2020 May

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3. $f(z) = z^4 + az^3 + bz^2 + cz + d$

where a, b, c and d are integers.

The complex numbers $3 + i$ and $-1 - 2i$ are roots of the equation $f(z) = 0$

(a) Write down the other roots of this equation.

(2)

(b) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

(c) Determine the values of a, b, c and d.

(5)

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Question 3 continued

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2. Given that $x = \frac{3}{8} + \frac{\sqrt{71}}{8}\mathrm{i}$ is a root of the equation

$$4x^3 - 19x^2 + px + q = 0$$

(a) write down the other complex root of the equation.

(1)

Given that $x = 4$ is also a root of the equation,

(b) find the value of p and the value of q.

(4)

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Question 2 continued

Q2

(Total 5 marks)

2021 Jan

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6. The complex number z is defined by

$$z = -\lambda + 3\mathrm{i} \quad \text{where } \lambda \text{ is a positive real constant}$$

Given that the modulus of z is 5

(a) write down the value of λ

(1)

(b) determine the argument of z, giving your answer in radians to one decimal place.

(2)

In part (c) you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

(c) Express in the form $a + \mathrm{i}b$ where a and b are real,

(i) $\dfrac{z + 3\mathrm{i}}{2 - 4\mathrm{i}}$

(ii) z^2

(5)

(d) Show on a single Argand diagram the points A, B, C and D that represent the complex numbers

$$z,\ z^*,\ \frac{z + 3\mathrm{i}}{2 - 4\mathrm{i}} \text{ and } z^2$$

(3)

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Question 6 continued

2021 June

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2. The complex numbers z_1, z_2 and z_3 are given by

$$z_1 = 2 - \mathrm{i} \quad z_2 = p - \mathrm{i} \quad z_3 = p + \mathrm{i}$$

where p is a real number.

(a) Find $\dfrac{z_2 z_3}{z_1}$ in the form $a + b\mathrm{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(3)

Given that $\left|\dfrac{z_2 z_3}{z_1}\right| = 2\sqrt{5}$

(b) find the possible values of p.

(4)

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Question 2 continued

2021 June

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5. $$f(x) = (9x^2 + d)\left(x^2 - 8x + (10d + 1)\right)$$

where d is a positive constant.

(a) Find the four roots of $f(x)$ giving your answers in terms of d.

(3)

Given $d = 4$

(b) Express these four roots in the form $a + ib$, where $a, b \in \mathbb{R}$.

(2)

(c) Show these four roots on a single Argand diagram.

(2)

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Question 5 continued

2021 Oct

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4. $f(z) = 2z^3 - z^2 + az + b$

where a and b are integers.

The complex number $-1 - 3i$ is a root of the equation $f(z) = 0$

(a) Write down another complex root of this equation.

(1)

(b) Determine the value of a and the value of b.

(4)

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

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Question 4 continued

2022 Jan

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2. The complex numbers z_1 and z_2 are given by

$$z_1 = 3 + 5\text{i} \quad \text{and} \quad z_2 = -2 + 6\text{i}$$

(a) Show z_1 and z_2 on a single Argand diagram.

(2)

(b) **Without using your calculator and showing all stages of your working,**

(i) determine the value of $|z_1|$

(1)

(ii) express $\frac{z_1}{z_2}$ in the form $a + b\text{i}$, where a and b are fully simplified fractions.

(3)

(c) Hence determine the value of $\arg\frac{z_1}{z_2}$

Give your answer in radians to 2 decimal places.

(2)

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Question 2 continued

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4. The equation

$$x^4 + Ax^3 + Bx^2 + Cx + 225 = 0$$

where A, B and C are real constants, has

- a complex root 4 + 3i
- a repeated positive real root

(a) Write down the other complex root of this equation.

(1)

(b) Hence determine a quadratic factor of $x^4 + Ax^3 + Bx^2 + Cx + 225$

(2)

(c) Deduce the real root of the equation.

(2)

(d) Hence determine the value of each of the constants A, B and C

(3)

Leave blank

Question 4 continued

2022 June

1. $z_1 = 3 + 3\mathrm{i} \qquad z_2 = p + q\mathrm{i} \qquad p, q \in \mathbb{R}$

Given that $|z_1 z_2| = 15\sqrt{2}$

(a) determine $|z_2|$

(2)

Given also that $p = -4$

(b) determine the possible values of q

(2)

(c) Show z_1 and the possible positions for z_2 on the same Argand diagram.

(2)

Question 1 continued

2022 June

4. $f(z) = 2z^4 - 19z^3 + Az^2 + Bz - 156$

where A and B are constants.

The complex number $5 - i$ is a root of the equation $f(z) = 0$

(a) Write down another complex root of this equation.

(1)

(b) Solve the equation $f(z) = 0$ completely.

(5)

(c) Determine the value of A and the value of B.

(2)

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Question 4 continued

2023 Jan

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(z) = 4z^3 + pz^2 - 24z + 108$$

where p is a constant.

Given that -3 is a root of the equation $f(z) = 0$

(a) determine the value of p

(2)

(b) using algebra, solve $f(z) = 0$ completely, giving the roots in simplest form,

(4)

(c) determine the modulus of the complex roots of $f(z) = 0$

(2)

(d) show the roots of $f(z) = 0$ on a single Argand diagram.

(2)

Question 3 continued

2023 June

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given that $x = 2 + 3i$ is a root of the equation

$$2x^4 - 8x^3 + 29x^2 - 12x + 39 = 0$$

(a) write down another complex root of this equation.

(1)

(b) Use algebra to determine the other 2 roots of the equation.

(4)

(c) Show all 4 roots on a single Argand diagram.

(2)

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Question 2 continued

2023 June

6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$z_1 = 3 + 2\mathrm{i} \qquad z_2 = 2 + 3\mathrm{i} \qquad z_3 = a + b\mathrm{i} \qquad a,b \in \mathbb{R}$$

(a) Determine the exact value of $|z_1 + z_2|$

(2)

Given that $w = \dfrac{z_2 z_3}{z_1}$

(b) determine w in terms of a and b, giving your answer in the form $x + \mathrm{i}y$, where $x, y \in \mathbb{R}$

(4)

Given also that $w = \dfrac{4}{13} + \dfrac{58}{13}\mathrm{i}$

(c) determine the value of a and the value of b

(2)

(d) determine arg w, giving your answer in radians to 4 significant figures.

(2)

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Question 6 continued

2024 Jan

2.

$$f(z) = 2z^3 + pz^2 + qz - 41$$

where p and q are integers.

The complex number $5 - 4i$ is a root of the equation $f(z) = 0$

(a) Write down another complex root of this equation.

(1)

(b) Solve the equation $f(z) = 0$ completely.

(4)

(c) Determine the value of p and the value of q.

(2)

When plotted on an Argand diagram, the points representing the roots of the equation $f(z) = 0$ form the vertices of a triangle.

(d) Determine the area of this triangle.

(2)

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Question 2 continued

2024 Jan

9. Given that

$$\frac{3z-1}{2} = \frac{\lambda + 5\mathrm{i}}{\lambda - 4\mathrm{i}}$$

where λ is a real constant,

(a) determine z, giving your answer in the form $x + y\mathrm{i}$, where x and y are real and in terms of λ.

(4)

Given also that $\arg z = \frac{\pi}{4}$

(b) find the possible values of λ.

(2)

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Question 9 continued

(Total for Question 9 is 6 marks)

2024 June

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(z) = z^3 - 13z^2 + 59z + p \qquad p \in \mathbb{Z}$$

Given that $z = 3$ is a root of the equation $f(z) = 0$

(a) show that $p = -87$

(2)

(b) Use algebra to determine the other roots of $f(z) = 0$, giving your answers in simplest form.

(4)

On an Argand diagram

- the root $z = 3$ is represented by the point P
- the other roots of $f(z) = 0$ are represented by the points Q and R
- the number $z = -9$ is represented by the point S

(c) Show on a single Argand diagram the positions of P, Q, R and S

(1)

(d) Determine the perimeter of the quadrilateral $PQSR$, giving your answer as a simplified surd.

(2)

Question 2 continued

2024 June

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The complex number z is defined by

$$z = -3 + 4i$$

(a) Determine $\left|z^2 - 3\right|$

(3)

(b) Express $\frac{50}{z^*}$ in the form kz, where k is a positive integer.

(3)

(c) Hence find the value of $\arg\left(\frac{50}{z^*}\right)$

Give your answer in radians to 3 significant figures.

(2)

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Question 4 continued

(Total for Question 4 is 8 marks)

2025 Jan

4. $f(z) = 6z^3 + Az^2 + Bz + C$

where A, B and C are integers.

Given that $z = \frac{2}{3} + \frac{\sqrt{17}}{3}i$ is a root of the equation $f(z) = 0$

(a) write down the other complex root of the equation $f(z) = 0$

(1)

Given that $z = -\frac{3}{2}$ is also a root of the equation $f(z) = 0$

(b) determine the value of A, the value of B and the value of C.

(5)

(c) Show all the roots of the equation $f(z) = 0$ on a single Argand diagram.

(2)

Question 4 continued

2025 June

5: The complex number z_1 is given by

$$z_1 = r\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$$

where r is a positive constant.

The complex number z_2 has modulus 5

Given that $|z_1 z_2| = 15$

(a) state the value of r.

(1)

Given further that $z_1 + z_2$ is a real number,

(b) determine the possible complex numbers z_2 in the form $a + bi$ where a and b are constants.

(5)

(c) Show z_1 and the possible complex numbers z_2 on a single Argand diagram.

(2)

Question 5 continued

2025 June

7: $f(z) = Pz^4 - 36z^3 + Qz^2 + 192z + 68$

where P and Q are real constants.

Given that $3 + 5i$ is a root of the equation $f(z) = 0$

(a) write down another root of the equation $f(z) = 0$

(1)

(b) Hence determine a quadratic factor, with real coefficients, of $f(z)$

(2)

(c) Determine the value of P and the value of Q.

(5)

Without using a calculator,

(d) determine the other roots of the equation $f(z) = 0$ giving your answers in simplest form.

(2)

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Question 7 continued