave blank

1. (i) $f(x) = x^3 + 4x - 6$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[1, 1.5]$

(2)

(b) Taking 1.5 as a first approximation, apply the Newton Raphson process twice to $f(x)$ to obtain an approximate value of α. Give your answer to 3 decimal places. Show your working clearly.

(4)

(ii) $g(x) = 4x^2 + x - \tan x$

where x is measured in radians.

The equation $g(x) = 0$ has a single root β in the interval $[1.4, 1.5]$

Use linear interpolation on the values at the end points of this interval to obtain an approximation to β. Give your answer to 3 decimal places.

(4)

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Question 1 continued

2021 Oct

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2. $$f(x) = 7\sqrt{x} - \frac{1}{2}x^3 - \frac{5}{3x} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval [2.8, 2.9]

(2)

(b) (i) Find $f'(x)$.

(ii) Hence, using $x_0 = 2.8$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to calculate a second approximation to α, giving your answer to 3 decimal places.

(4)

(c) Use linear interpolation once on the interval [2.8, 2.9] to find another approximation to α. Give your answer to 3 decimal places.

(3)

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Question 2 continued

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8. $$f(x) = 2x^{-\frac{2}{3}} + \frac{1}{2}x - \frac{1}{3x-5} - \frac{5}{2} \qquad x \neq \frac{5}{3}$$

The table below shows values of f(x) for some values of x, with values of f(x) given to 4 decimal places where appropriate.

x	1	2	3	4	5
f(x)	0.5		–0.2885		0.5834

(a) Complete the table giving the values to 4 decimal places.

(2)

The equation f(x) = 0 has exactly one positive root, α.

Using the values in the completed table and explaining your reasoning,

(b) determine an interval of width one that contains α.

(2)

(c) Hence use interval bisection twice to obtain an interval of width 0.25 that contains α.

(3)

Given also that the equation f(x) = 0 has a negative root, β, in the interval [–1, –0.5]

(d) use linear interpolation once on this interval to find an approximation for β.

Give your answer to 3 significant figures.

(3)

Leave blank

Question 8 continued

2022 June

2. $$f(x) = 10 - 2x - \frac{1}{2\sqrt{x}} - \frac{1}{x^3} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [0.4, 0.5]

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 0.5$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(2)

The equation $f(x) = 0$ has another root β in the interval [4.8, 4.9]

(d) Use linear interpolation once on the interval [4.8, 4.9] to find an approximation to β, giving your answer to 3 decimal places.

(2)

Question 2 continued

2023 Jan

4.

$$f(x) = 1 - \frac{1}{8x^4} + \frac{2}{7\sqrt{x^7}} \qquad x > 0$$

The equation $f(x) = 0$ has a single root, α, that lies in the interval $[0.15, 0.25]$

(a) (i) Determine $f'(x)$

(ii) Explain why 0.25 cannot be used as an initial approximation for α in the Newton-Raphson process.

(iii) Taking 0.15 as a first approximation to α apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α
Give your answer to 3 decimal places.

(5)

(b) Use linear interpolation once on the interval $[0.15, 0.25]$ to find another approximation to α
Give your answer to 3 decimal places.

(3)

Question 4 continued

2023 June

7. $$f(x) = x^{\frac{3}{2}} + x - 3$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval [1, 2]

(2)

(b) Starting with the interval [1, 2], use interval bisection twice to show that α lies in the interval [1.25, 1.5]

(3)

(c) (i) Determine $f'(x)$

(ii) Using 1.375 as a first approximation for α, apply the Newton-Raphson process once to $f(x)$ to determine a second approximation for α, giving your answer to 3 decimal places.

(3)

(d) Use linear interpolation once on the interval [1.25, 1.5] to obtain a different approximation for α, giving your answer to 3 decimal places.

(3)

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Question 7 continued

2024 Jan

6. (i)

$$f(x) = x - 4 - \cos\left(5\sqrt{x}\right) \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [2.5, 3.5]

(2)

(b) Use linear interpolation once on the interval [2.5, 3.5] to find an approximation to α, giving your answer to 2 decimal places.

(2)

(ii)

$$g(x) = \frac{1}{10}x^2 - \frac{1}{2x^2} + x - 11 \qquad x > 0$$

(a) Determine $g'(x)$.

(2)

The equation $g(x) = 0$ has a root β in the interval [6, 7]

(b) Using $x_0 = 6$ as a first approximation to β, apply the Newton–Raphson procedure once to $g(x)$ to find a second approximation to β, giving your answer to 3 decimal places.

(2)

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Question 6 continued

2024 June

3. $f(x) = x^3 - 5\sqrt{x} - 4x + 7 \qquad x \geqslant 0$

The equation $f(x) = 0$ has a root α in the interval [0.25, 1]

(a) Use linear interpolation once on the interval [0.25, 1] to determine an approximation to α, giving your answer to 3 decimal places.

(3)

The equation $f(x) = 0$ has another root β in the interval [1.5, 2.5]

(b) Determine $f'(x)$

(2)

(c) Hence, using $x_0 = 1.75$ as a first approximation to β, apply the Newton–Raphson process once to $f(x)$ to determine a second approximation to β, giving your answer to 3 decimal places.

(2)

Question 3 continued

(Total for Question 3 is 7 marks)

2025 Jan

2. $$f(x) = x^2 - \frac{7x - 4\sqrt{x}}{x^3} \qquad x > 0$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [0.3, 0.4]

(2)

(b) Determine $f'(x)$.

(3)

(c) Using $x_0 = 0.3$ as a first approximation for α, apply the Newton–Raphson procedure once to $f(x)$ to determine a second approximation for α, giving your answer to 3 decimal places.

(2)

The equation $f(x) = 0$ has another root β in the interval [1.3, 1.5]

(d) Use linear interpolation once on the interval [1.3, 1.5] to determine an approximation for β, giving your answer to 3 decimal places.

(2)

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Question 2 continued

2025 June

3: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) $f(x) = x^2 + 5 - 8^{5x}$

Given that the equation $f(x) = 0$ has a single root, α, in the interval [0, 1]

use interval bisection to determine an interval of width 0.25 that contains α.

(3)

(ii) $g(x) = 3^{\sin x} - 3\cos x$

where x is in radians.

(a) Show that the equation $g(x) = 0$ has a root, β, in the interval [4, 5]

(2)

(b) Use linear interpolation once on the interval [4, 5] to determine an approximation for β.
Give your answer to 4 significant figures.

(2)

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Question 3 continued