2020 May

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5. The rectangular hyperbola H has equation $xy = 64$

The point $P\left(8p, \frac{8}{p}\right)$, where $p \neq 0$, lies on H.

(a) Use calculus to show that the normal to H at P has equation

$$p^3x - py = 8(p^4 - 1)$$

(5)

The normal to H at P meets H again at the point Q.

(b) Determine, in terms of p, the coordinates of Q, giving your answers in simplest form.

(4)

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Question 5 continued

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7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The line l with equation $3x - 4y + 48 = 0$ is a tangent to C at the point P.

(a) Show that $a = 9$

(4)

(b) Hence determine the coordinates of P.

(2)

Given that the point S is the focus of C and that the line l crosses the directrix of C at the point A,

(c) determine the exact area of triangle PSA.

(4)

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Question 7 continued

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8. The hyperbola H has Cartesian equation $xy = 25$

The parabola P has parametric equations $x = 10t^2$, $y = 20t$

The hyperbola H intersects the parabola P at the point A

(a) Use algebra to determine the coordinates of A

(3)

The point B with coordinates $(10, 20)$ lies on P

(b) Find an equation for the normal to P at B

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers to be determined.

(5)

(c) Use algebra to determine, in simplest form, the exact coordinates of the points where this normal intersects the hyperbola H

(6)

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Question 8 continued

2021 June

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4. A rectangular hyperbola H has equation $xy = 25$

The point $P\left(5t, \frac{5}{t}\right)$, $t \neq 0$, is a general point on H.

(a) Show that the equation of the tangent to H at P is $t^2y + x = 10t$

(4)

The distinct points Q and R lie on H. The tangent to H at the point Q and the tangent to H at the point R meet at the point $(15, -5)$.

(b) Find the coordinates of the points Q and R.

(4)

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Question 4 continued

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6. The parabola C has Cartesian equation $y^2 = 8x$

The point $P(2p^2, 4p)$ and the point $Q(2q^2, 4q)$, where $p, q \neq 0$, $p \neq q$, are points on C.

(a) Show that an equation of the normal to C at P is

$$y + px = 2p^3 + 4p$$

(5)

(b) Write down an equation of the normal to C at Q

(1)

The normal to C at P and the normal to C at Q meet at the point N

(c) Show that N has coordinates

$$\left(2(p^2 + pq + q^2 + 2), -2pq(p + q)\right)$$

(5)

The line ON, where O is the origin, is perpendicular to the line PQ

(d) Find the value of $(p + q)^2 - 3pq$

(5)

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Question 6 continued

2021 Oct

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6. The curve H has equation

$$xy = a^2 \qquad x > 0$$

where a is a positive constant.

The line with equation $y = kx$, where k is a positive constant, intersects H at the point P

(a) Use calculus to determine, in terms of a and k, an equation for the tangent to H at P

(4)

The tangent to H at P meets the x-axis at the point A and meets the y-axis at the point B

(b) Determine the coordinates of A and the coordinates of B, giving your answers in terms of a and k

(2)

(c) Hence show that the area of triangle AOB, where O is the origin, is independent of k

(2)

Leave blank

Question 6 continued

Leave blank

8. The parabola C has equation $y^2 = 20x$

The point P on C has coordinates $(5p^2, 10p)$ where p is a non-zero constant.

(a) Use calculus to show that the tangent to C at P has equation

$$py - x = 5p^2$$

(3)

The tangent to C at P meets the y-axis at the point A.

(b) Write down the coordinates of A.

(1)

The point S is the focus of C.

(c) Write down the coordinates of S.

(1)

The straight line l_1 passes through A and S.

The straight line l_2 passes through O and P, where O is the origin.

Given that l_1 and l_2 intersect at the point B,

(d) show that the coordinates of B satisfy the equation

$$2x^2 + y^2 = 10x$$

(5)

Leave blank

Question 8 continued

Leave blank

3. The parabola C has equation $y^2 = 18x$

The point S is the focus of C

(a) Write down the coordinates of S

(1)

The point P, with $y > 0$, lies on C

The shortest distance from P to the directrix of C is 9 units.

(b) Determine the exact perimeter of the triangle OPS, where O is the origin.

Give your answer in simplest form.

(4)

Leave blank

Question 3 continued

Leave blank

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The rectangular hyperbola H has equation $xy = 36$

The point $P(4, 9)$ lies on H

(a) Show, using calculus, that the normal to H at P has equation

$$4x - 9y + 65 = 0$$

(4)

The normal to H at P crosses H again at the point Q

(b) Determine an equation for the tangent to H at Q, giving your answer in the form $y = mx + c$ where m and c are rational constants.

(5)

Leave blank

Question 7 continued

6. The parabola C has equation $y^2 = 36x$

The point $P(9t^2, 18t)$, where $t \neq 0$, lies on C

(a) Use calculus to show that the normal to C at P has equation

$$y + tx = 9t^3 + 18t$$

(4)

(b) Hence find the equations of the two normals to C which pass through the point (54, 0), giving your answers in the form $y = px + q$ where p and q are constants to be determined.

(4)

Given that

- the normals found in part (b) intersect the directrix of C at the points A and B
- the point F is the focus of C

(c) determine the area of triangle AFB

(3)

Question 6 continued

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The rectangular hyperbola H has equation $xy = 20$

The point $P\left(2t\sqrt{a}, \frac{2\sqrt{a}}{t}\right)$, $t \neq 0$, where a is a constant, is a general point on H

(a) State the value of a

(1)

(b) Show that the normal to H at the point P has equation

$$ty - t^3x - 2\sqrt{5}\left(1 - t^4\right) = 0$$

(4)

The points A and B lie on H

The point A has parameter $t = c$ and the point B has parameter $t = -\frac{1}{2c}$, where c is a constant.

The normal to H at A meets H again at B

(c) Determine the possible values of c

(4)

Question 6 continued

2023 Jan

8. A parabola C has equation $y^2 = 4ax$ where a is a positive constant.

The point S is the focus of C

The line l_1 with equation $y = k$ where k is a positive constant, intersects C at the point P

(a) Show that

$$PS = \frac{k^2 + 4a^2}{4a}$$

(3)

The line l_2 passes through P and intersects the directrix of C on the x-axis.

The line l_2 intersects the y-axis at the point A

(b) Show that the y coordinate of A is $\dfrac{4a^2k}{k^2 + 4a^2}$

(3)

The line l_1 intersects the directrix of C at the point B

Given that the areas of triangles BPA and OSP, where O is the origin, satisfy the ratio

$$\text{area } BPA : \text{area } OSP = 4k^2 : 1$$

(c) determine the exact value of a

(5)

Question 8 continued

2023 June

3. The rectangular hyperbola H has Cartesian equation $xy = 9$

The point P with coordinates $\left(3t, \frac{3}{t}\right)$, where $t \neq 0$, lies on H

(a) Use calculus to determine an equation for the normal to H at the point P

Give your answer in the form $ty - t^3x = \mathrm{f}(t)$

(4)

Given that $t = 2$

(b) determine the coordinates of the point where the normal meets H again.

Give your answer in simplest form.

(3)

Question 3 continued

(Total for Question 3 is 7 marks)

2023 June

8. The point $P(2p^2, 4p)$ lies on the parabola with equation $y^2 = 8x$

(a) Show that the point $Q\left(\frac{2}{p^2}, \frac{-4}{p}\right)$, where $p \neq 0$, lies on the parabola.

(1)

(b) Show that the chord PQ passes through the focus of the parabola.

(4)

The tangent to the parabola at P and the tangent to the parabola at Q meet at the point R

(c) Determine, in simplest form, the coordinates of R

(8)

Question 8 continued

3. The hyperbola H has equation $xy = c^2$ where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, where $t > 0$, lies on H.

The tangent to H at P meets the x-axis at the point A and meets the y-axis at the point B.

(a) Determine, in terms of c and t,

(i) the coordinates of A,

(ii) the coordinates of B.

(4)

Given that the area of triangle AOB, where O is the origin, is 90 square units,

(b) determine the value of c, giving your answer as a simplified surd.

(2)

Question 3 continued

(Total for Question 3 is 6 marks)

2024 Jan

7. The parabola C has equation $y^2 = \frac{4}{3}x$

The point $P\left(\frac{1}{3}t^2, \frac{2}{3}t\right)$, where $t \neq 0$, lies on C.

(a) Use calculus to show that the normal to C at P has equation

$$3tx + 3y = t^3 + 2t$$

(3)

The normal to C at the point where $t = 9$ meets C again at the point Q.

(b) Determine the exact coordinates of Q.

(4)

Question 7 continued

(Total for Question 7 is 7 marks)

2024 June

9. The rectangular hyperbola H has equation $xy = c^2$ where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, where $t > 0$, lies on H

(a) Use calculus to show that an equation of the normal to H at P is

$$t^3x - ty = c(t^4 - 1)$$

(4)

The parabola C has equation $y^2 = 6x$

The normal to H at the point with coordinates (8, 2) meets C at the point Q where $y > 0$

(b) Determine the exact coordinates of Q

(4)

Given that

- the point R is the focus of C
- the line l is the directrix of C
- the line through Q and R meets l at the point S

(c) determine the exact length of QS

(5)

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Question 9 continued

6. The rectangular hyperbola H has equation $xy = 100$

The point $P\left(10t, \frac{10}{t}\right)$, where $t \neq 0$, lies on H.

(a) Use calculus to show that the normal to H at P has equation

$$t^3x - ty = 10\left(t^4 - 1\right)$$

(4)

The normal to H at P meets the y-axis at the point Q.

Given that the area of triangle OPQ, where O is the origin, is 750

(b) determine both possible pairs of coordinates of the point P.

(5)

Question 6 continued

2025 Jan

9. The parabola C has equation $y^2 = \frac{1}{2}x$

The point $P\left(\frac{t^2}{8}, \frac{t}{4}\right)$, where $t \neq 0$, lies on C.

(a) Use calculus to show that the tangent to C at P has equation

$$8yt - 8x = t^2$$

(3)

Given that

- the tangent to C at P meets the y-axis at the point A
- the line l_1 is the perpendicular bisector of the line segment OA where O is the origin
- the line l_2 is the perpendicular bisector of the line segment AP
- l_1 and l_2 intersect at the point Q

(b) show that, as t varies, the coordinates of Q satisfy the equation

$$y^2 = \alpha x + \beta$$

where α and β are constants to be determined.

(5)

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Question 9 continued

2025 June

4: The rectangular hyperbola C has equation $xy = 81$

The point $P\left(9t, \frac{9}{t}\right)$, $t \neq 0$, lies on C.

(a) Use calculus to show that the normal to C at P has equation

$$ty = t^3x + 9\left(1 - t^4\right)$$

(4)

The normal to C at P meets the y-axis at the point A.

(b) Determine the exact coordinates of A in terms of t.

(2)

Given that $t = \frac{1}{3}$

(c) find the exact area of triangle OPA, where O is the origin.

(2)

Question 4 continued

9:

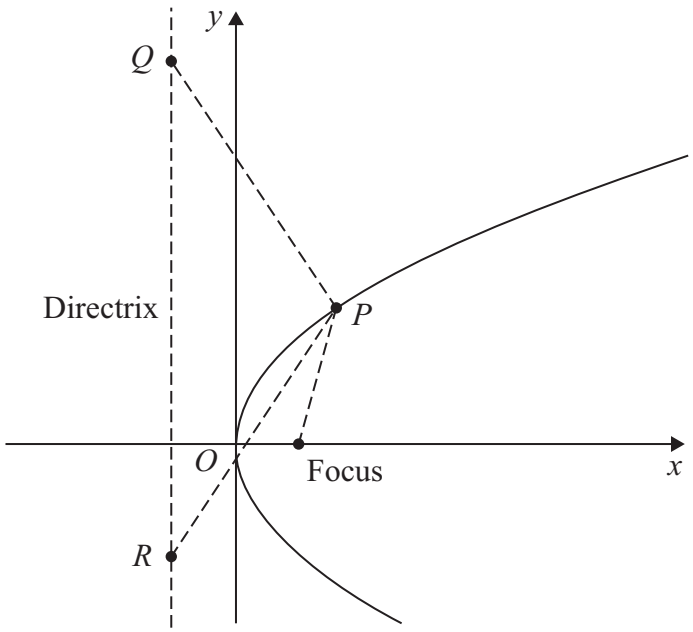

Figure 1

Figure 1 shows a sketch of a parabola C.

The point P lies on C at a distance 5 units from the focus of C.

The points Q and R lie on the directrix of C, where $PQ = PR = 10$ units.

Determine the exact length of QR giving the answer in simplest form.

(4)

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Question 9 continued