Edexcel International A Level (IAL) Physics

Your notes

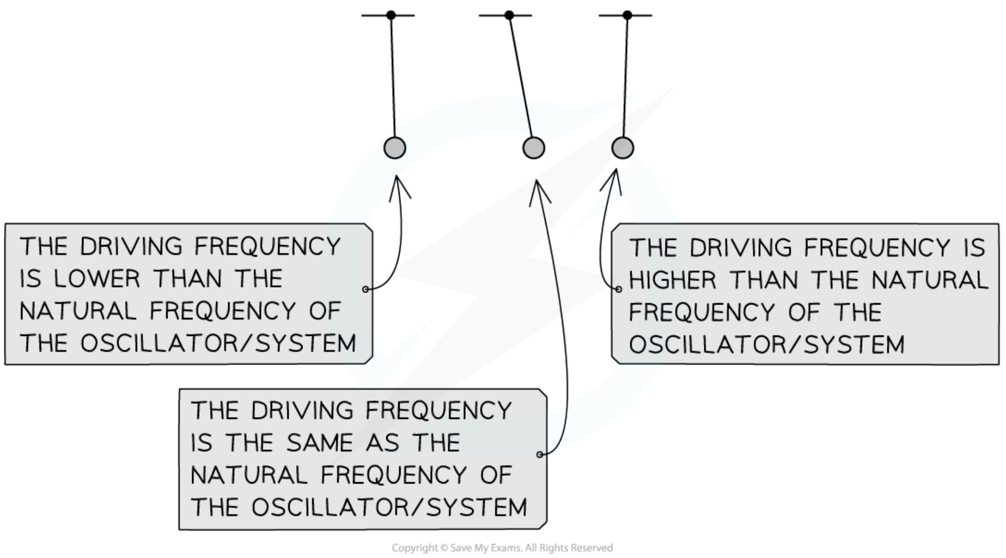
Resonance

Contents

Your notes

Resonance

- The frequency of forced oscillations is referred to as the **driving frequency**, f, or the frequency of the applied force
- All oscillating systems have a **natural frequency**, f_0, this is defined as the frequency of an oscillation when the oscillating system is allowed to oscillate freely
  - Oscillating systems can exhibit a property known as resonance



- When the **driving frequency** approaches the **natural frequency** of an oscillator, the system gains more energy from the driving force
  - Eventually, when they are equal, the oscillator vibrates with its maximum amplitude, this is **resonance**
- Resonance is defined as:

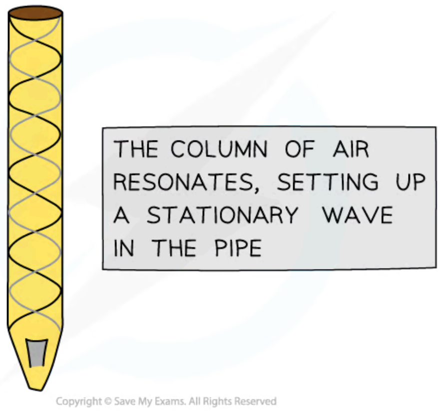
  When the frequency of the applied force to an oscillating system is equal to its natural frequency, the amplitude of the resulting oscillations increases significantly
- For example, when a child is pushed on a swing:
  - The swing plus the child has a fixed natural frequency
  - A small push after each cycle increases the amplitude of the oscillations to swing the child higher. This frequency at which this push happens is the driving frequency
  - When the driving frequency is exactly equal to the natural frequency of the swing oscillations, resonance occurs

- If the driving frequency does not quite match the natural frequency, the amplitude will increase but not to the same extent as when resonance is achieved

- This is because, at resonance, energy is transferred from the driver to the oscillating system **most efficiently**
  - Therefore, at resonance, the system will be transferring the maximum kinetic energy possible

Resonance Effects

- Resonance occurs for any forced oscillation where the frequency of the driving force is equal to the natural frequency of the oscillator
- Examples include:
  - An organ pipe, where air resonates down an air column setting up a stationary wave in the pipe
  - Glass smashing from a high pitched sound wave at the right frequency
  - A radio tuned so that the electric circuit resonates at the same frequency as the specific broadcast



Standing waves forming inside an organ pipe from resonance

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Core Practical 16: Investigating Resonance

Aim of the Experiment

- Determine the value of an unknown mass by a graphical method by using the resonant frequencies of the oscillation of known masses

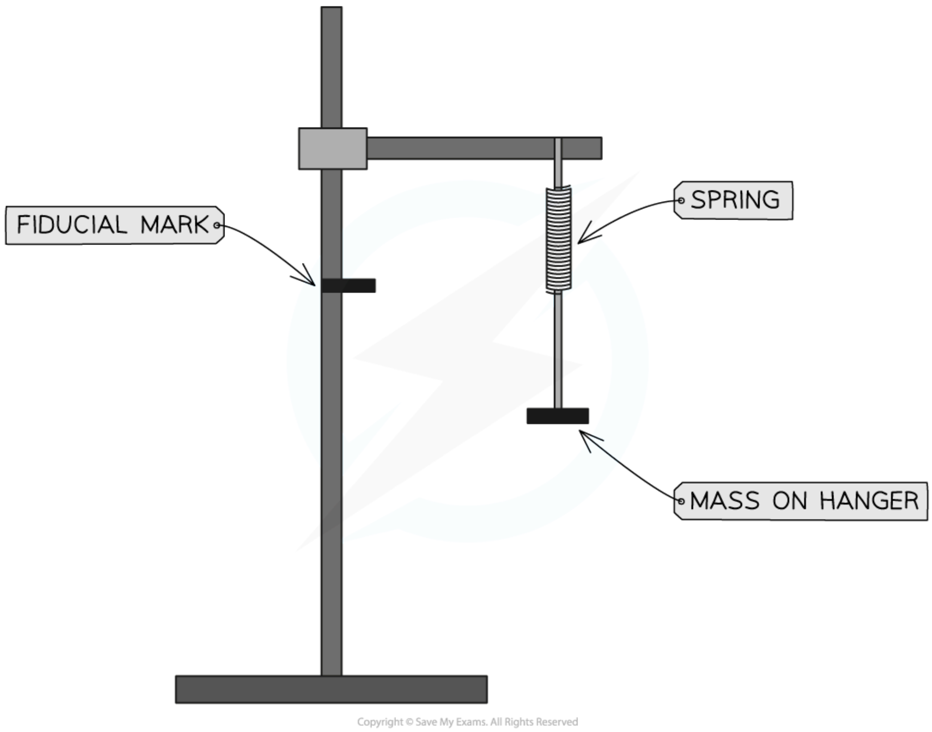
Variables

- Independent variable = mass (kg)
- Dependent variable = time period (s)
- Control variables:
  - The spring / oscillator

Equipment

- Spring (standard 20–25 mm spring)
- Slotted 100g masses and hanger
- Retort stand and clamp
- Digital timer
- Unknown test mass
- Digital scales

Method



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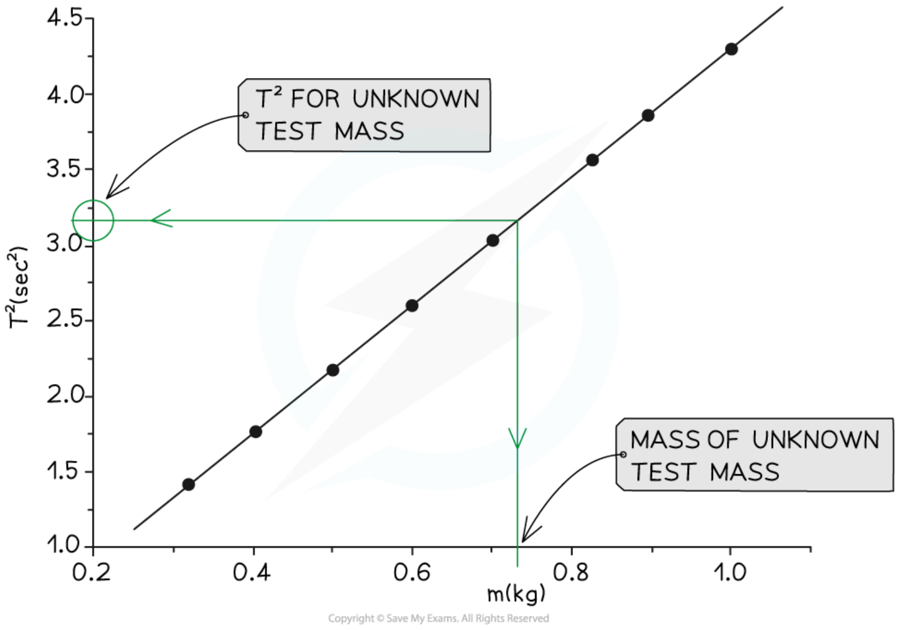
1. Set up the spring with 100 g mass attached
2. On the stand make a clear fiducial mark about 5 cm below the bottom of the spring
3. Extend the spring so that the bottom is level with the fiducial marker, release and start timing
4. Measure time for 10 oscillations
5. Repeat with the same mass two more times
6. Find the average time period of one oscillation
7. Add 100 g and adjust the fiducial mark downwards so that it is 5 cm below the new level of the spring
8. Repeat steps 3–7 until the total mass is 500 g
9. Plot a graph of T^2 on the y-axis against m on the x-axis

Testing the unknown mass

- Follow steps 2 - 6 for the test mass
- Find the value of the time period, T and square it to find T^2
- On the graph mark a horizontal from T^2 to the graph line and where they intersect, take the arrow vertically down to meet the x-axis
- The value of m which this line coincides with is the mass of the test mass
- Check the result using digital scales



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Analysis

- Analysis for this graph is based on three equations related to simple harmonic motion;
  - Angular velocity, $\omega = \sqrt{\frac{k}{m}}$ **(equation 1)**
  - Where k = spring constant (N kg^{-1}) and m = mass (kg)
  - Angular velocity, $\omega = 2\pi f$ **(equation 2)**
  - Where f = frequency of oscillations (Hz)
  - Frequency, $f = \frac{1}{T}$ **(equation 3)**
  - Where T = time period for one oscillation (s)
- Substitute **equations 2** into **equation 1**;

$$2\pi f = \sqrt{\frac{k}{m}}$$

- Substitute **equation 3** into **equation 2**

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

- Square both sides

$$\frac{4\pi^2}{T^2} = \frac{k}{m}$$

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- Make T^2 the subject

$$T^2 = m\left(\frac{4\pi^2}{k}\right)$$

- Plot a graph of T^2 on the y-axis against m on the x-axis to get a straight line through the origin with;

$$\text{gradient} = \left(\frac{4\pi^2}{k}\right)$$

Safety Considerations

- Clamp stand to the desk for stability
- Wear safety glasses in case the spring flies off or snaps
- Place a cushion or catch-mat in case of falling masses

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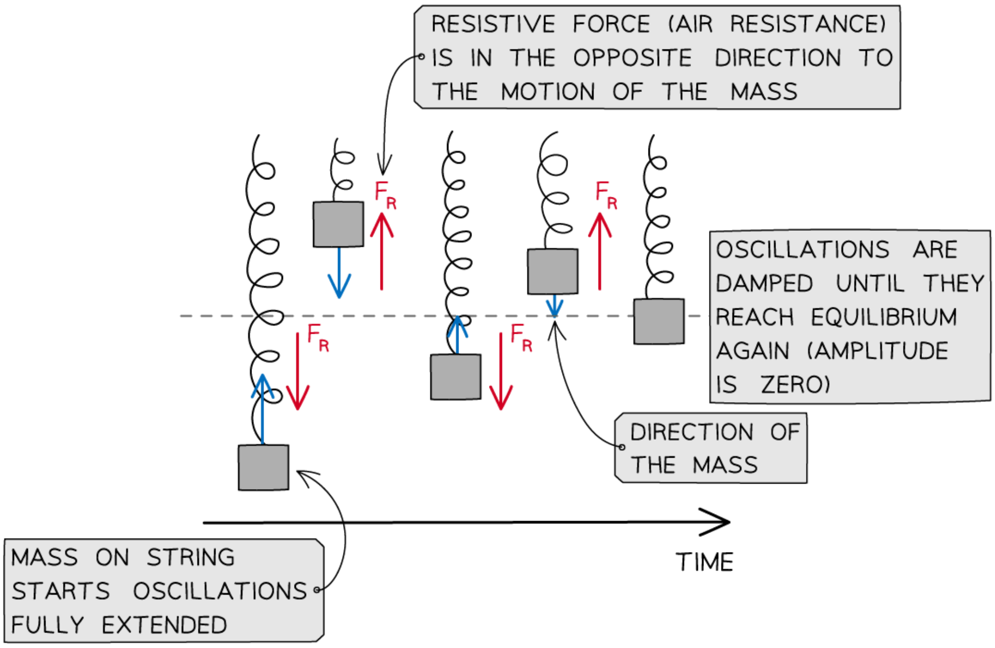
Damped & Undamped Oscillating Systems

- In practice, all oscillators eventually stop oscillating
  - Their amplitudes decrease rapidly, or gradually
- This happens due to **resistive forces**, such friction or air resistance, which act in the opposite direction to the motion of an oscillator
- Resistive forces acting on an oscillating simple harmonic system cause **damping**
  - These are known as **damped** oscillations
- Damping is defined as:

The reduction in energy and amplitude of oscillations due to resistive forces on the oscillating system

- Damping continues until the oscillator comes to rest at the equilibrium position
- A key feature of simple harmonic motion is that the **frequency** of damped oscillations **does not change** as the amplitude decreases
  - For example, a child on a swing can oscillate back and forth once every second, but this time remains the same regardless of the amplitude



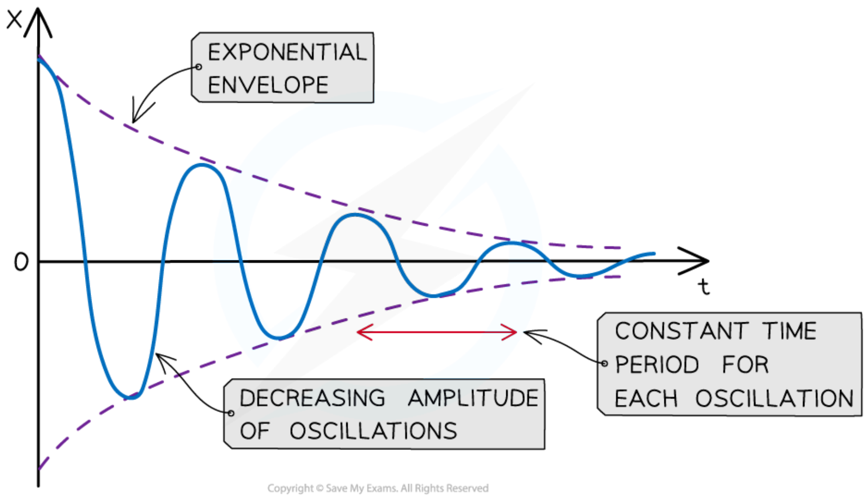
Damping on a mass on a spring is caused by a resistive force acting in the opposite direction to the motion. This continues until the amplitude of the oscillations reaches zero

Types of Damping

- There are three degrees of damping depending on how quickly the amplitude of the oscillations decrease:
  - Light damping
  - Critical damping
  - Heavy damping

Light Damping

- When oscillations are lightly damped, the amplitude does not decrease linearly
  - It decays exponentially with time
- When a lightly damped oscillator is displaced from the equilibrium, it will oscillate with gradually decreasing amplitude
  - For example, a swinging pendulum decreasing in amplitude until it comes to a stop



A graph for a lightly damped system consists of oscillations decreasing exponentially

- **Key features of a displacement-time graph for a lightly damped system:**
  - There are many oscillations represented by a sine or cosine curve with gradually decreasing amplitude over time
  - This is shown by the height of the curve decreasing in both the positive and negative displacement values
  - The amplitude decreases exponentially
  - The frequency of the oscillations remain constant, this means the time period of oscillations must stay the same and each peak and trough is equally spaced

Critical Damping

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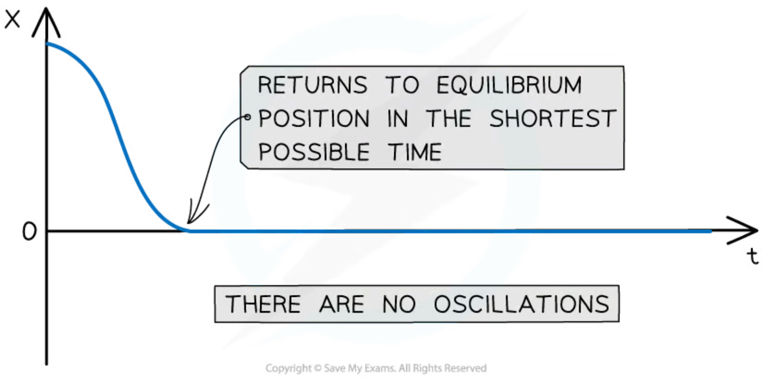
- When a critically damped oscillator is displaced from the equilibrium, it will return to rest at its equilibrium position in the shortest possible time **without** oscillating
    - For example, car suspension systems prevent the car from oscillating after travelling over a bump in the road

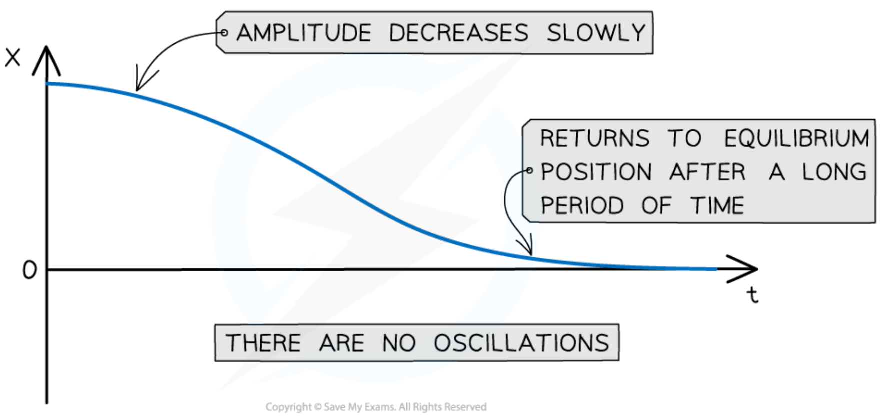


The graph for a critically damped system shows no oscillations and the displacement returns to zero in the quickest possible time

- **Key features of a displacement-time graph for a critically damped system:**
    - This system does **not** oscillate, meaning the displacement falls to 0 straight away
    - The graph has a fast decreasing gradient when the oscillator is first displaced until it reaches the x axis
    - When the oscillator reaches the equilibrium position (x = 0), the graph is a horizontal line at x = 0 for the remaining time

Heavy Damping

- When a heavily damped oscillator is displaced from the equilibrium, it will take a long time to return to its equilibrium position **without** oscillating
- The system returns to equilibrium more slowly than the critical damping case
    - For example, door dampers to prevent them from slamming shut



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A heavy damping curve has no oscillations and the displacement returns to zero after a long period of time

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- **Key features of a displacement-time graph for a heavily damped system:**
  - There are no oscillations. This means the displacement does not pass 0
  - The graph has a slow decreasing gradient from when the oscillator is first displaced until it reaches the x axis
  - The oscillator reaches the equilibrium position (x = 0) after a long period of time, after which the graph remains a horizontal line for the remaining time

Worked Example

A mechanical weighing scale consists of a needle which moves to a position on a numerical scale depending on the weight applied. Sometimes, the needle moves to the equilibrium position after oscillating slightly, making it difficult to read the number on the scale to which it is pointing to. Suggest, with a reason, whether light, critical or heavy damping should be applied to the mechanical weighing scale to read the scale more easily.

Answer:

- Ideally, the needle should not oscillate before settling
  - This means the scale should have either **critical** or **heavy damping**
- Since the scale is read straight away after a weight is applied, ideally the needle should settle as quickly as possible
- Heavy damping would mean the needle will take some time to settle on the scale
- Therefore, **critical damping** should be applied to the weighing scale so the **needle can settle as quickly as possible** to read from the scale

Examiner Tips and Tricks

Make sure not to confuse **resistive** force and **restoring** force:

- Resistive force is what **opposes the motion** of the oscillator and causes damping
- Restoring force is what brings the oscillator **back to the equilibrium position**

Free & Forced Oscillations

Free Oscillations

- Free oscillations occur when there is no transfer of energy to or from the surroundings
  - This happens when an oscillating system is displaced and then left to oscillate
- In practice, this only happens in a vacuum. However, anything vibrating in air is still considered a free vibration as long as there are **no external forces** acting upon it
- Therefore, a **free oscillation** is defined as:

  An oscillation where there are only internal forces (and no external forces) acting and there is no energy input
- A free vibration always oscillates at its **resonant frequency**

Forced Oscillations

- In order to sustain oscillations in a simple harmonic system, a periodic force must be applied to replace the energy lost in damping
  - This periodic force **does work** against the resistive force responsible for decreasing the oscillations
  - It is sometimes known as an external **driving** force
- These are known as **forced oscillations** (or vibrations), and are defined as:

  Oscillations acted on by a periodic external force where energy is given in order to sustain oscillations
- Forced oscillations are made to oscillate at the same frequency as the oscillator creating the external, periodic driving force
- For example, when a child is on a swing, they will be pushed at one end after each cycle in order to keep swinging and prevent air resistance from damping the oscillations
  - These extra pushes are the forced oscillations, without them, the child will eventually come to a stop

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Worked Example

State whether the following are free or forced oscillations:

(i) Striking a tuning fork

(ii) Breaking a glass from a high pitched sound

(iii) The interior of a car vibrating when travelling at a high speed

(iv) Playing the clarinet

Answer:

Part (i)

Striking a tuning fork

This is a free vibration. When a tuning fork is struck, it will vibrate at its natural frequency and there are no other external forces

Part (ii)

Breaking a glass from a high pitched sound

This is a forced vibration. The glass is forced to vibrate at the same frequency as the sound until it breaks. The frequency of the high-pitched sound is the external driving frequency

Part (iii)

The interior of a car vibrating when travelling at a particular speed

This is a forced vibration. The interior of the car vibrates at the same frequency as the wheels travelling over a rough surface at a high speed

Part (iv)

Playing the clarinet

This is a forced vibration. The air from the player's lungs is used to sustain the vibration in the air column in a clarinet to create and hold a sound. The air column inside the clarinet mimics the vibrations at the same frequency as the air forced into the mouthpiece of the clarinet (the reed).

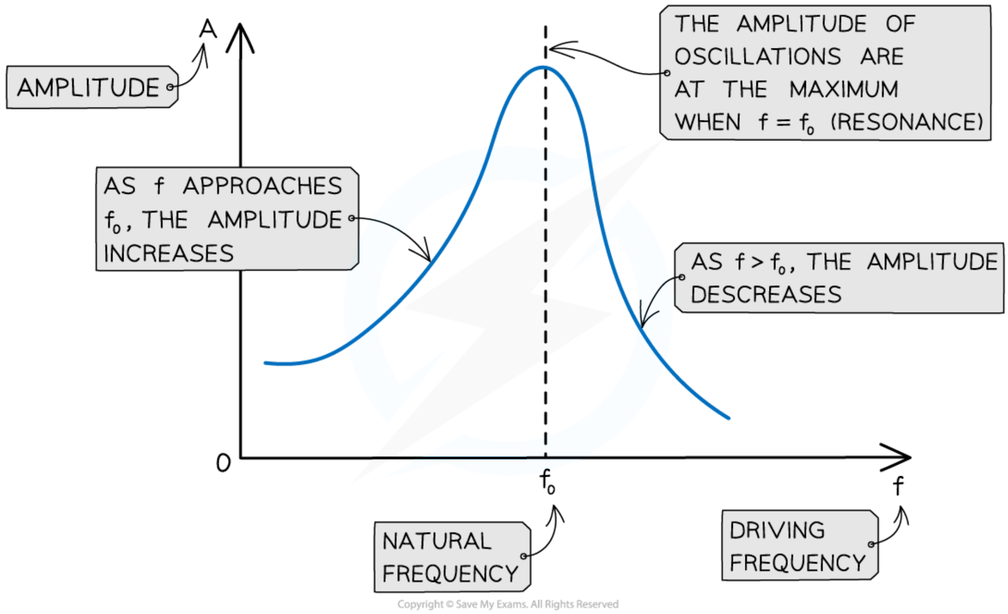
Your notes

Examiner Tips and Tricks

Avoid writing 'a free oscillation is not forced to oscillate'. Mark schemes are mainly looking for a reference to internal and external forces and energy transfers

Resonance Graphs

- A graph of driving frequency f against amplitude A of oscillations is called a **resonance curve**. It has the following key features:
  - When $f < f_0$, the amplitude of oscillations increases
  - At the peak where $f = f_0$, the amplitude is at its maximum. This is **resonance**
  - When $f > f_0$, the amplitude of oscillations starts to decrease



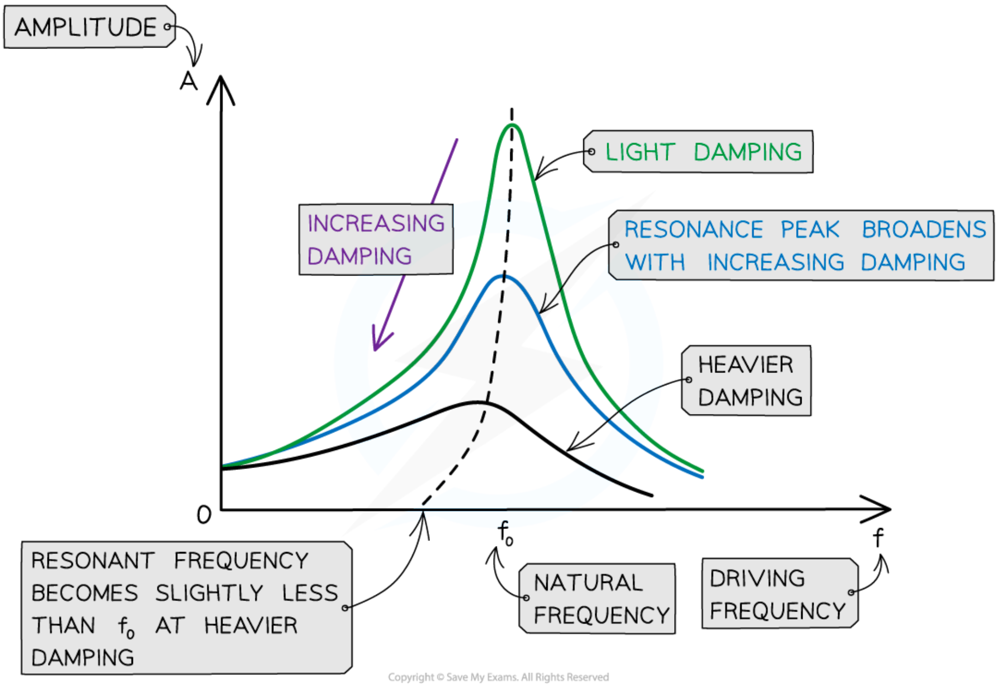
The maximum amplitude of the oscillations occurs when the driving frequency is equal to the natural frequency of the oscillator

Damping & Resonance

- Damping **reduces** the amplitude of resonance vibrations
- The height and shape of the resonance curve will therefore change slightly depending on the degree of damping
  - **Note:** the natural frequency f_0 of the oscillator will remain the same
- As the degree of damping is increased, the resonance graph is altered in the following ways:
  - The amplitude of resonance vibrations **decrease**, meaning the peak of the curve lowers
  - The resonance peak **broadens**

- The resonance peak moves slightly to the **left** of the natural frequency when heavily damped

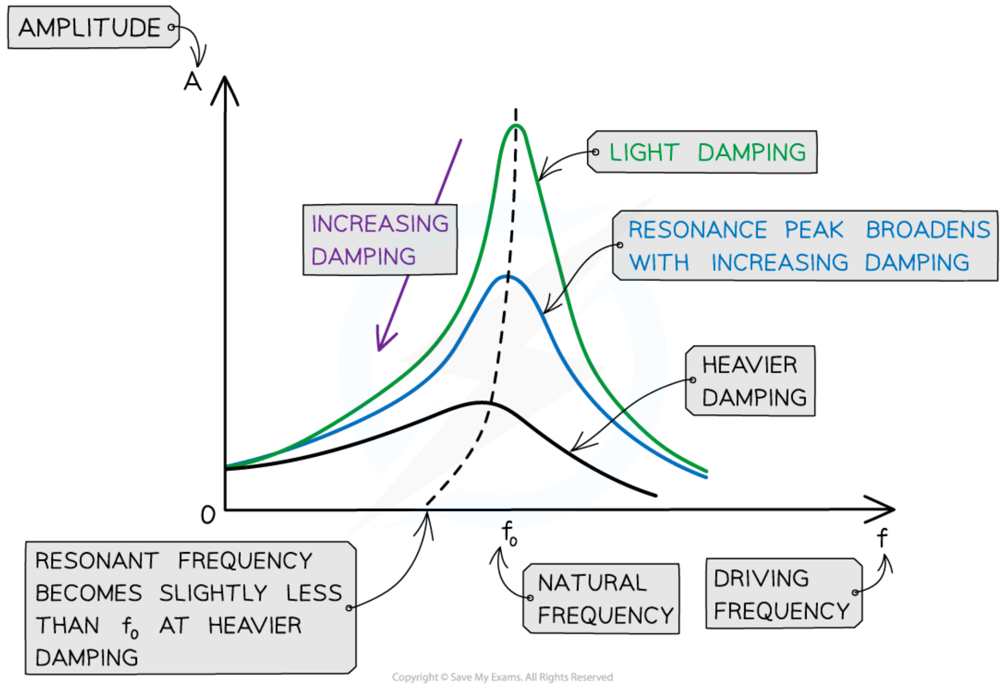
- Therefore, damping reduced the sharpness of resonance and reduces the amplitude at resonant frequency



As damping is increased, resonance peak lowers, the curve broadens and moves slightly to the left

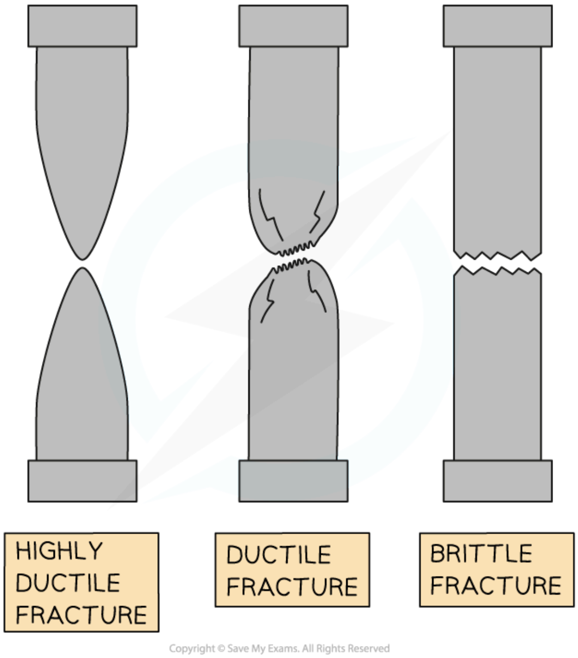
Damping & Plastic Deformation

- **Damping** an oscillator affects its **amplitude** of oscillation:
  - When **damping** is **increased** the **amplitude decreased**
  - **damping** and **amplitude** are **inversely proportional** to each other

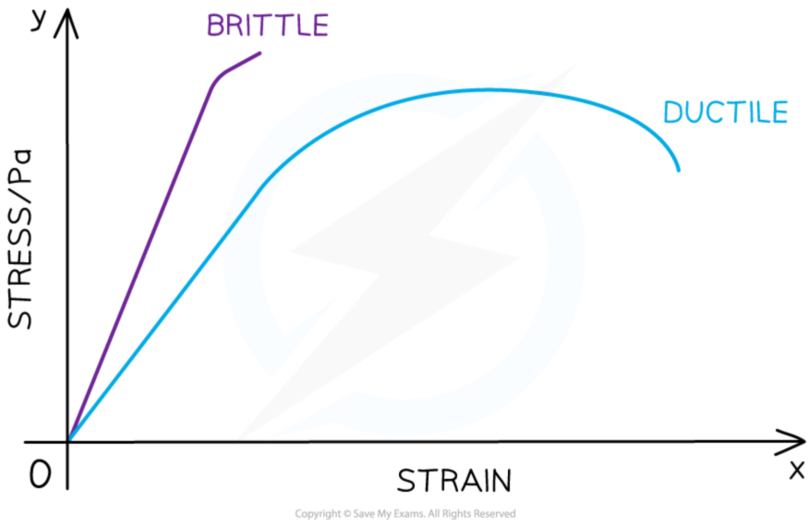


As damping is increased, resonance peak lowers, the curve broadens and moves slightly to the left

- A **Ductile** material can be stretched for a long time before it snaps
  - We can say it undergoes a large amount of **plastic deformation** before it is **permanently deformed**



- Examples of ductile materials include:
  - Most metals (particularly copper, gold and silver)
  - Non-metals are generally not ductile



Brittle and ductile materials on a stress-strain graph. These are the same on a force-extension graph too

- The **amplitude of oscillations** can be reduced due to the **plastic deformation** of a **ductile material**
- This happens because energy from the oscillations is used to deform the material

- The kinetic energy of the oscillator is reduced and transferred into the deformation of the material

- A climbing rope is different from a rescue rope or a bungee cord:
  - A climbing rope is designed to extend when loaded suddenly
  - The rope stretches to reduce the amplitude of the oscillation when a climber falls onto it
  - It provides critical damping by immediately stopping the climber from bouncing

A climber uses a dynamic rope that stretches when she falls onto it. This reduces the amplitude of her oscillation and the force she experiences reducing injury.

Your notes