Edexcel International A Level (IAL) Physics

Your notes

Stretching Materials

Contents

Your notes

Hooke's Law

- When a force F is added to the bottom of a vertical metal wire of length L, the wire **stretches**
- A material obeys Hooke's Law if:

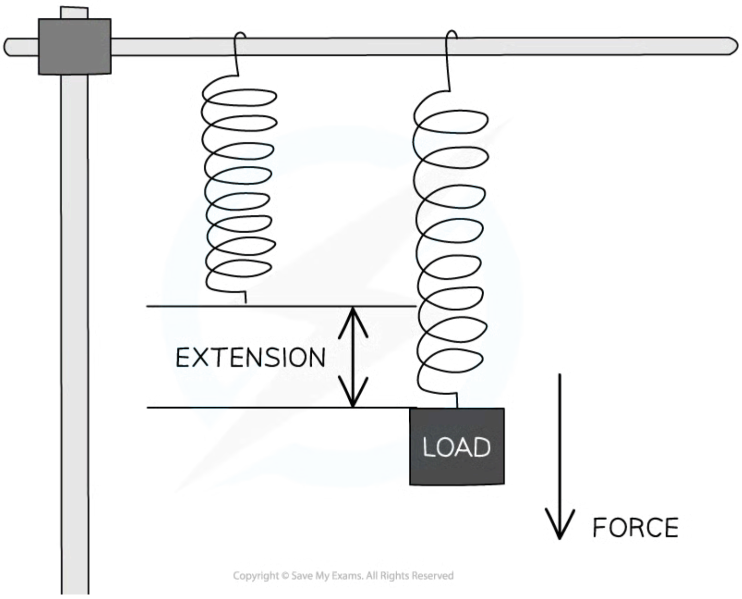
  The extension of the material is directly proportional to the applied force (load) up to the limit of proportionality

- This linear relationship is represented by the Hooke's law equation:

$$\Delta F = k\Delta x$$

- Where:
  - F = applied force (N)
  - k = spring constant (N m^{-1})
  - Δx = extension (m)

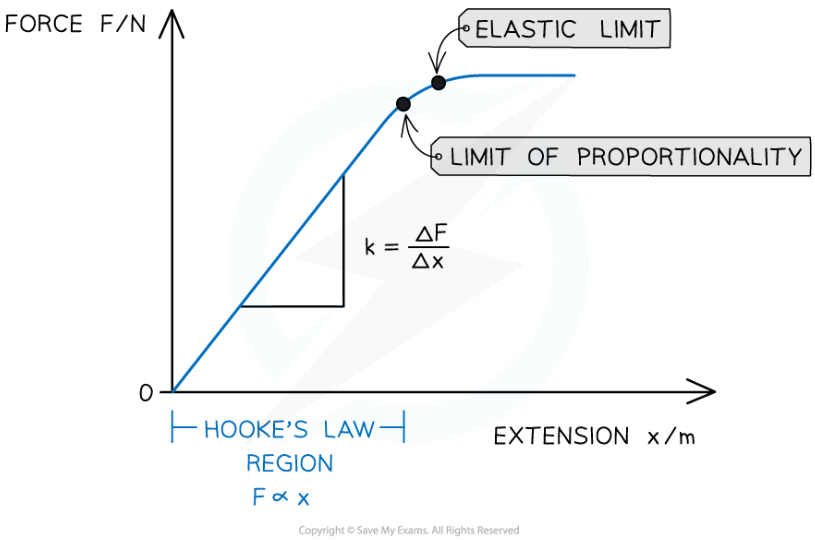
- The spring constant is a property of the material being stretched and measures the **stiffness** of a material
  - The larger the spring constant, the stiffer the material
- Hooke's Law applies to both **extensions** and **compressions**:
  - The extension of an object is determined by how much it has **increased** in length
  - The compression of an object is determined by how much it has **decreased** in length



Stretching a spring with a load produces a force that leads to an extension

Force–Extension Graphs

- The way a material responds to a given force can be shown on a force-extension graph
- A material may obey Hooke's Law up to a point
  - This is shown on its force-extension graph by a **straight line through the origin**
- As more force is added, the graph may start to curve slightly



The Hooke's Law region of a force-extension graph is a straight line. The spring constant is the gradient of that region

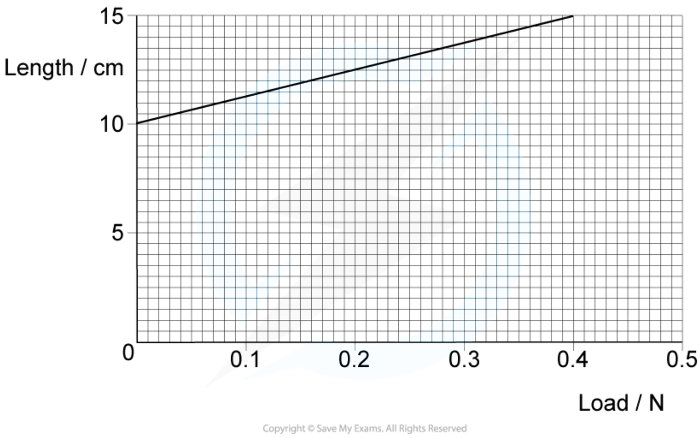
- The key features of the graph are:
  - **The limit of proportionality:** The point beyond which Hooke's law is no longer true when stretching a material i.e. the extension is no longer proportional to the applied force
    - The point is identified on the graph where the line starts to curve (flattens out)
  - **Elastic limit:** The maximum amount a material can be stretched and still return to its original length (above which the material will no longer be elastic). This point is always **after** the limit of proportionality
    - The **gradient** of this graph is equal to the **spring constant** ***k***

Worked Example

A spring was stretched with increasing load.

The graph of the results is shown below.

Your notes



What is the spring constant?

Answer:

Step 1: Rearrange Hooke's Law to make the spring constant the subject

$$k = \frac{\Delta F}{\Delta x}$$

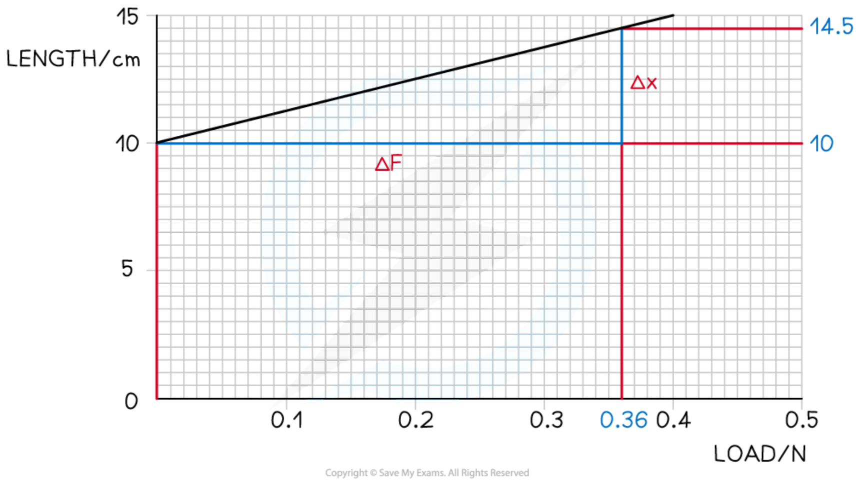
Step 2: Compare the gradient to the equation in Step 1

- This graph is length - extension, so the gradient gives:

$$\frac{increase\ in\ y}{increase\ in\ x} = \frac{\Delta x}{\Delta F}$$

- Therefore k is the reciprocal of the gradient

Step 3: Find the gradient



Step 4: Calculate gradient

Your notes

$$gradient = \frac{\Delta x}{\Delta F} = \frac{(0.145 - 0.10)}{0.36} = 0.125$$

Step 5: Calculate the spring constant by finding the reciprocal of the gradient

$$k = \frac{1}{0.125} = 8.0$$

Step 6: Write the answer, including units

- Spring constant, k = 8.0 N m^{-1}

Examiner Tips and Tricks

Always double check the axes before finding the spring constant as the gradient of a force-extension graph. Exam questions often swap the force (or load) onto the x-axis and extension (or length) on the y-axis.

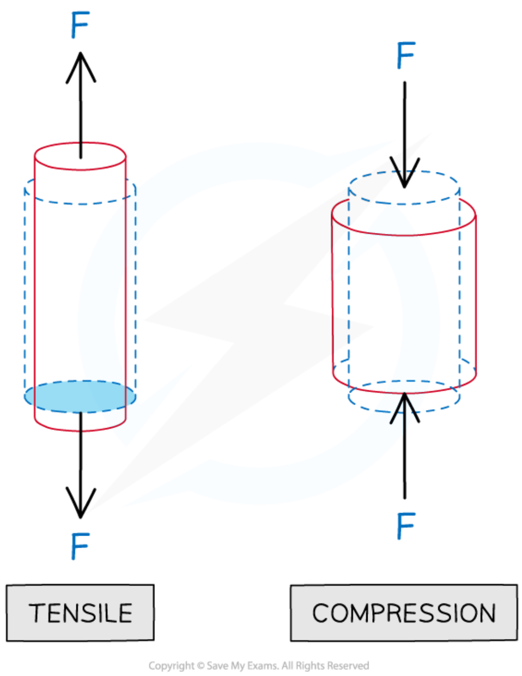
In this case, the gradient is **not** the spring constant, it is **1 ÷ gradient** instead.

Your notes

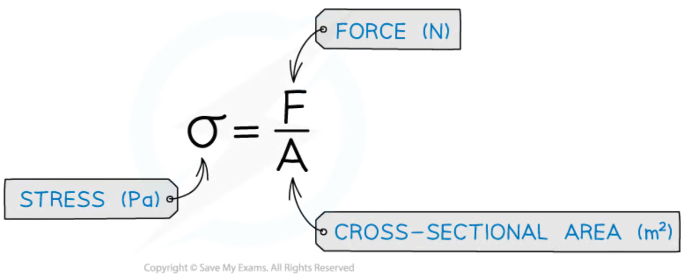
Stress & Strain

Stress

- Stress is the **applied force per unit cross sectional area** of a material
- Forces can be;
  - Tensile forces, which pull on an object and extend it
  - Compressive forces, which push onto an object and compress (or squash) it



- The equation for stress is the force per unit area, and so the units are N m^{-2}, or Pascals, the same unit as pressure

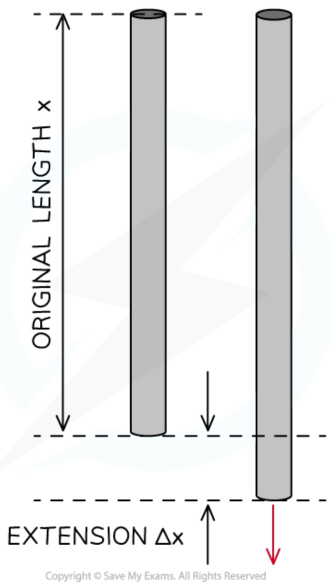


Stress equation

- The **ultimate tensile stress** is the **maximum** force per original cross-sectional area a wire is able to support until it breaks

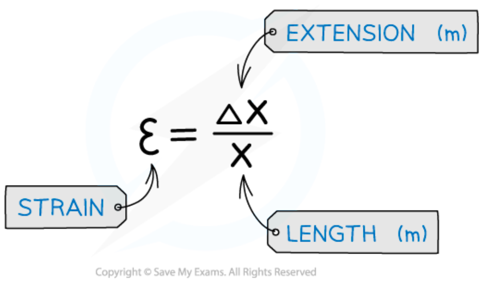
Strain

- Strain is the **extension per unit length**



Strain is the ratio of the extension (or compression) and the original length

- This is a deformation of a solid due to stress in the form of elongation or contraction
- Note that strain is a **dimensionless** unit because it's the ratio of lengths



Strain equation

The Young Modulus

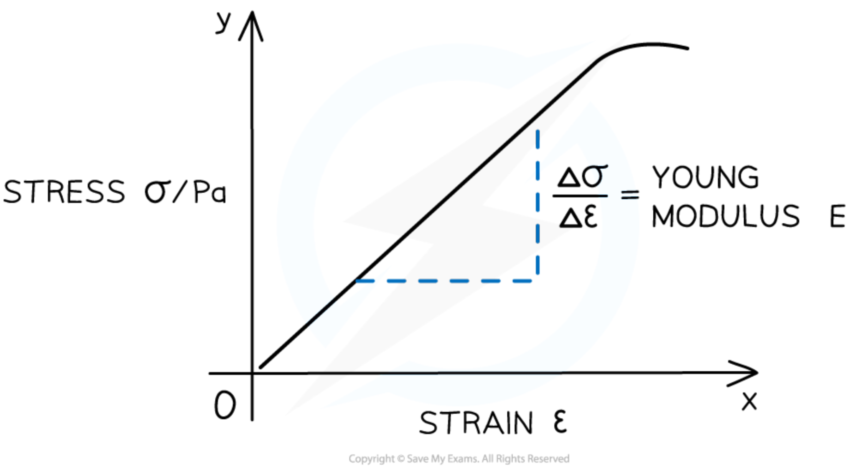
Young Modulus

- The Young modulus (sometimes called Young's Modulus) is the measure of the ability of a material to withstand changes in length with an added load ie. how stiff a material is
- This gives information about the elasticity of a material
- The Young Modulus is defined as the **ratio of stress and strain**

$$\text{YOUNG MODULUS } \underset{(\text{Pa})}{E} = \frac{\text{STRESS } \sigma}{\text{STRAIN } \varepsilon} = \frac{Fx}{A \Delta x}$$

Young Modulus equation

- Its unit is the same as stress: **Pa** (since strain is unitless)
- Just like the Force-Extension graph, stress and strain are directly proportional to one another for a material exhibiting elastic behaviour



A stress-strain graph is a straight line with its gradient equal to Young modulus

- The **gradient** of a stress-strain graph when it is linear is the **Young Modulus**

Worked Example

Your notes

A metal wire that is supported vertically from a fixed point has a load of 92 N applied to the lower end.

The wire has a cross-sectional area of 0.04 mm^2 and obeys Hooke's law.

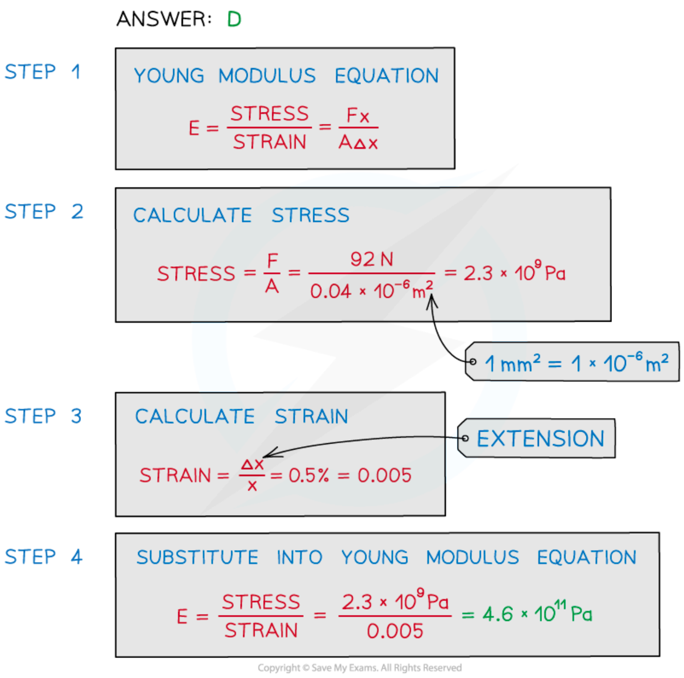
The length of the wire increases by 0.50%. What is the Young modulus of the metal wire?

A. 4.6×10^7 Pa

B. 4.6×10^{12} Pa

C. 4.6×10^9 Pa

D. 4.6×10^{11} Pa



Examiner Tips and Tricks

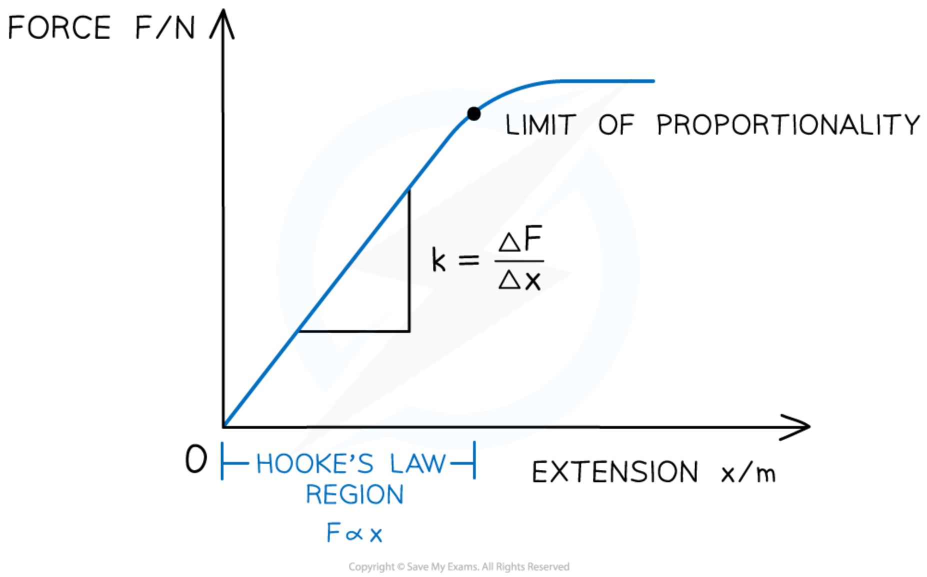
To remember whether stress or strain comes first in the Young modulus equation, try thinking of the phrase 'When you're stressed, you show the strain' i.e. Stress ÷ strain.

Your notes

Force-Extension Graphs

- The way a material responds to a tensile or compressive force can be shown on a force-extension, or a force-compression graph
    - Although compression can be put into **equations** as a **negative** value, the graphs have the same shaped curves
    - Compression is plotted on the **graph** as a **positive**, increasing value
- Every material will have a unique force-extension graph depending on how **brittle** or **ductile** it is
- In the same way, materials have unique force-compression graphs, which will not be the same as their force-extension graph
    - This is because materials behave differently under tensile and compressive strain

Simple Force-Extension Graphs



Simple force-extension graph showing the Hooke's Law region, and the calculation to find k, the spring constant

- A material may obey Hooke's Law up to a point
    - This is shown on its force-extension graph by a **straight line through the origin**
- As more force is added, the graph may start to curve slightly

The key features of the graph are:

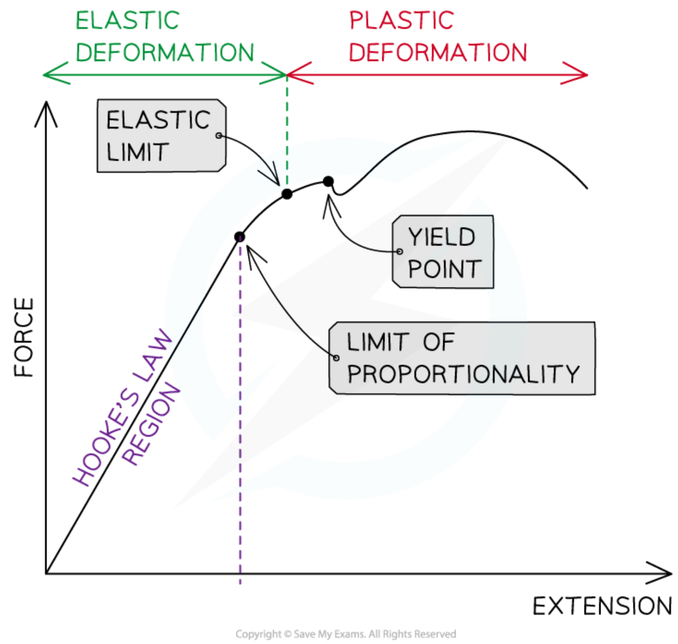
- **The limit of proportionality**

- The point **beyond** which Hooke's law is no longer true when stretching or compressing a material i.e. the extension/ compression is no longer proportional to the applied force
- The point is identified on the graph where the line starts to curve

- **The elastic limit**
  - The point **before** which a material will **return to its original length or shape** when the deforming force is removed
  - This point is always **after** the limit of proportionality
- **The spring constant *k*** is found from the **gradient** of the straight part of the graph

More Detailed Force-Extension Graphs

- Graphs of applied load-extension can give more detailed information about materials
  - This will apply when loads were continued well past the elastic limit



Detailed force-extension graph showing a material under loads which exceed the elastic limit

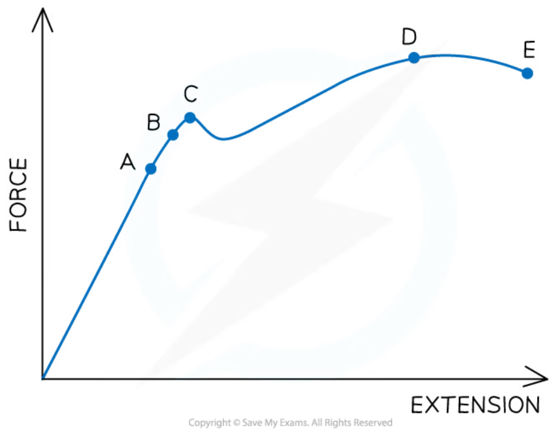
- **The yield point** is where the material continues to stretch even though no extra force is being applied to it
- **Elastic deformation** is a change of shape where the material **will return to its original shape** when the load is removed
- **Plastic deformation** occurs after the yield point
  - It is a change of shape where the material will **not return to its original shape** when the load is removed

Your notes

Your notes

Worked Example

The graph below shows force plotted against extension for a material under compressive forces.



Use the letters marked to identify the elastic limit and yield point, and define both terms.

Answer:

Step 1: Define both terms

- Elastic limit: point before which a material will return to its original length or shape when the deforming force is removed
- Yield point: the point at which the continues to stretch even though no extra force is being applied to it

Step 2: Using your definitions as prompts, select the correct point on the graph

- Elastic limit is at **B** because the line has curved slightly after the Hooke's law region but not started to extend with no extra force
- Yield point is at **C** because the line increases in the x-direction (extension is increasing) but not in the y-direction (no extra force is being added)

Examiner Tips and Tricks

The detailed graph looks confusing at first because it is such an unusual shape. Try tracing out the line with your finger to see how force and extension are related at each point on the graph.

When defining the terms, be very careful to state whether the material behaves in certain ways either **before** or **after** reaching that point.

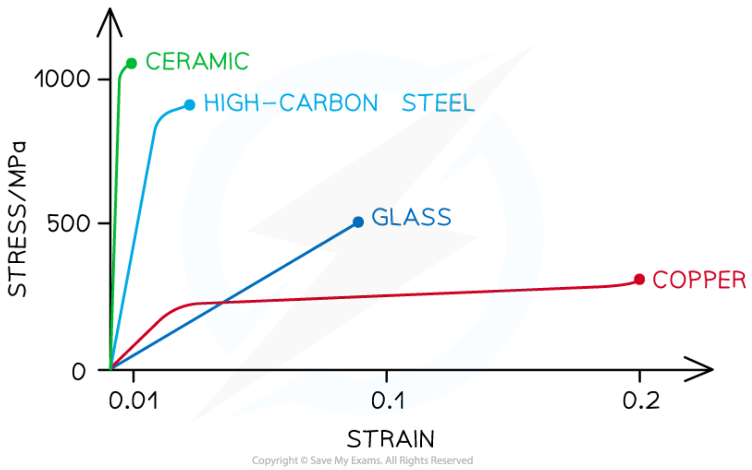
For example, the limit of proportionality is the point '**beyond which'** Hooke's Law no longer applies, **not '**at which'.

Your notes

Your notes

Stress-Strain Graphs

- Stress-strain curves give an indication of the properties of materials such as
  - Up to what stress and strain they obey Hooke's Law
  - Whether they exhibit elastic and/or plastic behaviour
  - The value of their Young Modulus
  - The value of their **breaking stress**
- Each material has a unique stress-strain curve



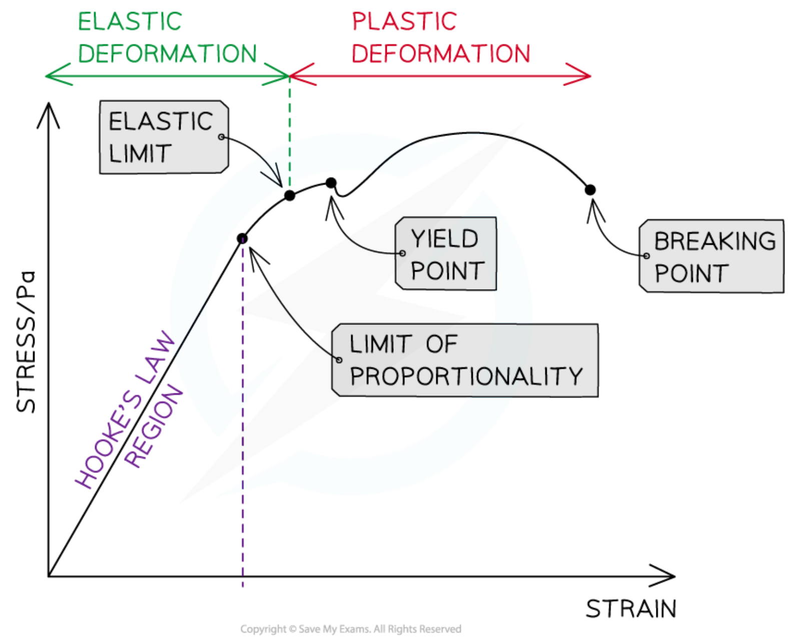
Stress-strain graph for different materials up to their breaking stress

Comparing Force-Extension to Stress-Strain Graphs

The key features of the graph which are also on the force-extension graph are:

- **Limit of proportionality**, beyond which Hooke's law no longer applies
- **The elastic limit**, before which a material returns to its original length or shape when the deforming force is removed
- The **yield point** beyond which the material continues to stretch (more strain is seen) even though no extra force is being applied to it (without additional stress)
- **Elastic deformation** where the material **will return to its original shape** when the load is removed
- **Plastic deformation** where the material will **not return to its original shape** when the load is removed

Your notes



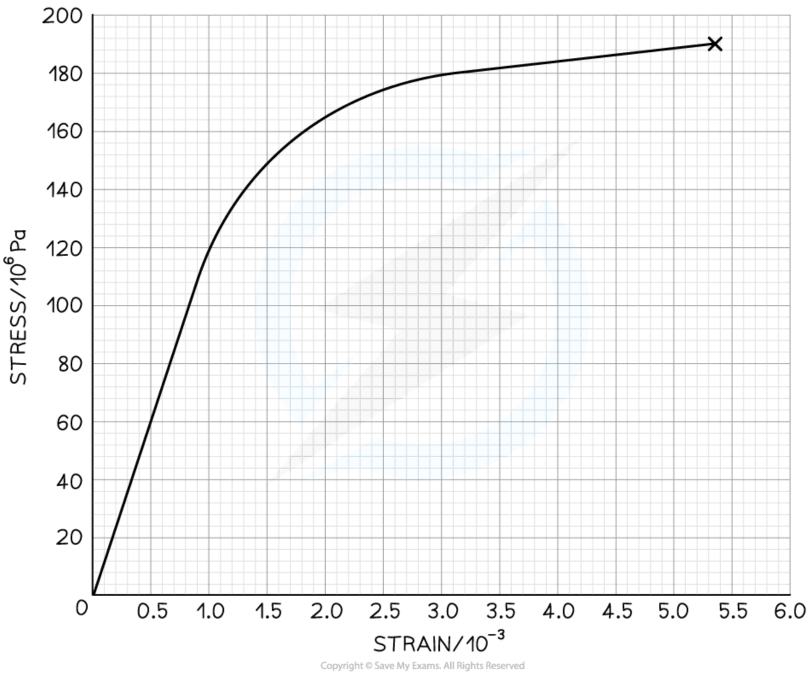
The important points shown on a stress-strain graph

The stress-strain graph is also used to find;

- The **Young Modulus** is found from the gradient of the straight part of the graph
- **Breaking stress** (also called fracture stress) is the stress at the point where the material breaks
  - At the yield point the atoms in the material had started to move relative to each other, at the breaking stress they separate completely
  - Breaking stress is not the same as ultimate tensile stress which is marked on many graphs

Worked Example

The graph below shows a stress-strain curve for a copper wire.



From the graph, state the value of:

(a) The breaking stress

(b) The stress at which plastic deformation begins

Answer:

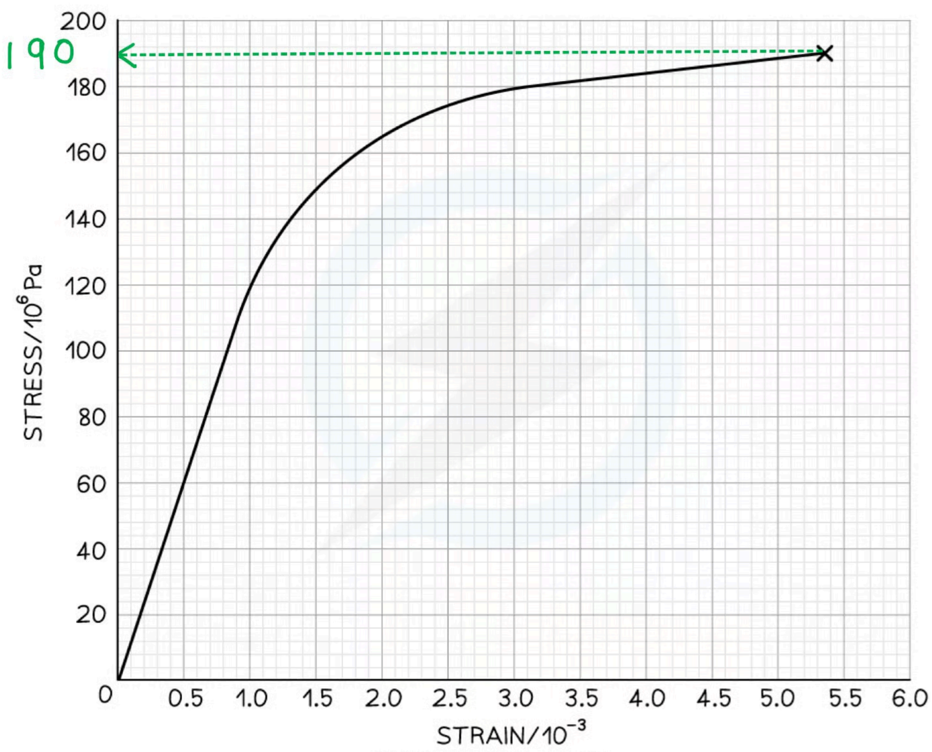
Part (a)

Step 1: Define breaking stress

- The breaking stress is the maximum stress a material can stand before it fractures. This is the stress at the final point on the graph

Step 2: Determine breaking stress from the graph

- Draw a line to the y axis at the point of fracture



The breaking stress is 190 MPa

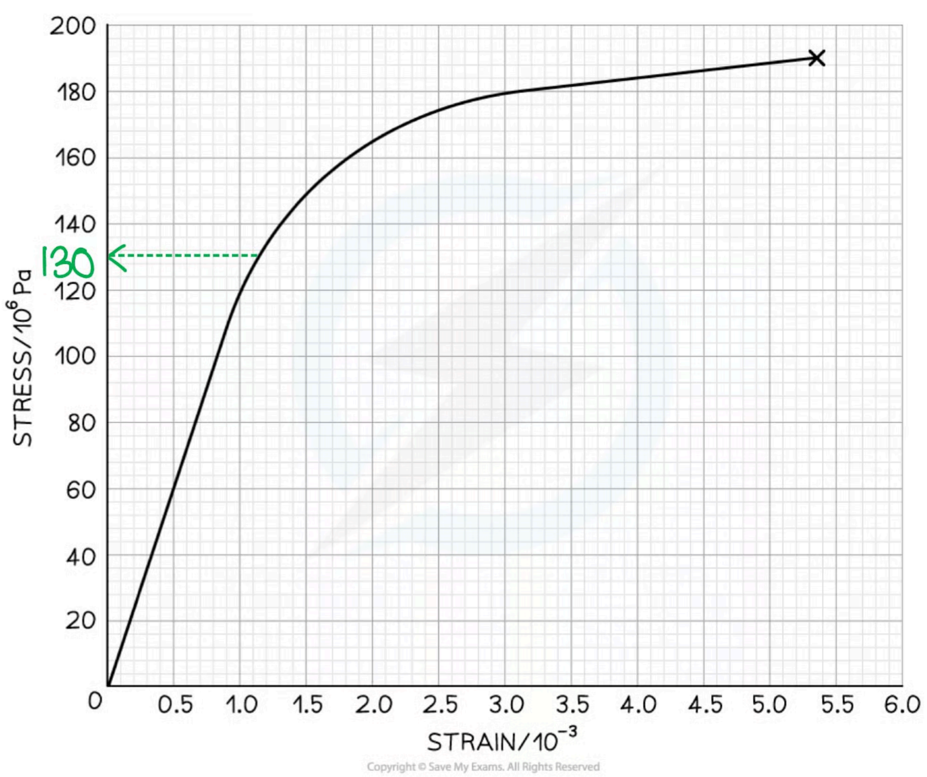
Part (b)

Step 1: Define plastic deformation

- Plastic deformation is when the material is deformed permanently and will not return to its original shape once the applied force is removed
- This is shown on the graph where it is curved

Step 2: Determine the stress of where plastic deformation beings on the graph

- Draw a line to the y axis at the point where the graph starts to curve



Plastic deformation begins at a stress of 130 MPa

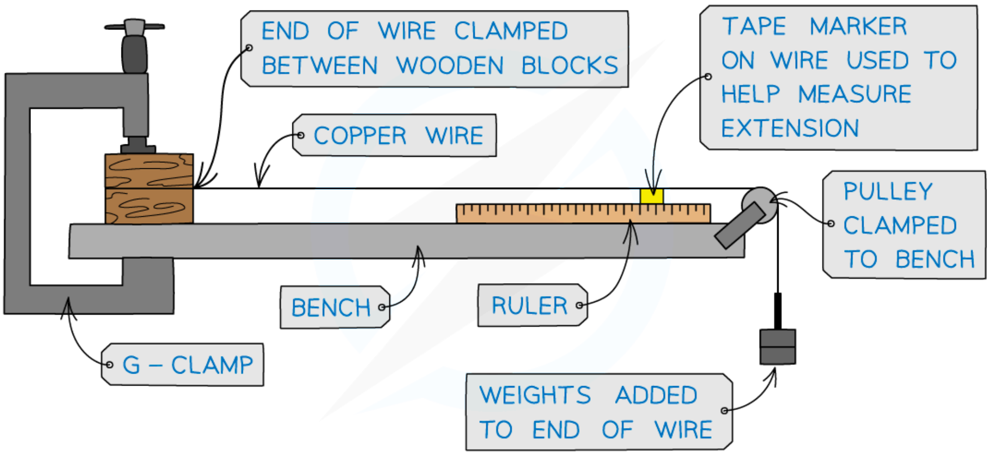
Examiner Tips and Tricks

It is a very common exam question to be asked to define some of the key points on this graph, or to identify them from a graph.

Make sure you have cleared up exactly what the differences are between the limit of proportionality, elastic limit and yield point. It's easy to get confused between them so practice sketching the graph, and labelling the points with their definitions.

Core Practical 3: Investigating Young Modulus of a Material

- To measure the Young Modulus of a metal in the form of a wire requires a clamped horizontal wire over a pulley (or vertical wire attached to the ceiling with a mass attached) as shown in the diagram below



- A reference marker is needed on the wire. This is used to accurately measure the extension with the applied load
- The independent variable is the **load**
- The dependent variable is the **extension**

Method

1. Measure the original length of the wire using a metre ruler and mark this reference point with tape
2. Measure the diameter of the wire with micrometer screw gauge or digital calipers
3. Measure or record the mass or weight used for the extension e.g. 300 g
4. Record initial reading on the ruler where the reference point is
5. Add mass and record the new scale reading from the metre ruler
6. Record final reading from the new position of the reference point on the ruler
7. Add another mass and repeat method

Your notes

Reducing Uncertainty

- To reduce the uncertainty in the final answer, take the following precautions when measuring
  - Take pairs of readings of the diameter right angles to each other, to ensure the wire is circular
  - Six to ten readings altogether is enough to get an average value
  - Remove the load and check the wire returns to the original limit after each reading. A little 'creep' is acceptable but a large amount indicates that the elastic limit has been exceeded
  - Take several readings with different loads and find average
  - Use a Vernier scale to measure the extension of the wire

Measurements to Determine the Young Modulus

1. Determine extension *x* from final and initial readings

Example table of results:

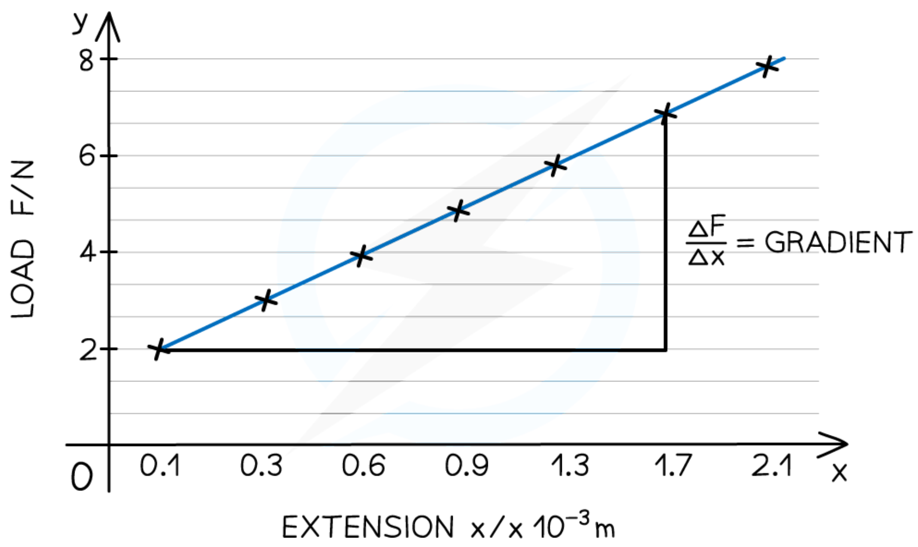
Mass m / g	Load F / N	Initial length / mm	Final length / mm	Extension x ($\times 10^{-3}$) / m
200	2.0	500	500.1	0.1
300	2.9	500.1	500.4	0.3
400	3.9	500.4	501.0	0.6
500	4.9	501.0	501.9	0.9
600	5.9	501.9	503.2	1.3
700	6.9	503.2	504.9	1.7
800	7.8	504.9	507.0	2.1

Table with additional data

Length l / m	1.382
Diameter 1 / mm	0.277
Diameter 2 / mm	0.280
Diameter 3 / mm	0.275
Average Diameter d / mm	0.277
Cross-sectional area A / m^2	6.03×10^{-8}

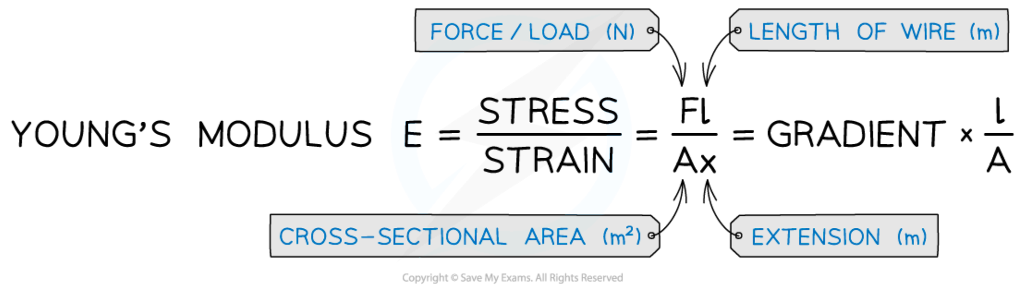
2. Plot a graph of force against extension and draw line of best fit

3. Determine gradient of the force v extension graph



4. Calculate cross-sectional area from:

5. Calculate the Young modulus from:



Safety Considerations

- Safety glasses should be worn in case of the wire snapping
- Protect feet and the floor from falling weights by cushioning the area underneath the weights

Examiner Tips and Tricks

Although every care should be taken to make the experiment as reliable as possible, you will be expected to suggest improvements in producing more accurate and reliable results

Good examples of improvements in any experiment are:

- Take repeat readings and take an average to improve accuracy
- Measure longer distances, such as using a longer length of wire, to reduce percentage error

Your notes

Area Under a Force-Extension Graph

- For a material which obeys Hooke's law, the elastic strain energy, E_{el} can be determined by finding the area under the force-extension graph
  - Since this area will be a triangle with sides *F* (force) and *x* (extension) the equation is:

$$\Delta E_{el} = \frac{1}{2} F \Delta x$$

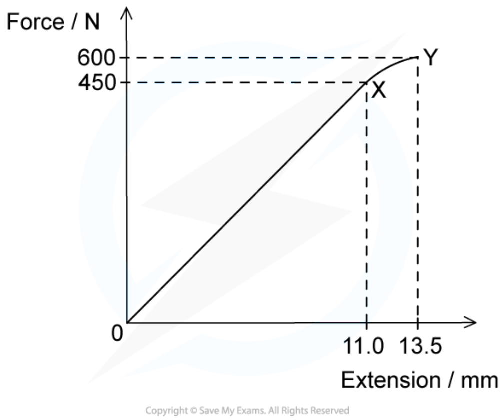
- Where:
  - E_{el} = elastic strain energy (or work done) (J)
  - F = average force (N)
  - Δx = extension (m)
- Since Hooke's Law states that $\boldsymbol{F = k\Delta x}$, the elastic strain energy can also be written as:

$$\Delta E_{el} = \frac{1}{2} k(\Delta x)^2$$

- Where:
  - k = spring constant (N m^{-1})

Worked Example

The graph shows the behaviour of a sample of a metal when it is stretched until it starts to undergo plastic deformation.



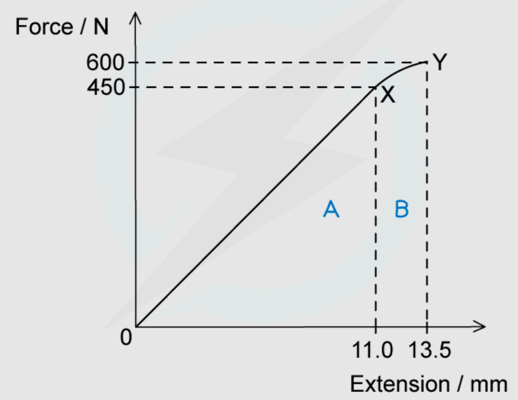
What is the total work done in stretching the sample from zero to 13.5 mm extension?

Simplify the calculation by treating the curve XY as a straight line.

Answer:

STEP 1: WORK DONE = AREA UNDER THE FORCE–EXTENSION GRAPH

STEP 2: SPLIT GRAPH INTO THE TWO AREAS



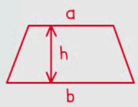
STEP 3: CALCULATE AREA A

AREA OF A RIGHT ANGLED TRIANGLE = $\frac{1}{2}$ × BASE × HEIGHT

AREA = $\frac{1}{2} \times 11 \times 10^{-3} \times 450 = 2.475$ J

STEP 4: CALCULATE AREA B

AREA OF TRAPEZIUM = $\left(\frac{a+b}{2}\right) \times h$



AREA = $\left(\frac{450 + 600}{2}\right) \times 2.5 \times 10^{-3} = 1.313$ J

STEP 5: TOTAL AREA = 2.475 + 1.313 = 3.79 J (3 s.f.)

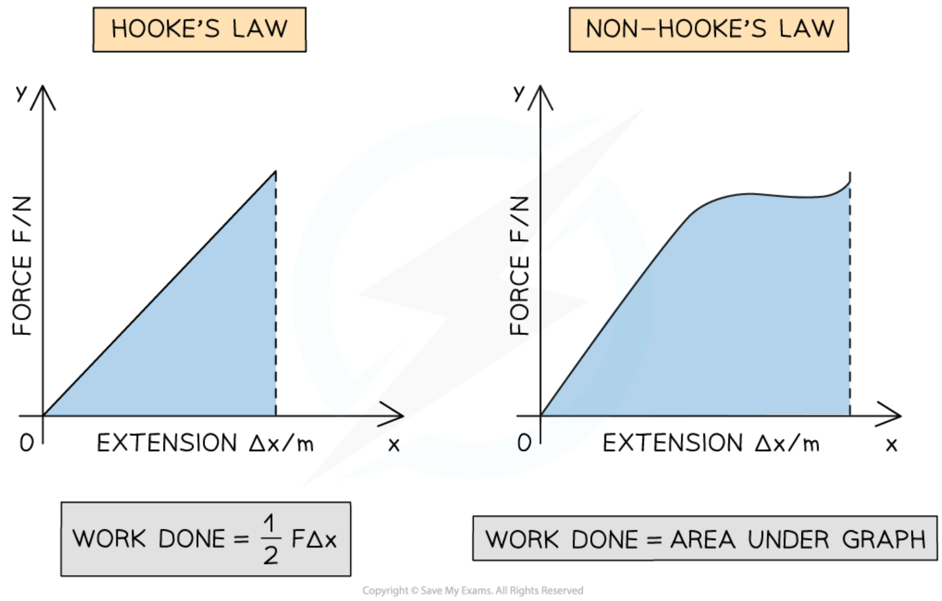
Examiner Tips and Tricks

Make sure that you are familiar with the area of common 2D shapes such as a square, rectangle, right-angled triangle and trapezium. Don't forgot to split the area of the graph up into these easier shapes and add up the area of each section for more complicated graphs! Always look at the units of the extension and the force, and check any unit conversions before giving your final answer.

Your notes

Elastic Strain Energy

- Work has to be done to stretch a material
- Before a material reaches its elastic limit (whilst it obeys Hooke's Law), all the work is done is stored as **elastic strain energy**
- The work done, or the elastic strain energy is the **area under the force-extension graph**



Work done is the area under the force-extension graph

- This is true for whether the material obeys Hooke's law or not

Linear Graphs

- For the region where the material **obeys** Hooke's law, the work done is the area of a **right-angled triangle** under the graph

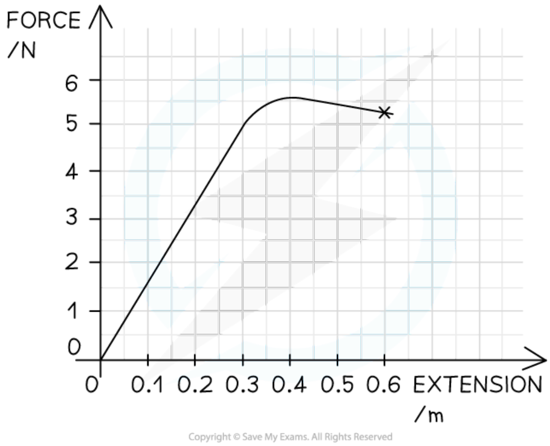
Non-linear Graphs

- For the region where the material **doesn't obey Hooke's law, the area is the full region** under the graph.
- To calculate this area, split the graph into separate segments and add up the individual areas of each
- For the remaining part, **count the squares** left over
  - Before adding squares to the total they must be converted using the values on the axes
  - For example, if each division on the y-axis = 0.1 N and each division on the x-axis = 0.2 m, then each square = 0.1 × 0.2 N m = 0.02 N m

Your notes

Worked Example

For the force extension graph shown, determine the work done in stretching the material.

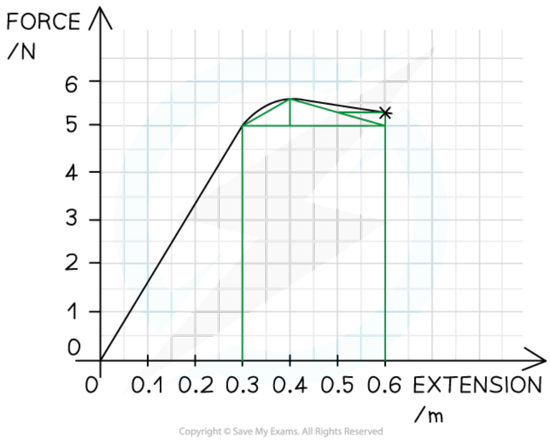


Answer:

Step 1: Choose which aspect of the graph will give the answer

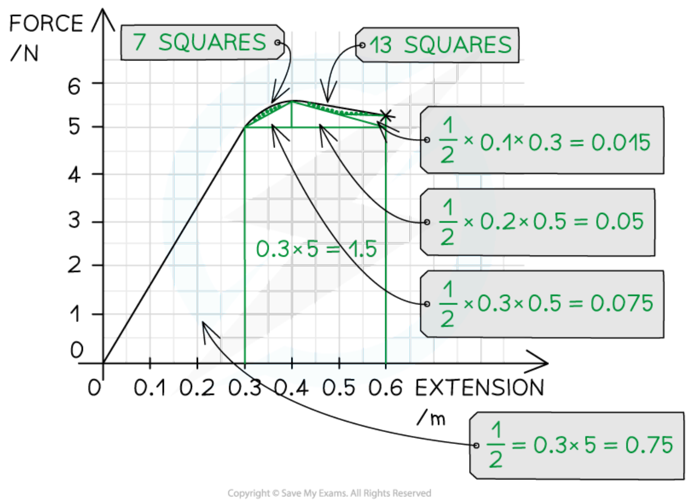
- This is a force-extension graph,
- Since work done = force × distance, find the area under the graph

Step 2: Divide the area under the line into geometric shapes, using as much of the space as possible



Step 3: For each shape calculate the area and find the total

Total of geometric shapes = 0.025 + 0.05 + 0.75 + 1.5 +0.015 = 2.34



Step 4: For the remaining area count squares then convert

- Total of counted squares

$$7 + 13 = 20$$

- Value of each small square

$$0.1\,\text{N} \times 0.01\,\text{m} = 0.001\,\text{N m}$$

- Multiply to find value of area of counted squares

$$20 \times 0.001 = 0.02$$

Step 5: Add the totals together to find the area

$$2.34 + 0.02 = 2.36$$

Step 6: Write the final answer to the same significant figures as the data from the graph and include units

- Work done stretching the material = 2.4 J

Examiner Tips and Tricks

As you add up your final totals you may find that the value of the left over squares is more than the rest of the graph. This means you have forgotten to convert the number of squares into their actual value. Go back and do that step and your answer will come out right!

Your notes