Edexcel International A Level (IAL) Physics

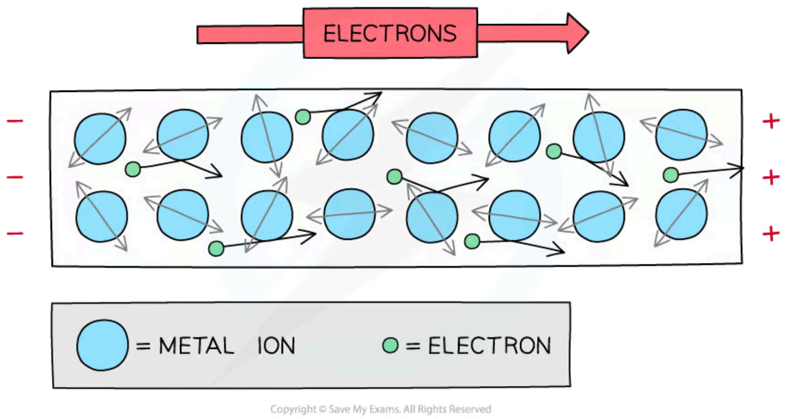
Your notes

Resistance, Resistivity & Potential Dividers

Contents

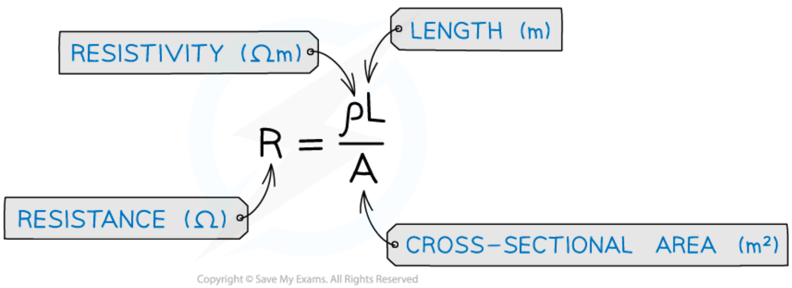
Electrical Resistivity

- All materials have some **resistance** to the flow of charge
- As **free electrons move** through a metal wire, they collide with ions which get in their way
- As a result, they **transfer** some, or all, of their **kinetic energy** on **collision**, which causes electrical **heating**



Free electrons collide with ions which resist their flow

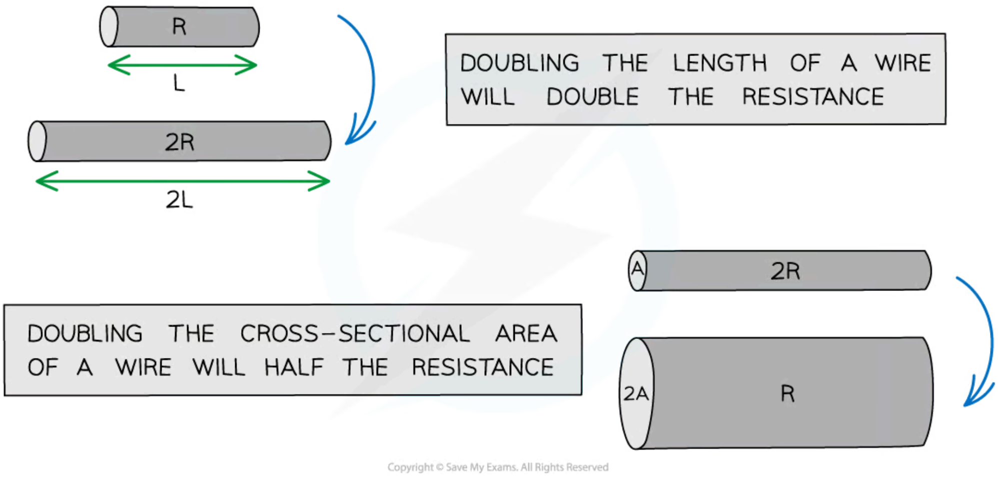
- Since **current** is the **flow** of **charge**, the ions resisting their flow cause **resistance**
- Resistance depends on the **length** of the wire, the **cross-sectional area** through which the current is passing and the **resistivity** of the material



Electrical resistance equation

- The resistivity equation shows that:
  - The **longer** the wire, the **greater** its resistance

- The **thicker** the wire, the **smaller** its resistance



The length and width of the wire affect its resistance

- Resistivity is a property that describes the extent to which a material opposes the flow of electric current through it
- It is a property of the material, and is dependent on temperature
- Resistivity is measured in **Ω m**

	Material	Resistivity ρ / Ωm
Metals	Copper	1.7×10^{-8}
	Gold	2.4×10^{-8}
	Aluminium	2.6×10^{-8}
Semiconductors	Germanium	0.6
	Silicon	2.3×10^{3}
Insulators	Glass	10^{12}
	Sulfur	10^{15}

Your notes

Resistivity of some materials at room temperature

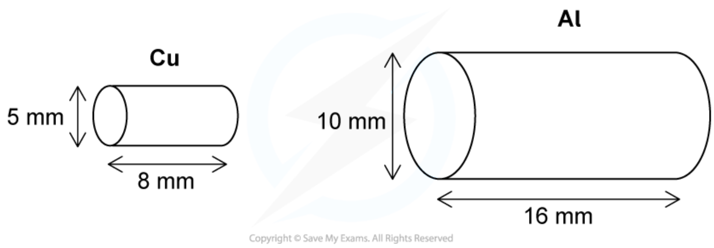
- The higher the resistivity of a material, the higher its resistance
  - Copper has a relatively low resistivity at room temperature, and so is used for electrical wires
  - This is because current flows through it very easily
- Insulators have such a high resistivity that virtually no current will flow through them

Your notes

Worked Example

Two electrically-conducting cylinders made from copper and aluminium respectively.

Their dimensions are shown below.

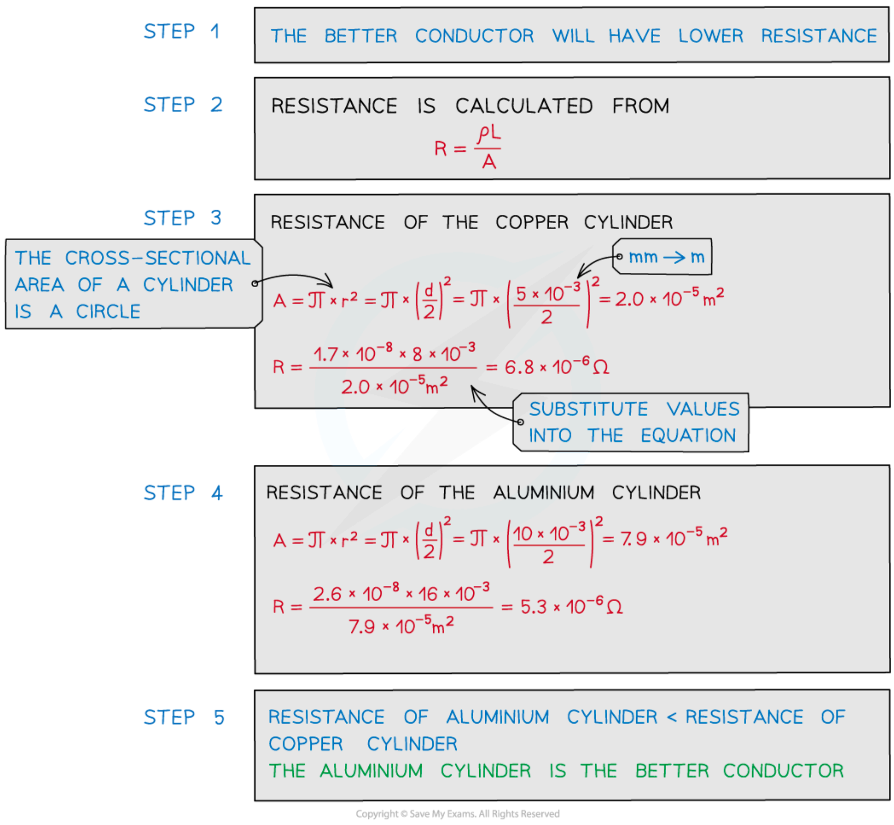


Copper resistivity = $1.7 \times 10^{-8}\ \Omega\,m$

Aluminium resistivity = $2.6 \times 10^{-8}\ \Omega\,m$

Which cylinder is the better conductor?

Answer:



Examiner Tips and Tricks

You don't need to memorise the value of the resistivity of any material; these will be given in the exam question.

Remember if the cross-sectional area is a circle e.g. in a wire, then area is proportional to the radius or diameter squared. This means if the diameter doubles, the area will quadruple. This causes the resistance to drop by a quarter. Considering how changing one property affects the value of the others is a common exam question.

Your notes

Core Practical 7: Investigating Resistivity

Aims of the Experiment

- The aim of the experiment is to determine the resistivity of a length of wire

Variables:

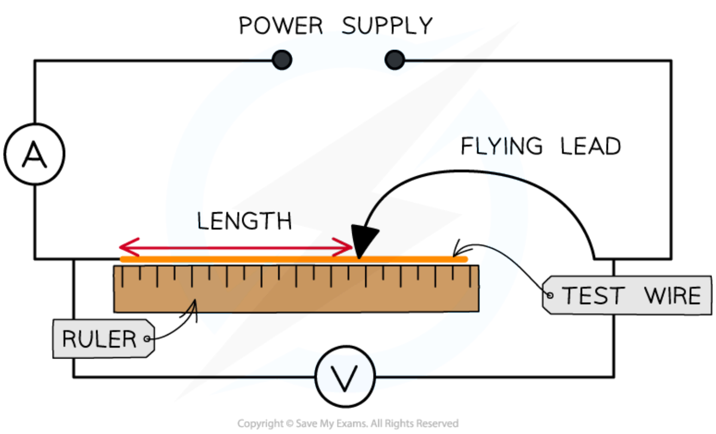
- Independent variable = Length, L, of the wire (m)
- Dependent variable = The current, I, through the wire (A)
- Control variables:
  - Voltage across the wire
  - The material the wire is made from

Equipment List

Equipment	Purpose
Ammeter	To determine the current through the wire
Voltmeter	To determine the voltage across the wire
2.0 m of constantan wire (22–36 swg)	To calculate its resistivity
Flying lead	A wire with a crocodile clip at one end to allow connection at any point along the test wire
Metre ruler	To measure the length of the wire
Micrometer	To measure the diameter of the wire
Power supply	To provide the voltage through the wire

- Resolution of measuring equipment:
  - Metre ruler = 1 mm
  - Micrometer screw gauge = 0.01 mm
  - Voltmeter = 0.1 V
  - Ammeter = 0.01 A

Method



1. Measure the diameter of the wire using a micrometer.
   - The measurement should be taken between 5–10 times randomly along the wire.
   - Calculate the mean diameter from these values
2. Set up the equipment so the wire is taped or clamped to the ruler with one end of the circuit attached to the wire where the ruler reads 0.
   - The ammeter is connected in series and the voltmeter in parallel with the wire
3. Attach the flying lead to the test wire at 0.25 m and set the power supply at a voltage of 6.0 V.
   - Check that this is the voltage across the wire on the voltmeter
4. Read and record the current from the ammeter, then switch off the current immediately after the reading
   - This is to prevent the wire from heating up and changing the resistivity
5. Vary the distance between the fixed end of the wire and the flying lead in 0.25 m intervals (0.25 m, 0.50 m, 0.75 etc.) until the full length
   - In this example, a 2.0 m wire is used.
   - The original length and the intervals can be changed (e.g. start at 0.1 m and increase in 0.1 m intervals), as long as there are 8–10 readings
6. Record the current for each length at least 3 times and calculate an average current, *I*
7. For each length, calculate the average resistance of the length of the wire using the equation

$$R = \frac{V}{I}$$

- Where:
  - R = average resistance of the length of the wire (Ω)

Your notes

- V = potential difference across the circuit (V)
- I = the average current through the wire for the chosen length (A)

- An example of a table of results might look like this:

LENGTH OF WIRE L / m	CURRENT I_1 / A	CURRENT I_2 / A	CURRENT I_3 / A	AVERAGE CURRENT I / A	RESISTANCE R / Ω
0.25					
0.50					
0.75					
1.00					
1.25					
1.50					
1.75					
2.00					

Analysis of Results

- The resistivity, ρ, of the wire is equal to

$$\rho = \frac{RA}{L}$$

- Where:
  - ρ = resistivity (Ω m)
  - R = resistance (Ω)
  - A = cross-sectional area of the wire (m^2)
  - L = length of wire (m)
- Rearranging for the resistance, R, gives:

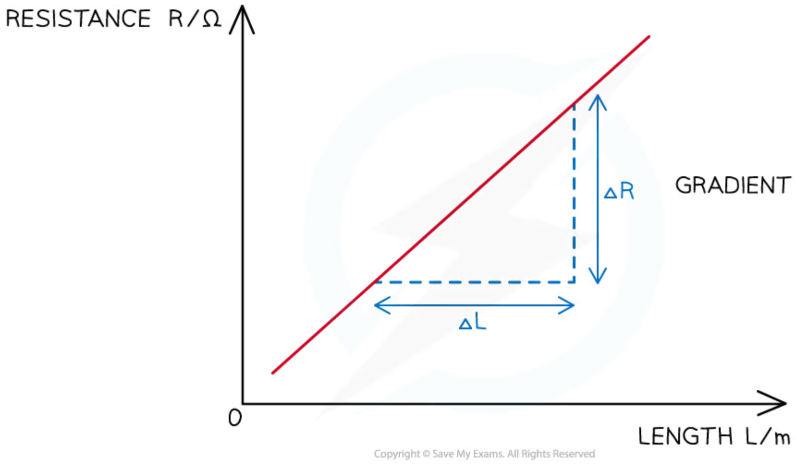
$$R = \frac{\rho L}{A}$$

- Comparing this to the equation of a straight line: y = mx
  - y = R
  - x = L
  - Gradient, m = ρ / A
- Therefore, to find resistivity:

Your notes

- Plot a graph of the length of the wire, *L*, against the average resistance of the wire
- Draw a line of best fit
- Calculate the gradient
- Multiply the gradient by cross-sectional area, A

$$\rho = \text{gradient} \times A$$



- To calculate the cross-sectional area, *A*, of the wire

$$\text{Cross-sectional area } A = \frac{\pi d^2}{4}$$

Evaluating the Experiment

Systematic Errors:

- The end of the wire that is attached to the circuit (not the flying lead) must start at 0 on the ruler
  - Otherwise, this could cause a zero error in your measurements of the length

Random Errors:

- Only allow small currents to flow through the wire
  - The resistivity of a material depends on its temperature
  - The current flowing through the wire will cause its temperature to increase
  - Therefore the temperature is kept constant by small currents
- The current should be switched off between readings
  - So that there isn't a temperature rise
- Calculate an average diameter

Your notes

- This will reduce random errors in the reading
- Make at least 5–10 measurements of the diameter of the wire with the micrometer

Your notes

Safety Considerations

- When there is a high current, and a thin wire, the wire will become very hot.
    - Make sure never to touch the wire directly when the circuit is switched on
- Switch off the power supply right away if you smell burning
- Make sure there are no liquids close to the equipment,
    - This could damage the electrical equipment
    - Or cause a short circuit which will affect the results

Worked Example

A student conducts an experiment to find the resistivity of a constantan wire.

They attach one end of the wire to a circuit that contains a 6.0 V battery. The other end of the wire is attached by a flying lead to the wire at different lengths.

They obtain the following table of results:

Length of wire L / m	Current I_1 / A	Current I_2 / A	Current I_3 / A	Average Current I / A	Resistance R / Ω
0.25	1.34	1.34	1.35		
0.50	0.85	0.85	0.83		
0.75	0.51	0.51	0.50		
1.00	0.35	0.36	0.35		
1.25	0.30	0.31	0.31		
1.50	0.27	0.27	0.27		
1.75	0.23	0.21	0.21		
2.00	0.18	0.17	0.18		

The following additional data for the wire is:

										Average diameter / mm
0.19	0.19	0.20	0.19	0.18	0.19	0.20	0.18	0.20	0.19	0.19

Calculate the resistivity of the wire.

Answer:

Step 1: Complete the average current and resistance columns in the table

- The resistance is calculated using the equation

$$R = \frac{V}{I}$$

Length of wire L / m	Current I_1 / A	Current I_2 / A	Current I_3 / A	Average Current I / A	Resistance R / Ω
0.25	1.34	1.34	1.35	1.34	4.48
0.50	0.85	0.85	0.83	0.84	7.14
0.75	0.51	0.51	0.50	0.51	11.76
1.00	0.35	0.36	0.35	0.35	17.14
1.25	0.30	0.31	0.31	0.31	19.35
1.50	0.27	0.27	0.27	0.27	22.22
1.75	0.23	0.21	0.21	0.22	27.27
2.00	0.18	0.17	0.18	0.18	33.33

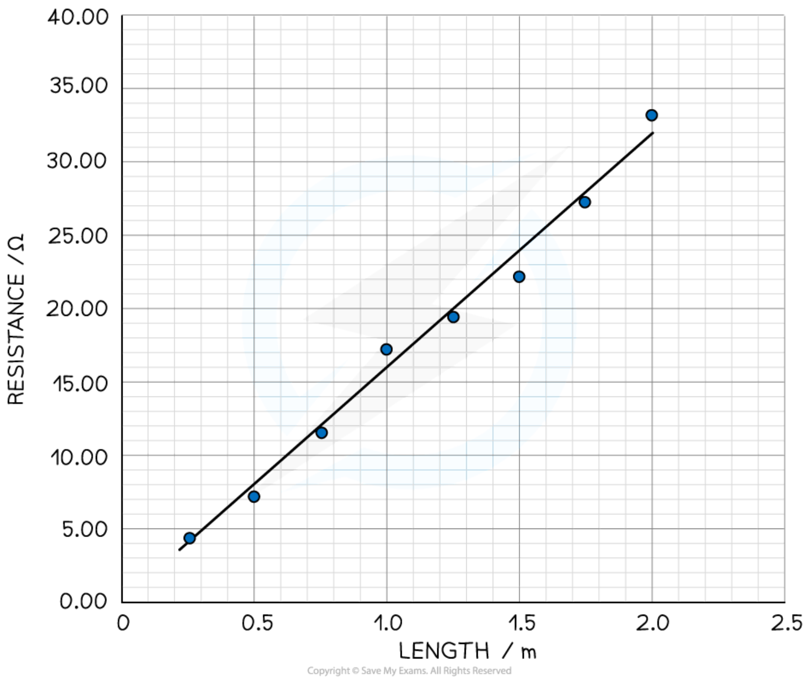
Step 2: Calculate the cross-sectional area of the wire from the diameter

- The average diameter is 0.191 mm = 0.191×10^{-3} m
- The cross-sectional area is equal to

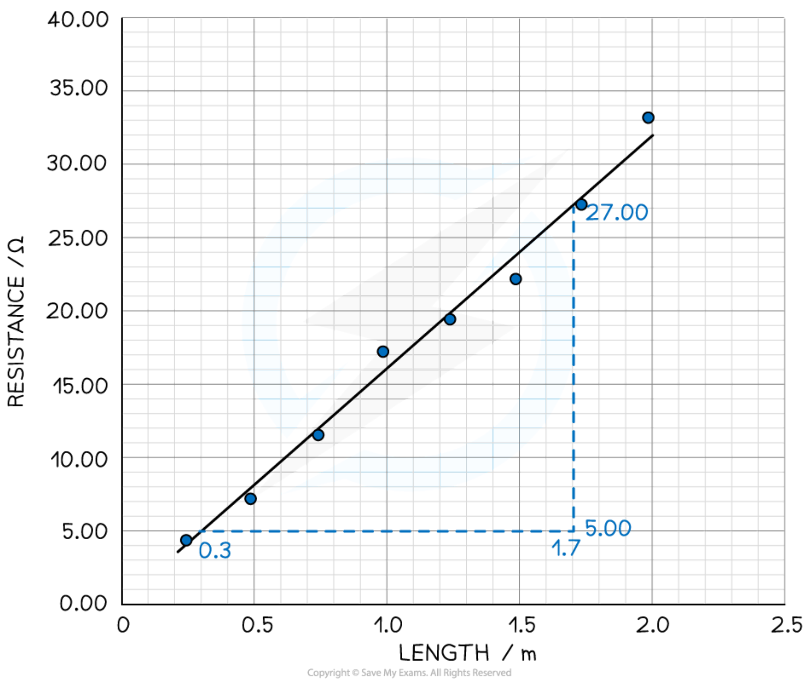
$$A = \frac{\pi\left(0.191 \times 10^{-3}\right)^2}{4} = 2.87 \times 10^{-8}\ \text{m}^2$$

Step 3: Plot a graph of the length L against the resistance R

Your notes



Step 4: Calculate the gradient of the graph



$$\frac{\Delta R}{\Delta L} = \frac{\rho}{A} = \frac{27.00 - 5.00}{1.7 - 0.3} = 15.71$$

Step 5: Calculate the resistivity of the wire

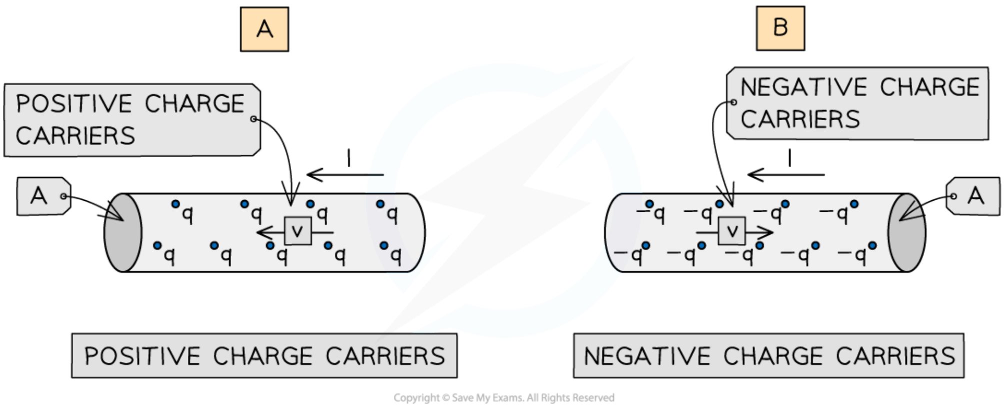
$$\rho = gradient \times A = 15.71 \times (2.87 \times 10^{-8}) = 4.51 \times 10^{-7}\ \Omega\text{m}$$

Your notes

Calculating Current & Drift Velocity

Drift Velocity

- In a conductor, the current is due to the movement of charge carriers
  - The charge carriers can be negative or positive
  - However current is always taken to be in the **same** direction
- **Drift velocity** is the **average** velocity of the charge carriers travelling through the conductor
- In conductors, the charge carrier is usually free electrons
  - Free electrons only travel small distances before colliding with a metal ion
  - Therefore they have a relatively slow drift velocity of ~ 10^{-3} m s^{-1}
- In the diagram below, the current in each conductor is from right to left
  - In diagram **A** (positive charge carriers), the drift velocity is in the **same** direction as the current
  - In diagram **B** (negative charge carriers), the drift velocity is in the **opposite** direction to the current



Conduction in a current-carrying conductor

- The density *n* represents the number of free charge carriers (electrons) **per unit volume**
  - **Conductors**, such as metals, have a **high** value of *n*
  - **Insulators**, such as plastics, have a **low** value of *n*
- Since the density of charge carriers is so large in conductors, the flow of current flow appears to happen instantaneously

The Transport Equation

- The current can be expressed in the transport equation:

$$I = nqvA$$

- Where:
  - I = current (A)
  - n = number density (m^{-3})
  - q = the charge of the charge carrier (C)
  - v = drift velocity ($m\ s^{-1}$)
  - A = cross sectional area of the wire (m^2), calculated using $\mathbf{A = \pi r^2}$
- The **same equation** is used whether the charge carriers are positive or negative
  - A negative value for v will indicate current in the opposite direction to the charge carriers
- The transport equation shows that v is **inversely proportional** to n
  - Since the more charge carriers available per unit volume the more the density will slow down their speed through the conductor
- The transport equation also shows that I is **directly proportional** to n
  - Greater n means a greater charge is flowing and therefore a larger current I
- When the value of n is lower, the charge carriers must **travel faster** to carry the same current

Worked Example

The number density of conduction electrons in a copper wire is $9.2 \times 10^{28}\ m^{-3}$. The wire carries a current of 3.5 A and it has a cross-sectional area of 1.5 mm^2.

Determine the average drift velocity of the electrons.

Answer:

Step 1: Consider the situation

- A copper wire is a conductor, and the free electrons are charge carriers
- Use the transport equation $I = nqvA$

Step 2: Rearrange the equation for drift speed *v*

$$v = \frac{I}{nqA}$$

Step 3: Substitute in values

Your notes

- Current, $I = 3.5$ A
- Cross-sectional area, $A = 1.5\ \text{mm}^2 = 1.5 \div 1000^2 = 1.5 \times 10^{-6}\ \text{m}^2$
- Number density of conduction electrons, $n = 9.2 \times 10^{28}\ \text{m}^{-3}$
- Charge on an electron, $q = 1.60 \times 10^{-19}$ C (From the data sheet)

$$v = \frac{3.5}{(9.2 \times 10^{28}) \times (1.60 \times 10^{-19}) \times (1.5 \times 10^{-6})} = 0.16 \times 10^{-3}\ \text{m s}^{-1}$$

$$v = 0.16\ \text{mm s}^{-1}\ (2\ \text{s.f.})$$

The Large Range of Material Resistivities

Resistivity

- The transport equation tells us that current, $I \propto$ number of charge carriers, n
  - Therefore, the **larger** the number of charge carriers, the **greater** the current will be for the same applied voltage
  - This is because resistivity has decreased with more charge carriers available
- Different materials have different numbers of charge carriers
- Insulators have **few** charge carriers:
  - They have such a high resistivity that virtually no current will flow through them
  - A perfect insulator would have no charge carriers, $n = 0$
  - A perfect insulator would have a current of zero regardless of the voltage applied
- Conductors have a **large number** of charge carriers
  - Metals are good conductors because they have **free electrons**
  - Free electrons are the atoms from the outer shell of each atom
  - Therefore there are lots of charge carriers per unit volume
  - This means resistivity is low
- Semiconductors have a **small number** of free electrons
  - There are **fewer** delocalised electrons in a semiconductor than in a metal
  - There are a greater number of free electrons at a higher temperature
  - Resistivity changes in a semiconductor, due to the variation with temperature in free electrons which are available as charge carriers
  - Silicon is an example of a semiconductor

	Material	Resistivity ρ/Ωm
Metals	Copper	1.7×10^{-8}
	Gold	2.4×10^{-8}
	Aluminium	2.6×10^{-8}
Semiconductors	Germanium	0.6
	Silicon	2.3×10^{3}
Insulators	Glass	10^{12}
	Sulfur	10^{15}

The resistivity of some materials at room temperature

Examiner Tips and Tricks

Remember that the cross-sectional area is in m^2, the drift velocity is in m s^{-1} and the number density is in m^{-3}.

Therefore, sometimes unit conversions from cm or mm may be required, so make sure you're comfortable with these.

Your notes

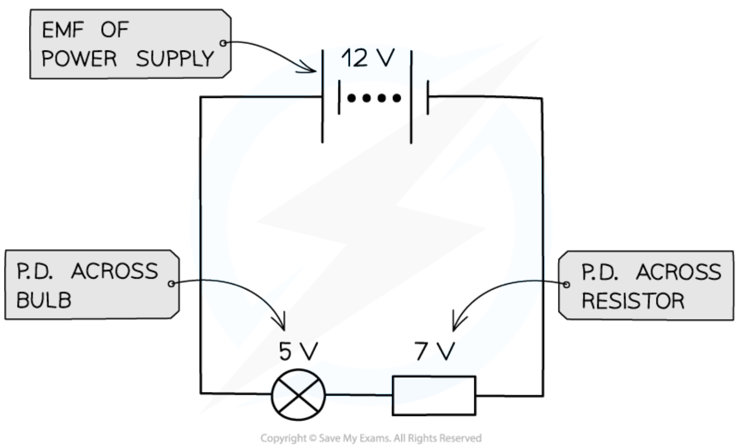
Potential Difference & Conductor Length

Potential difference

- A cell makes one end of the circuit positive and the other negative. This sets up a **potential difference** across the circuit
- The potential difference across a component in a circuit is defined as:

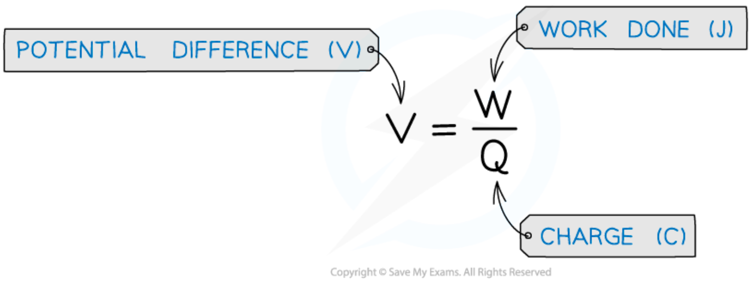
The energy transferred per unit charge

- The energy is transferred from electrical energy into other forms, depending on the component or device being used
- Potential difference is measured in **volts (V)**, which is equivalent to **joules per coulomb ($J\,C^{-1}$)**
- The potential difference of a power supply connected in series is always shared between all the components in the circuit



The potential difference is the voltage across each component in a circuit

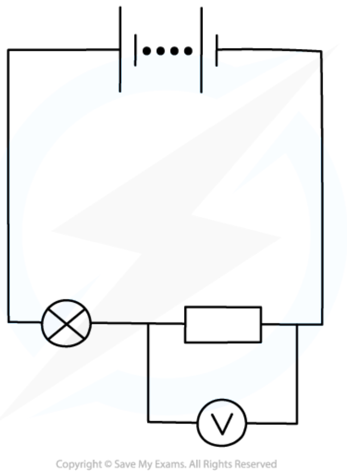
- Another description of energy transfer is **work done**
- Therefore, potential difference can also be defined as the **work done per unit charge**



Potential difference is the work done per unit charge

Measuring potential difference

- Potential difference or voltage is measured using a **voltmeter**
  - A voltmeter is always set up in parallel to the component being measured



Potential difference can be measured by connecting a voltmeter in parallel between two points in a circuit

Conductor length

- The equation for resistivity is

$$R = \frac{\rho l}{A}$$

- Where:
  - R = resistance (Ω)
  - ρ = resistivity (Ω m^{-1})
  - l = length (m)
  - A = area (m^{-2})

Your notes

- Therefore, as the length of a uniform conductor at constant temperature increases, resistance also increases
- Voltage and current are linked by Ohm's Law

$$V = IR$$

- Where
  - V = potential difference (V)
  - I = current (A)
  - R = resistance (Ω)
- Therefore, as R increases, so must potential difference across the wire
  - Potential difference increases uniformly with length

Your notes

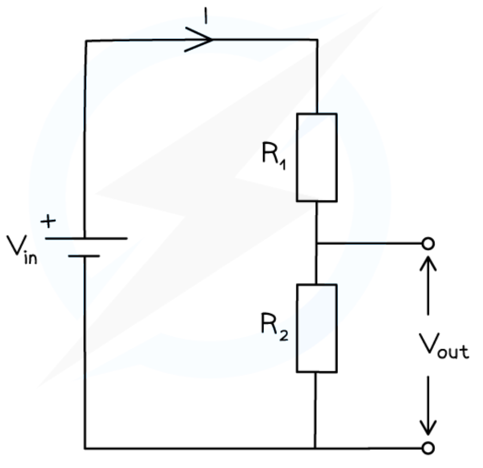
Potential Dividers

- The electrical voltages rule is defined as:

  The sum of the e.m.f.s in a closed circuit loop is equal to the sum of the potential differences around that loop

- Therefore, when two resistors are connected in series, the potential difference across the power source will be divided across the two resistors
- Potential dividers are circuits that produce an output voltage as a **fraction** of the input voltage
- This is done by using two resistors in series to split or divide the voltage of the supply in a chosen **ratio**
- Potential dividers have three main purposes:
  - To provide a variable potential difference
  - To enable a specific potential difference to be chosen
  - To split the potential difference of a power source between two or more components
- Potential dividers are used widely in volume controls and sensory circuits using LDRs and thermistors
- The link between the input voltage and the output voltage across each resistor is linked in an equation

POTENTIAL DIVIDER EQUATION: $V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$



Potential divider diagram and equation

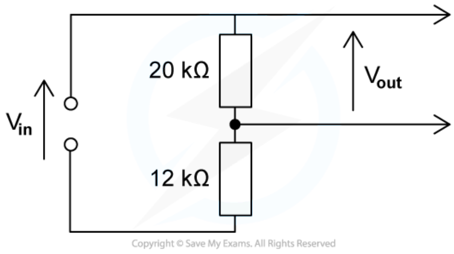
- The input voltage V_{in} is applied across both resistors, which are in series
- The output voltage V_{out} is measured across one of the resistors, in this case resistor R_2
- The potential difference *V* across each resistor depends upon its resistance *R*:
  - The resistor with the **largest resistance** will have the **greater** potential difference across it
  - This is shown as a greater V_{out}
  - This is from ***V = IR***
- If the resistance of one of the resistors is increased, it will get a greater share of the potential difference, whilst the other resistor will get a smaller share
- Since potential divider circuits are based on the ratio of voltage between components, and since V=IR, this is equal to the ratio of the resistances of the resistors
- Therefore, the ratios of the potential differences and resistances across each resistor can be linked

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

- Where:
  - V_1 = potential difference of R_1 (V)
  - V_2 = potential difference or R_2 (V)
- Using Ohm's Law, with a constant current, I, these can also be written as:
  - $V_1 = IR_1$
  - $V_2 = IR_2$

Worked Example

The circuit shown is designed to light up a lamp when the input voltage exceeds a preset value.



Your notes

V_{out} is equal to 5.3 V when the lamp lights.

Calculate the input voltage V_{in}.

Answer:

STEP 1

POTENTIAL DIVDER EQUATION

$$V_{out} = \left(\frac{R_1}{R_1 + R_2}\right) V_{in}$$

STEP 2

REARRANGE FOR INPUT VOLTAGE V_{in}

$$V_{in} = V_{out} \div \left(\frac{R_1}{R_1 + R_2}\right) = V_{out} \times \left(\frac{R_1 + R_2}{R_1}\right)$$

STEP 3

SUBSTITUTE IN VALUES

$R_1 = 20\ \text{k}\Omega$ $R_2 = 12\ \text{k}\Omega$ $V_{out} = 5.3\ \text{V}$

$$V_{in} = 5.3 \times \left(\frac{12 + 20}{20}\right) = 8.48\ \text{V}$$

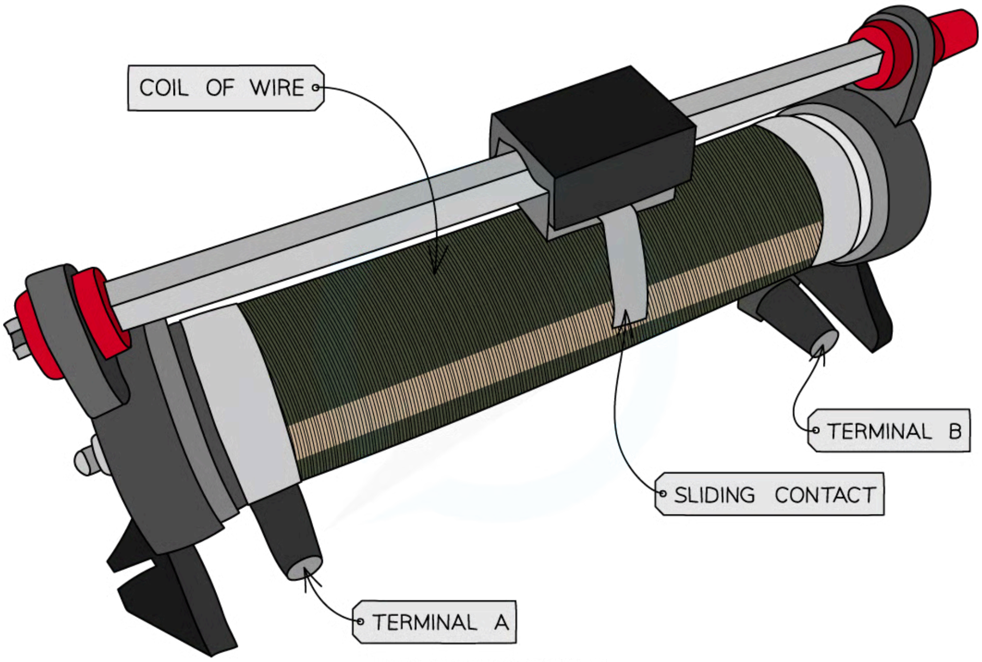
$V_{in} = 8.5\ \text{V}$ (2 s.f.)

Your notes

Potential Dividers & Variable Resistance

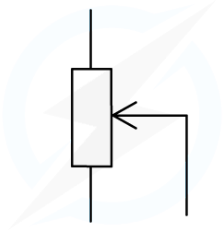
The Potentiometer

- A potentiometer is a single component which can act as a potential divider.
  - It consists of a coil of wire with a sliding contact
  - A variable output voltage can be varied by moving a slider along the component



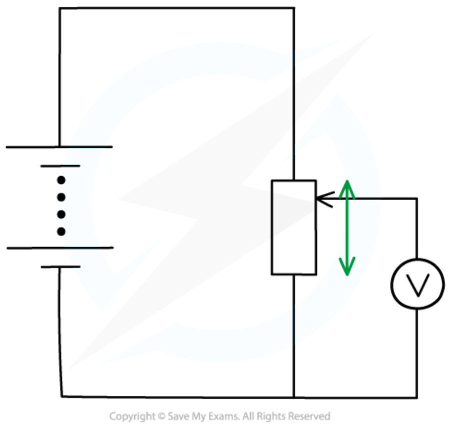
A potentiometer is a type of variable resistor

- The circuit symbol is drawn as an arrow next to the resistor, to represent the sliding contact



- The sliding contact has the effect of separating the potentiometer into two parts
  - Each part will have different resistances

- Therefore output voltage will change



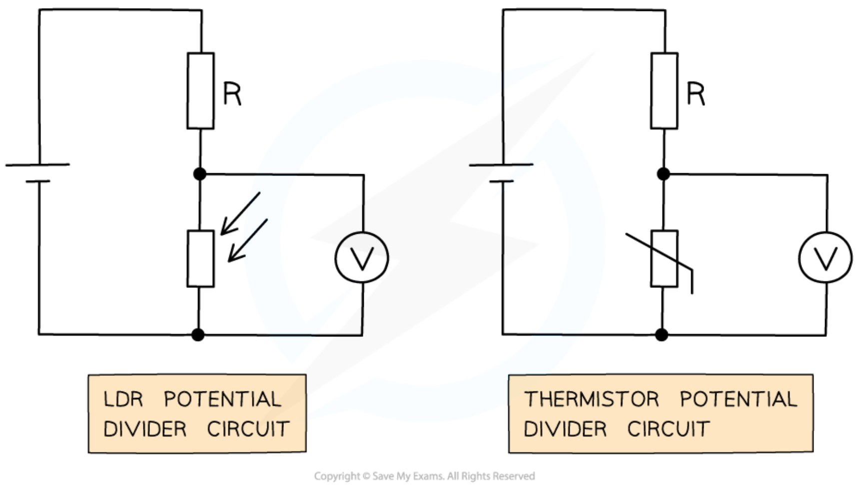
Moving the slider (the arrow in the diagram) changes the resistance (and hence potential difference) of the two parts of the potentiometer

- If the slider in the above diagram is moved upwards, the resistance of the lower part will increase and so the potential difference across it will also increase
- Therefore, the variable resistor obtains a maximum or minimum value for the output voltage
- If the resistance is 3 Ω:
  - Maximum voltage is when the resistance is 3 Ω
  - Minimum voltage is when the resistance is 0 Ω

Thermistors & Light Dependent Resistors (LDRs)

- Sensory resistors are used in potential dividers to vary the output voltage
  - This could cause an external component to switch on or off
  - For example, a heater switches off automatically when its surroundings are at room temperature
- Examples of the variable sensory resistors used are **thermistors** and **light-dependent resistors (LDRs)**

Your notes



LDR and thermistor in a potential divider circuit with a fixed resistor R

- The voltmeter in both circuits is measuring V_{out}
- From Ohm's law ***V = IR***, the potential difference V_{out} from a sensory resistor in a potential divider circuit is **proportional** to its resistance
  - If an LDR or thermistor's resistance **decreases**, the potential difference through it also **decreases**
  - If an LDR or thermistor's resistance **increases**, the potential difference through it also **increases**
- Since the total potential difference of the components must be equal to $V_{in:}$
  - If the potential difference of the sensory resistor **decreases** then the potential difference across the other resistor in the circuit must **increase**
  - If the potential difference of the sensory resistor **increases** then the potential difference across the other resistor in the circuit must **decrease**

The resistance of an LDR…

- Varies with **light intensity**
  - The higher the light intensity, the lower the resistance
  - The lower the light intensity, the higher the resistance
- Therefore:
  - If light intensity increases, V_{out} across the LDR will **decrease** because resistance has decreased
  - If light intensity decreases, V_{out} across the LDR will **increase** because resistance has increased
- An LDR circuit is often used for street and security lights

- When light intensity falls, V_{out} increases and so this can provide the voltage required to turn on a lamp

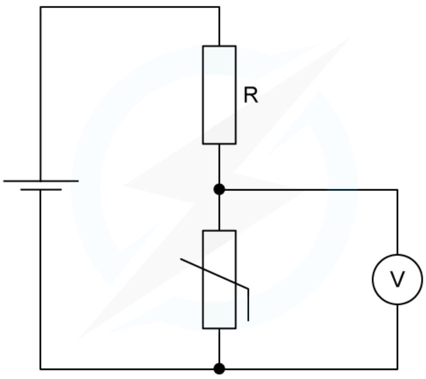
The resistance of a thermistor…

- Varies with **temperature**
  - The hotter the thermistor, the lower the resistance
  - The cooler the thermistor, the higher the resistance
- Therefore:
  - If temperature increases, V_{out} across the thermistor will **decrease** because resistance has decreased
  - If temperature decreases, V_{out} across the thermistor will **increase** because resistance has increased
- A thermistor circuit is used in fire alarms, ovens and digital thermometers
  - When temperature falls, V_{out} increases and so this can provide the voltage required to turn on a heater

Your notes

Worked Example

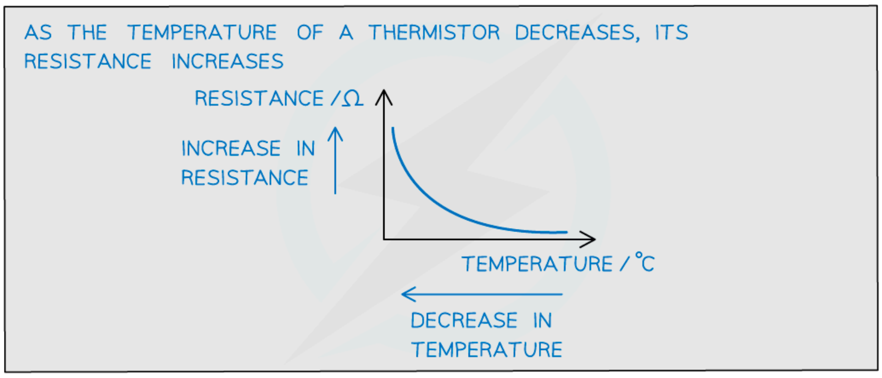
A potential divider consists of a fixed resistor R and a thermistor connected in series.



Which row of the table describes what happens to the potential difference across resistor R and the thermistor when the temperature of the thermistor decreases?

	P.d of Thermistor / V	P.d of Resistor R/V
A	Increases	Increases
B	Decreases	Increases
C	Decreases	Decreases
D	Increases	Decreases

Answer: D



Step 1: Consider Ohm's Law

- Due to Ohm's Law (V = IR), both the resistor and thermistor are connected in series and have the same current I
  - If resistance *R* increases, the potential difference across the thermistor also **increases**

Step 2: Consider the electrical voltages rule

- In series, the input potential difference is shared equally amongst the components due to the electrical voltages rule
  - Therefore, since the potential difference across the thermistor **increases**, the potential difference across the resistance R must **decrease**

Step 3: State the answer

- This is row **D**

Examiner Tips and Tricks

A potentiometer can also be used as a variable resistor if only two of the three possible terminals are used.

Your notes