2020 May

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6. (i) $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents a rotation of 45° clockwise about the origin.

(b) Write down the matrix **B**, giving each element of the matrix in exact form.

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

(c) Determine **C**.

(2)

(ii) The trapezium T has vertices at the points $(-2, 0)$, $(-2, k)$, $(5, 8)$ and $(5, 0)$, where k is a positive constant. Trapezium T is transformed onto the trapezium T' by the matrix

$$\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}$$

Given that the area of trapezium T' is 510 square units, calculate the exact value of k.

(5)

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Question 6 continued

2021 Jan

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3. The matrix **M** is defined by

$$\mathbf{M} = \begin{pmatrix} k+5 & -2 \\ -3 & k \end{pmatrix}$$

(a) Determine the values of k for which **M** is singular.

(2)

Given that **M** is non-singular,

(b) find $\mathbf{M}^{-1}$ in terms of k.

(2)

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Question 3 continued

Q3

(Total 4 marks)

2021 Jan

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7. The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 4 & -5 \\ -3 & 2 \end{pmatrix}$$

The transformation represented by **A** maps triangle T onto triangle T'

Given that the area of triangle T is 23 cm^2

(a) determine the area of triangle T'

(2)

The point P has coordinates $(3p + 2, 2p - 1)$ where p is a constant. The transformation represented by **A** maps P onto the point P' with coordinates $(17, -18)$

(b) Determine the value of p.

(2)

Given that

$$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(c) describe fully the single geometrical transformation represented by matrix **B**

(2)

The transformation represented by matrix **A** followed by the transformation represented by matrix **C** is equivalent to the transformation represented by matrix **B**

(d) Determine **C**

(3)

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Question 7 continued

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3. The triangle T has vertices $A(2, 1)$, $B(2, 3)$ and $C(0, 1)$.

The triangle T' is the image of T under the transformation represented by the matrix

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) Find the coordinates of the vertices of T'

(2)

(b) Describe fully the transformation represented by **P**

(2)

The 2×2 matrix **Q** represents a reflection in the x-axis and the 2×2 matrix **R** represents a rotation through 90° anticlockwise about the origin.

(c) Write down the matrix **Q** and the matrix **R**

(2)

(d) Find the matrix **RQ**

(2)

(e) Give a full geometrical description of the single transformation represented by the answer to part (d).

(2)

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Question 3 continued

2021 Oct

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1. $$\mathbf{A} = \begin{pmatrix} 3 & a \\ -2 & -2 \end{pmatrix}$$

where a is a non-zero constant and $a \neq 3$

(a) Determine $\mathbf{A}^{-1}$ giving your answer in terms of a.

(2)

Given that $\mathbf{A} + \mathbf{A}^{-1} = \mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a.

(3)

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Question 1 continued

Q1

(Total 5 marks)

2021 Oct

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7. In part (i), the elements of each matrix should be expressed in exact numerical form.

(i) (a) Write down the 2×2 matrix that represents a rotation of 210° anticlockwise about the origin.

(1)

(b) Write down the 2×2 matrix that represents a stretch parallel to the y-axis with scale factor 5

(1)

The transformation T is a rotation of 210° anticlockwise about the origin followed by a stretch parallel to the y-axis with scale factor 5

(c) Determine the 2×2 matrix that represents T

(2)

(ii)

$$\mathbf{M} = \begin{pmatrix} k & k+3 \\ -5 & 1-k \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Find det **M**, giving your answer in simplest form in terms of k.

(2)

A closed shape R is transformed to a closed shape R' by the transformation represented by the matrix **M**.

Given that the area of R is 2 square units and that the area of R' is $16k$ square units,

(b) determine the possible values of k.

(3)

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Question 7 continued

2022 Jan

Leave blank

1. $$\mathbf{M} = \begin{pmatrix} 3x & 7 \\ 4x+1 & 2-x \end{pmatrix}$$

Find the range of values of x for which the determinant of the matrix **M** is positive.

(5)

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Question 1 continued

Q1

(Total 5 marks)

Leave blank

5. $$\mathbf{P} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

The matrix **P** represents the transformation U

(a) Give a full description of U as a single geometrical transformation.

(2)

The transformation V, represented by the 2×2 matrix **Q**, is a reflection in the line $y = -x$

(b) Write down the matrix **Q**

(1)

The transformation U followed by the transformation V is represented by the matrix **R**

(c) Determine the matrix **R**

(2)

The transformation W is represented by the matrix $3\mathbf{R}$

The transformation W maps a triangle T to a triangle T'

The transformation W' maps the triangle T' back to the original triangle T

(d) Determine the matrix that represents W'

(3)

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Question 5 continued

2022 June

3. $\mathbf{M} = \begin{pmatrix} k & k \\ 3 & 5 \end{pmatrix}$ where k is a non-zero constant

(a) Determine $\mathbf{M}^{-1}$, giving your answer in simplest form in terms of k.

(2)

Hence, given that $\mathbf{N}^{-1} = \begin{pmatrix} k & k \\ 4 & -1 \end{pmatrix}$

(b) determine $(\mathbf{MN})^{-1}$, giving your answer in simplest form in terms of k.

(2)

Question 3 continued

(Total for Question 3 is 4 marks)

2022 June

7. $$\mathbf{A} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Determine the matrix $\mathbf{A}^2$

(1)

(b) Describe fully the single geometrical transformation represented by the matrix $\mathbf{A}^2$

(2)

(c) Hence determine the smallest positive integer value of n for which $\mathbf{A}^n = \mathbf{I}$

(1)

The matrix **B** represents a stretch scale factor 4 parallel to the x-axis.

(d) Write down the matrix **B**

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**

(e) Determine the matrix **C**

(2)

The parallelogram P is transformed onto the parallelogram P' by the matrix **C**

(f) Given that the area of parallelogram P' is 20 square units, determine the area of parallelogram P

(2)

Question 7 continued

2023 Jan

1. Given that

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 3 \\ -2 & 3 & 0 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 1 & k \\ 0 & -3 \\ 2k & 2 \end{pmatrix}$$

where k is a non-zero constant,

(a) determine the matrix **AB**

(2)

(b) determine the value of k for which $\det(\mathbf{AB}) = 0$

(3)

Question 1 continued

(Total for Question 1 is 5 marks)

2023 Jan

7. (i)

$$\mathbf{P} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

The matrix **P** represents a geometrical transformation U

(a) Describe U fully as a single geometrical transformation.

(2)

The transformation V, represented by the 2×2 matrix **Q**, is a rotation through 240° anticlockwise about the origin followed by an enlargement about (0, 0) with scale factor 6

(b) Determine the matrix **Q**, giving each entry in exact numerical form.

(2)

Given that U followed by V is the transformation T, which is represented by the matrix **R**

(c) determine the matrix **R**

(2)

(ii) The transformation W is represented by the matrix

$$\begin{pmatrix} -2 & 2\sqrt{3} \\ 2\sqrt{3} & 2 \end{pmatrix}$$

Show that there is a real number λ for which W maps the point $(\lambda, 1)$ onto the point $(4\lambda, 4)$, giving the exact value of λ

(5)

Question 7 continued

2023 June

4. (i) $\mathbf{A} = \begin{pmatrix} -3 & 8 \\ -3 & k \end{pmatrix}$ where k is a constant

The transformation represented by **A** transforms triangle T to triangle T'

The area of triangle T' is three times the area of triangle T

Determine the possible values of k

(4)

(ii) $\mathbf{B} = \begin{pmatrix} a & -4 \\ 2 & 3 \end{pmatrix}$ and $\mathbf{BC} = \begin{pmatrix} 2 & 5 & 1 \\ 1 & 4 & 2 \end{pmatrix}$ where a is a constant

Determine, in terms of a, the matrix **C**

(4)

Question 4 continued

2024 Jan

1.

$$\mathbf{M} = \begin{pmatrix} 2k+1 & k \\ k+7 & k+4 \end{pmatrix} \text{ where } k \text{ is a constant}$$

(a) Show that **M** is non-singular for all real values of k.

(3)

(b) Determine $\mathbf{M}^{-1}$ in terms of k.

(2)

Question 1 continued

(Total for Question 1 is 5 marks)

4.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

(a) Describe the single geometrical transformation represented by the matrix **A**.

(2)

The matrix **B** represents a rotation of 210° anticlockwise about centre (0, 0).

(b) Write down the matrix **B**, giving each element in exact form.

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

(c) Find **C**.

(2)

The hexagon H is transformed onto the hexagon H' by the matrix **C**.

(d) Given that the area of hexagon H is 5 square units, determine the area of hexagon H'

(2)

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Question 4 continued

(Total for Question 4 is 7 marks)

2024 June

1. (i) The matrix **A** is defined by

$$\mathbf{A} = \begin{pmatrix} 3k & 4k-1 \\ 2 & 6 \end{pmatrix}$$

where k is a constant.

(a) Determine the value of k for which **A** is singular.

(2)

Given that **A** is non-singular,

(b) determine $\mathbf{A}^{-1}$ in terms of k, giving your answer in simplest form.

(2)

(ii) The matrix **B** is defined by

$$\mathbf{B} = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$$

where p and q are integers.

State the value of p and the value of q when **B** represents

(a) an enlargement about the origin with scale factor –2

(b) a reflection in the y–axis.

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

2024 June

6. (a) Prove by induction that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 1 & r \\ 0 & 2 \end{pmatrix}^n = \begin{pmatrix} 1 & \left(2^n - 1\right)r \\ 0 & 2^n \end{pmatrix}$$

where r is a constant.

(4)

$$\mathbf{M} = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix} \qquad \mathbf{N} = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}^4$$

The transformation represented by matrix **M** followed by the transformation represented by matrix **N** is represented by the matrix **B**

(b) (i) Determine **N** in the form $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c and d are integers.

(ii) Determine **B**

(3)

Hexagon S is transformed onto hexagon S' by matrix **B**

(c) Given that the area of S' is 720 square units, determine the area of S

(2)

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Question 6 continued

2025 Jan

1. $\mathbf{P} = \begin{pmatrix} p-1 & p+1 \\ -3 & p \end{pmatrix}$ where p is a constant

(a) Determine det **P** in simplest form in terms of p.

(2)

(b) Hence show that **P** is non-singular for all real values of p.

(2)

(c) Determine $\mathbf{P}^{-1}$ in terms of p.

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

7. (i) $$\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents a stretch scale factor 2 parallel to the x-axis.

(b) Write down the matrix **B**.

(1)

The transformation represented by matrix **B** followed by the transformation represented by matrix **A** is the transformation represented by the matrix **C**.

(c) Determine **C**.

(2)

(ii) $$\mathbf{M} = \begin{pmatrix} k & -2 \\ -1 & 2k \end{pmatrix}$$ where k is a constant

Given that the transformation represented by matrix **M** maps the point (k, k) onto the point $(35, 91)$

(a) determine the value of k.

(4)

A quadrilateral Q is transformed to another quadrilateral Q' by the matrix **M**.

Given that Q' has area 336

(b) use the value of k found in part (ii)(a) to determine the area of Q.

(2)

Question 7 continued

8. (i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} -2 & 9 \\ -1 & 4 \end{pmatrix}^n = \begin{pmatrix} 1-3n & 9n \\ -n & 3n+1 \end{pmatrix}$$

(5)

(ii) A sequence of numbers is defined by

$$u_1 = 1 \qquad u_2 = 4$$

$$u_{n+2} = 6u_{n+1} - 9u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3^{n-2}(n+2)$$

(5)

Question 8 continued

2025 June

1: $$\mathbf{M} = \begin{pmatrix} 1 & a \\ 3 & -5 \end{pmatrix}$$

where a is a constant, $a \neq -\frac{5}{3}$

(a) Determine $\mathbf{M}^{-1}$ in terms of a.

(3)

Given that $\mathbf{M}^{-1} = 2\mathbf{M} + 8\mathbf{I}$ where $\mathbf{I}$ is the 2×2 identity matrix,

(b) determine the value of a.

(3)

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Question 1 continued

2025 June

2:

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

The matrix **P** represents the transformation P.

(a) Describe P fully as a single geometrical transformation.

(2)

$$\mathbf{Q} = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

The matrix **Q** represents the transformation Q.

(b) Describe Q fully as a single geometrical transformation.

(2)

The transformation R is a reflection in the line $y = -x$

(c) Write down the matrix representing R.

(1)

The transformation R maps the point $(4, 3)$ to the point A.

(d) Determine the coordinates of A.

(1)

The transformation P followed by the transformation Q maps the point B to the point A.

(e) Determine the coordinates of B.

(2)

Question 2 continued