Edexcel International A Level (IAL) Physics

Your notes

Simple Harmonic Motion

Contents

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Conditions for Simple Harmonic Motion

- **Simple harmonic motion (SHM)** is a specific type of oscillation
- An oscillation is said to be SHM when:
  - **The acceleration is proportional to the displacement**
  - **The acceleration is in the opposite direction to the displacement**

- Examples of oscillators that undergo SHM are:
  - The pendulum of a clock
  - A mass on a spring
  - Guitar strings
  - The electrons in alternating current flowing through a wire

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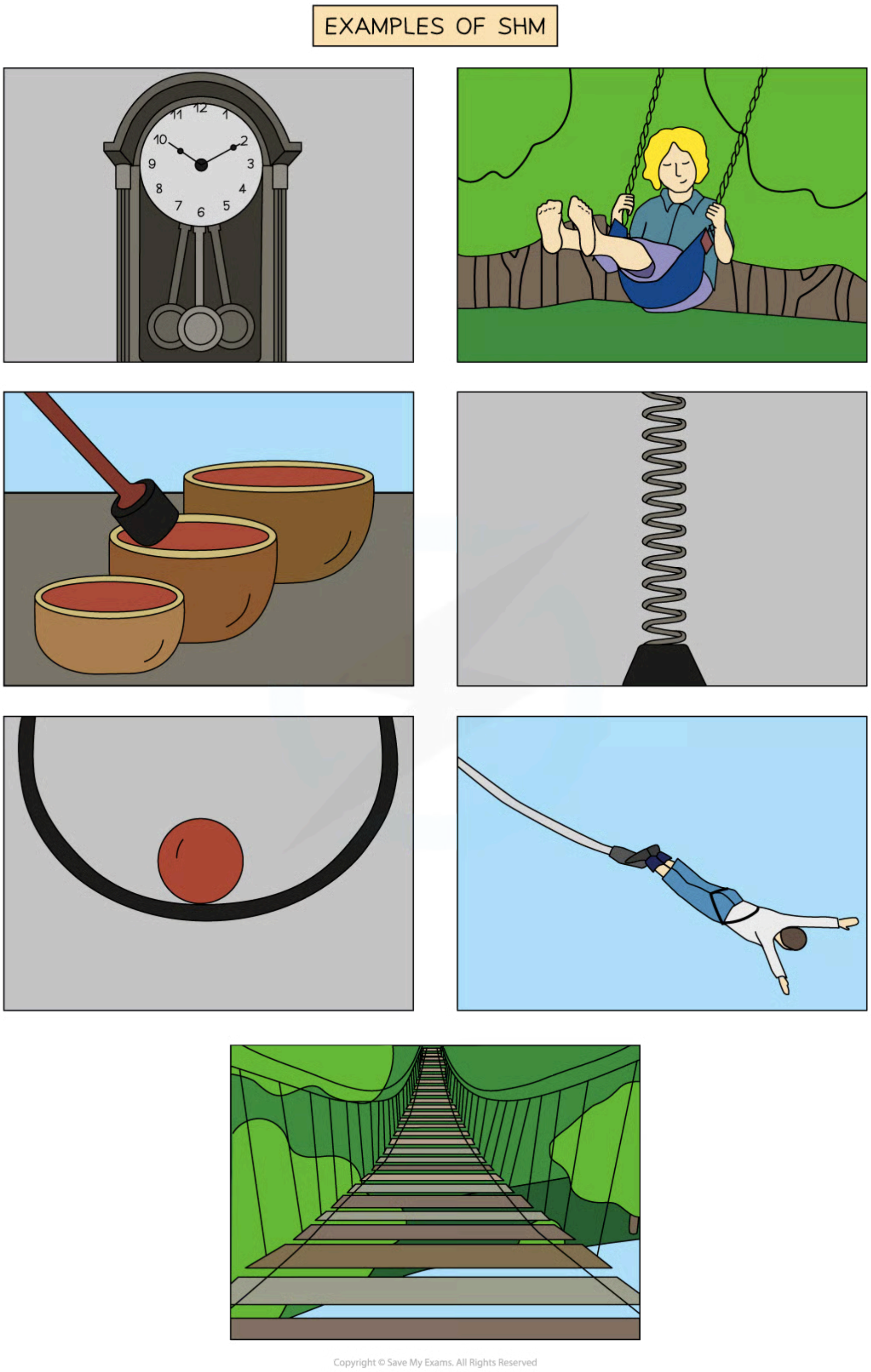

- **Time period, *T*:**
  - The objects swings are periodic, meaning they are repeated in regular intervals according to their frequency or time period
  - If an object swings freely it always takes the same time to complete one swing

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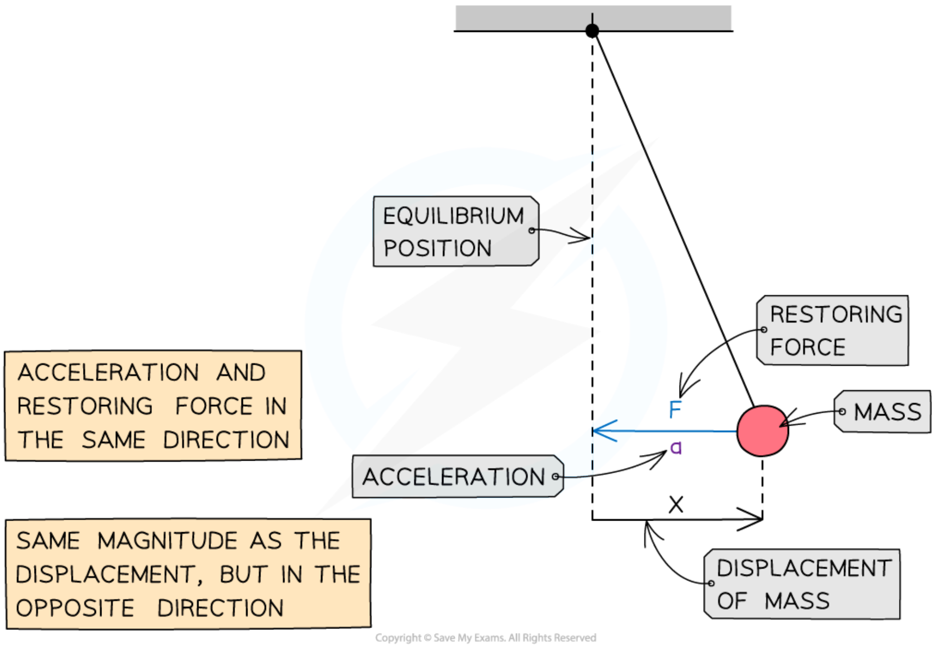
Restoring force

- When an object is moving in SHM a force, called the restoring force, *F*, is always trying to return the object back to its equilibrium position.
- The force is proportional to the displacement, *x*, from that equilibrium position

$$F = -kx$$

- Where:
  - *F* is the restoring force
  - *x* is the displacement of the object from the equilibrium position
  - *k* is a constant depending on the system
  - the negative sign shows that the acceleration will always be towards the centre of oscillation



Force, acceleration and displacement of a pendulum in SHM

- This is why a person jumping on a trampoline is not an example of simple harmonic motion:
  - The restoring force on the person is **not** proportional to their distance from the equilibrium position
  - When the person is not in contact with the trampoline, the restoring force is equal to their weight, which is constant
  - This does not change, even if they jump higher

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Worked Example

A 200g toy robot is attached to a pole by a spring, with a spring constant of 90 N m^{-1}, and made to oscillate horizontally.

(a) What force will act on the robot when it is at its amplitude position of 5 cm from equilibrium?

(b) How fast will the robot accelerate whilst at this amplitude position?

Answer:

Part (a)

Step 1: Convert amplitude into m

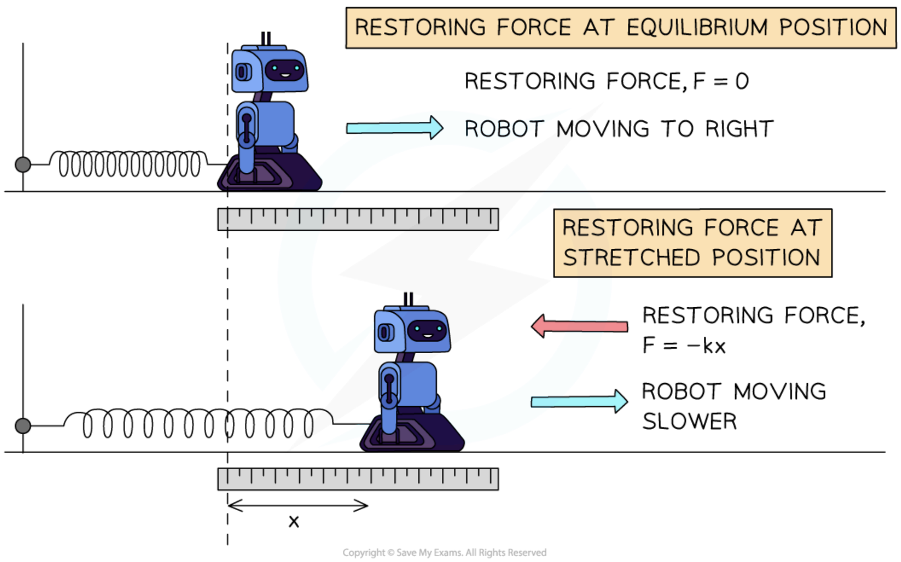
$$5 \text{ cm} = 0.05 \text{ m}$$

Step 2: Substitute values into the restoring force equation

$$F = -kx = -(90) \times (0.05) = -4.5 \text{ N}$$

Step 3: Explain the answer

A force of 4.5 newtons will act on the robot, trying to pull it back towards the equilibrium position.



Part (b)

Step 1: Convert mass of robot into kg

$$200 \text{ g} = 0.2 \text{ kg}$$

Step 2: Substitute values into Newton's second law equation:

$$F = ma$$

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So, $a = \frac{F}{m} = \frac{-4.5}{0.2}$ = -22.5 m s^{-2}

Step 3: Explain the answer

The robot will decelerate at a rate of 22.5 m s^{-2} when at this amplitude position

Examiner Tips and Tricks

Even with this topic you must make sure you convert all quantities into standard SI units

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Equations for Simple Harmonic Motion

Acceleration and SHM

- Acceleration a and displacement x can be represented by the defining equation of SHM:

$$a \propto -x$$

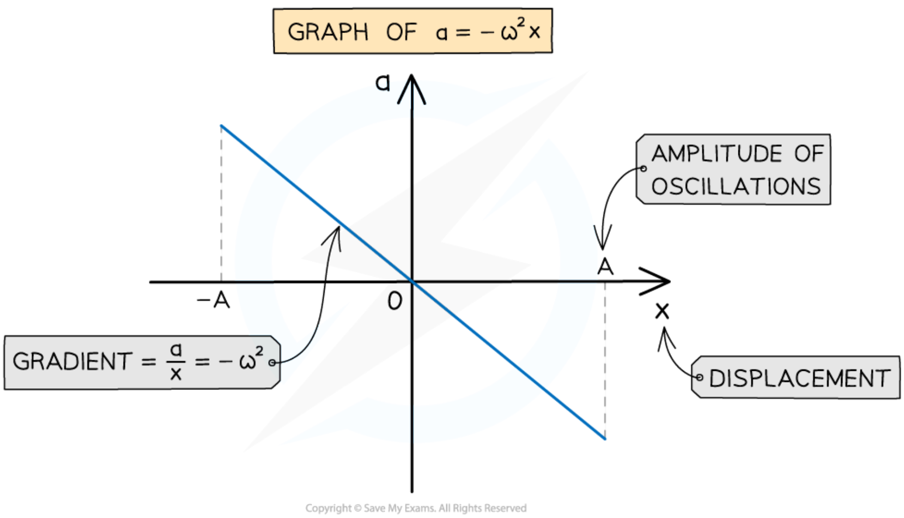
- The acceleration of an object oscillating in simple harmonic motion is:

$$a = -\omega^2 x$$

- Where:
  - a = acceleration (m s^{-2})
  - ω = angular frequency (rad s^{-1})
  - x = displacement (m)
- This is used to find the acceleration of an object with a particular angular frequency ω at a specific displacement x
- The equation demonstrates:
  - The acceleration reaches its **maximum** value when the displacement is at a **maximum** i.e.. x = A (amplitude)
  - The **minus** sign shows that when the object is displaced to the **right**, the direction of the acceleration is to the **left** and vice versa (a and x are always in opposite directions to each other)

Displacement and SHM

- The graph of acceleration against displacement is a straight line through the origin sloping downwards (similar to y = –x)



The acceleration of an object in SHM is directly proportional to the negative displacement

- The key features of the graph are:
  - The gradient is equal to $-\omega^2$
  - The maximum and minimum displacement x values are the amplitudes $-A$ and $+A$
- A solution to the SHM acceleration equation is the displacement equation:

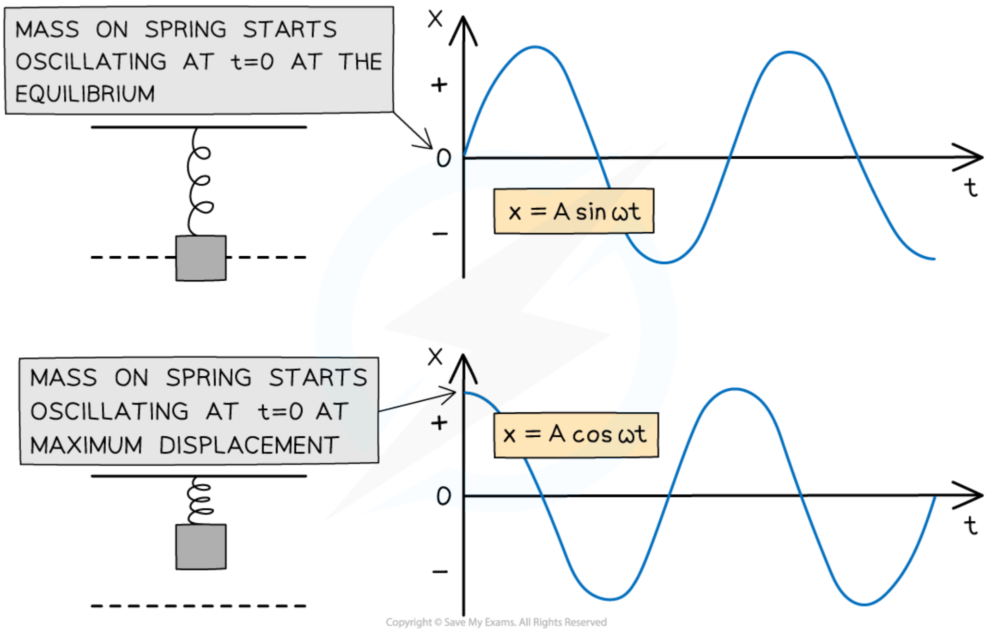
$$x = A\cos(\omega t)$$

- Where:
  - A = amplitude (m)
  - t = time (s)

- This occurs when:
  - An object is oscillating from its amplitude position ($x = A$ or $x = -A$ at $t = 0$)
  - The displacement will be at its maximum when $\cos(\omega t)$ equals 1 or −1, when $x = A$
- This equation can be used to find the position of an object in SHM with a particular angular frequency and amplitude at a moment in time
- If an object is oscillating from its equilibrium position ($x = 0$ at $t = 0$) then the displacement equation will be:

$$x = A\sin(\omega t)$$

- The displacement will be at its maximum when $\sin(\omega t)$ equals 1 or −1, when $x = A$
- This is because the sine graph starts at 0, whereas the cosine graph starts at a maximum



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These two graphs represent the same SHM. The difference is the starting position

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Worked Example

A mass of 55 g is suspended from a fixed point by means of a spring.

The stationary mass is pulled vertically downwards through a distance of 4.3 cm and then released at t = 0.

The mass is observed to perform simple harmonic motion with a period of 0.8 s.

Calculate the displacement x, in cm, of the mass at time t = 0.3 s.

Answer:

Step 1: Write down the SHM displacement equation

Since the mass is released at t = 0 at its maximum displacement, the displacement equation will be with the cosine function:

$$x = A\cos(\omega t)$$

Step 2: Calculate angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 7.85\ rad\ s^{-1}$$

Remember to use the value of the time period given, not the time passed

Step 3: Substitute values into the displacement equation

$$x = 4.3\cos(7.85 \times 0.3) = -3.0369... = -3.0 \text{ cm (2 s.f)}$$

Make sure the calculator is in **radians mode**

The negative value means the mass is 3.0 cm on the opposite side of the equilibrium position to where it started (3.0 cm above it)

Speed and SHM

- The speed of an object in simple harmonic motion varies as it oscillates back and forth
  - Its speed is the magnitude of its velocity
- The greatest speed of an oscillator is at the equilibrium position ie. when its displacement *x* = 0
- How the speed *v* changes with the oscillator's displacement *x* in SHM is defined by:

$$v = \pm\omega\sqrt{(A^2 - x^2)}$$

- Where:
  - *v* = speed ($m\ s^{-1}$)

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- A = amplitude (m)
- $\pm$ = 'plus or minus'. The value can be negative or positive
- ω = angular frequency (rad s^{-1})
- x = displacement (m)

- This equation shows that when an oscillator has a greater amplitude A, it has to travel a greater distance in the same time and hence has greater speed v
- Although the symbol v is commonly used to represent velocity, not speed, exam questions focus more on the magnitude of the velocity than its direction in SHM

Worked Example

A simple pendulum oscillates with simple harmonic motion with an amplitude of 15 cm.

The frequency of the oscillations is 6.7 Hz.

Calculate the speed of the pendulum at a position of 12 cm from the equilibrium position.

Answer:

Step 1: Write out the known quantities

- Amplitude of oscillations, A = 15 cm = 0.15 m
- Displacement at which the speed is to be found, x = 12 cm = 0.12 m
- Frequency, f = 6.7 Hz

Step 2: Oscillator speed with displacement equation

$$v = \pm\,\omega\sqrt{(A^2 - x^2)}$$

- Since the speed is being calculated, the $\pm$ sign can be removed as direction does not matter in this case

Step 3: Write an expression for the angular frequency

- Equation relating angular frequency and normal frequency:

$$\omega = 2\pi f = 2\pi \times 6.7 = 42.097...$$

Step 4: Substitute in values and calculate

$$v = (2\pi \times 6 \cdot 7) \times \sqrt{(0 \cdot 15)^2 - (0 \cdot 12)^2}$$

$v = 3.789 = 3.8$ m s^{-1} (2 s.f)

Examiner Tips and Tricks

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Since displacement is a vector quantity, remember to keep the minus sign in your solutions if they are negative, you could lose a mark if not! Also, remember that your calculator must be in **radians** mode when using the cosine and sine functions. This is because the angular frequency ω is calculated in rad s^{-1}, **not** degrees. You often have to convert between time period T, frequency f and angular frequency ω for many exam questions – so make sure you revise the equations relating to these.

Your notes

Period of a simple pendulum

- A simple pendulum is:
  - an object moving from side to side
  - attached to a string which is fixed to a point above
- The period of a simple pendulum can be calculated using the equation:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

- Where:
  - l = the length of the pendulum
  - g = the gravitational field strength on the planet on which the pendulum is set up

Worked Example

A child is sitting on a swing that is 200 cm long. What is the period of oscillation?

Answer:

Step 1: Convert length to meters

200 cm = 2 m

Step 2: Substitute the correct values

$$T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{2}{9.81}} = 2.84 \text{ s}$$

Step 3: Confirm the answer

The time period of 1 oscillation of the swing is 2.84 s

Period of a Mass-Spring System

- A mass-spring system is:
  - an object moving up and down, or side to side
  - attached to the end of a spring

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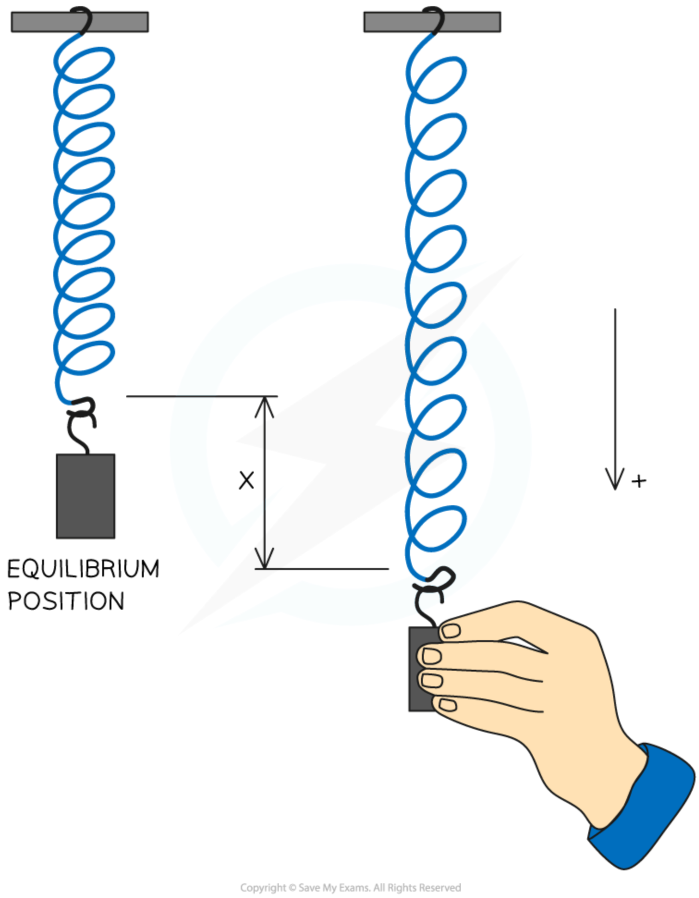


- The equation for the restoring force in SHM is the same as the equation for **Hooke's law**

$$F = -kx$$

- The **time period *T*** can be calculated using the equation:

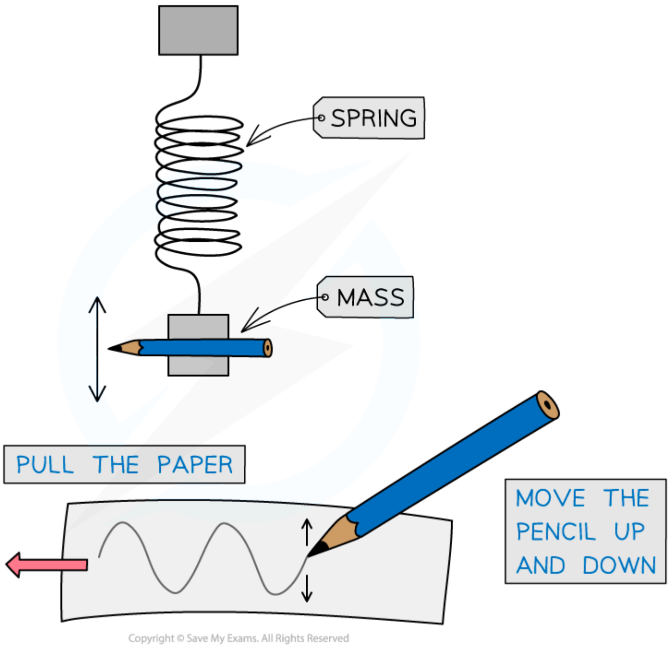
$$T = 2\pi\sqrt{\frac{m}{k}}$$

- Where:
  - m = the mass of the object on the end of the spring
  - k = the spring constant of the spring

Observing the Motion of a Mass-Spring System

- An experimental and graphical method can be used to observe the motion of a simple mass-spring system
  - Tie a pencil together with the mass and set the mass in free oscillations by displacing it downwards slightly
- The oscillations will move the pencil up and down

- On a piece of graph paper, allow the pencil to trace the path of the oscillations by pulling the paper sideways as the mass-spring system oscillates up and down
- The oscillations will produce a curved, periodic graph
  - This will decrease in amplitude as the mass-spring system slows down



The motion of an oscillator can be observed through a simple mass-spring system

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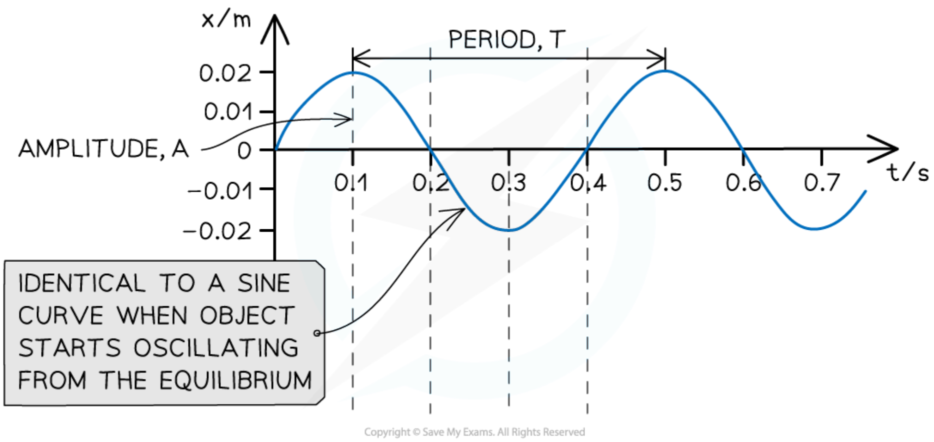
Displacement-Time graph for an oscillator

- The displacement of an object in simple harmonic motion can be represented by a graph of displacement against time
- All undamped SHM graphs are represented by **periodic functions**
  - This means they can all be described by sine and cosine curves



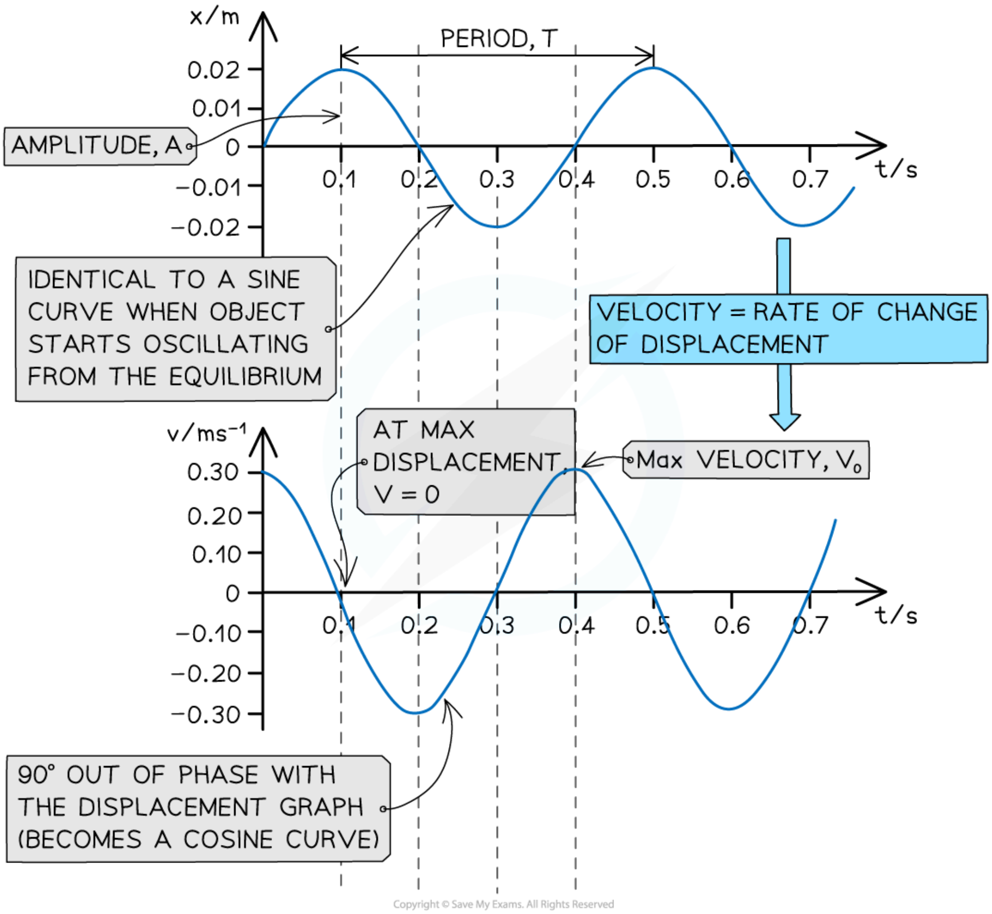
- **Key features of the displacement-time graph:**
  - The amplitude of oscillations *A* can be found from the maximum value of *x*
  - The time period of oscillations *T* can be found from reading the time taken for one full cycle
  - The graph might not always start at 0
  - If the oscillations starts at the positive or negative amplitude, the displacement will be at its maximum

Examiner Tips and Tricks

This graph might not look identical to what is in your textbook, depending on where the object starts oscillating from at t = 0 (on either side of the equilibrium, or at the equilibrium). However, if there is no damping, they will all always be a general sine or cosine curve.

Velocity-Time Graph for an Oscillator

- The velocity of an object in simple harmonic motion can be represented by a graph of velocity against time



- **Key features of the velocity-time graph:**
  - It is 90° out of phase with the displacement-time graph
  - Velocity is equal to the rate of change of displacement
  - So, the velocity of an oscillator at any time can be determined from the **gradient of the displacement-time graph:**

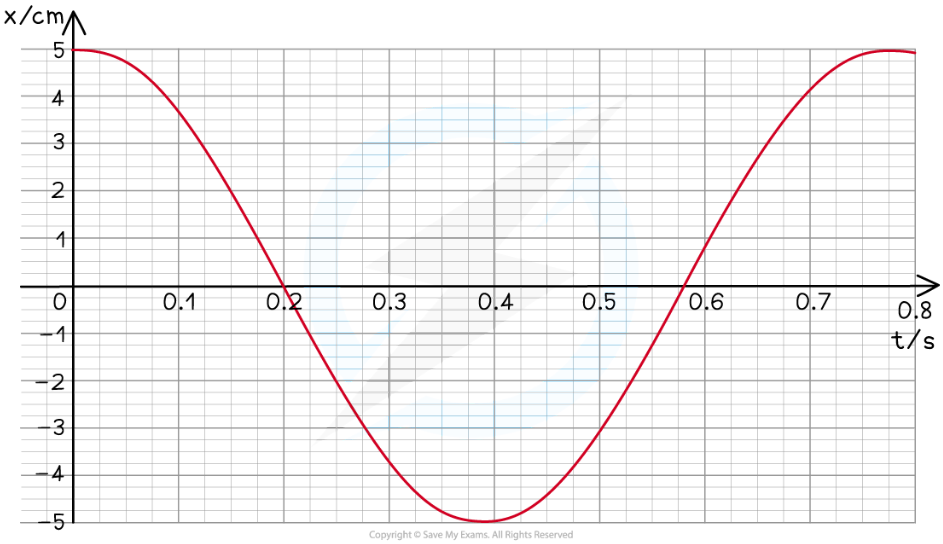
$$v = \frac{\Delta x}{\Delta t}$$

- An oscillator moves the fastest at its equilibrium position
  - Therefore, the velocity is at its **maximum** when the **displacement is zero**

Worked Example

A swing is pulled 5 cm and then released.

The variation of the horizontal displacement *x* of the swing with time t is shown on the graph below.



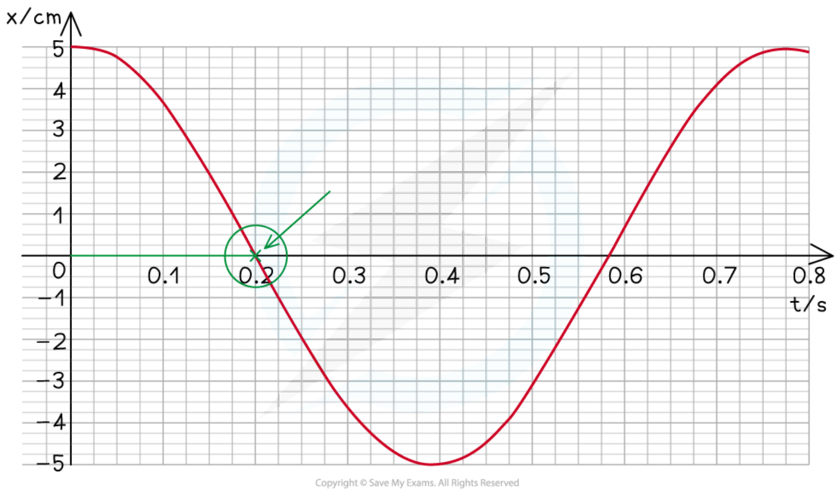
The swing exhibits simple harmonic motion.

Use data from the graph to determine at what time the velocity of the swing is first at its maximum.

Answer:

Step 1: The velocity is at its maximum when the displacement x = 0

Step 2: Reading value of time when x = 0



From the graph this is equal to 0.2 s

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Examiner Tips and Tricks

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