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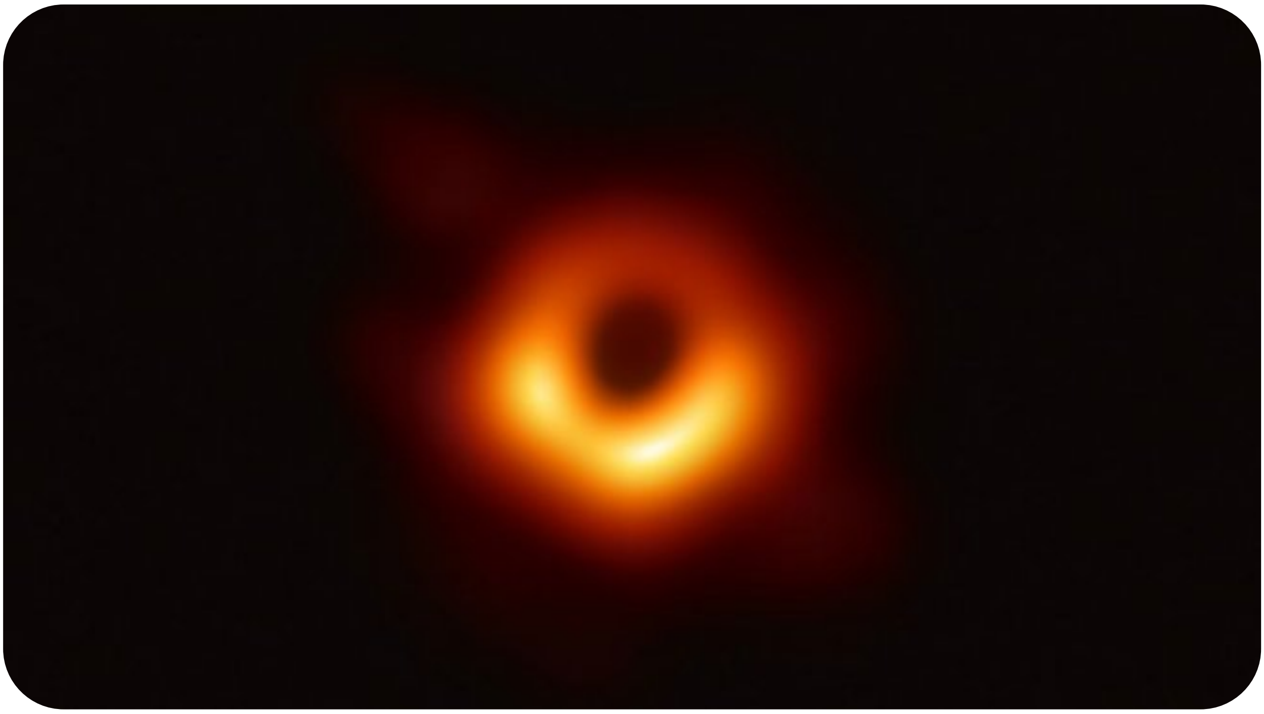

Edexcel IAL Physics Unit 5

Summary©

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HS

Thermal energy

1 HEAT AND TEMPERATURE

Heat is energy and is more accurately referred to as thermal energy; heating is a process which involves a flow of thermal energy. Heating can occur through 3 different processes and these are namely conduction, convection and radiation.

On the other hand, **temperature is related to the mean, random, kinetic energy of the vibrating atoms of a body**. According to the kinetic theory, when energy is supplied to an object, the particles in that object take up the energy as kinetic energy, and move faster. In solids, this motion is usually in the form of vibrations. If we are considering a gas, we imagine the molecules whizzing around their container at a greater speed. It is this kinetic energy that determines the temperature. **If the average kinetic energy of the molecules of a substance increases, then it is at a higher temperature**.

Temperature is one of the fundamental SI quantities. The base SI unit of temperature is the Kelvin, symbol K. This is defined in terms of what is called the absolute thermodynamic scale of temperature, which has absolute zero as its zero and defines the melting point of ice as 273 K. The formula for the interconversion between Celsius and Kelvin are as follows:

$$T/\mathrm{K} = \theta/{}^\circ\mathrm{C} + 273$$

A change in temperature of $\Delta\theta$ measured in °C is numerically the same as the corresponding change of temperature ΔT in K.

Absolute zero

Absolute zero is the lowest temperature that can theoretically exist and is given a temperature of 0 K. In terms of the kinetic theory, absolute zero is the temperature at which the molecules of matter have their lowest possible average kinetic energy. In a simplified model, the molecules are considered to have no average kinetic energy at absolute zero, in other words they have no random movement. In practice, quantum mechanics requires that they have a minimum kinetic energy, called the zero-point energy.

2 SPECIFIC HEAT CAPACITY

Transferring the same amount of heat energy to two different objects will increase their internal energy by the same amount. However, this will not necessarily cause the same rise in temperature in both. The effect that transferred heat energy has on the temperature of an object depends on three things:

1- The amount of heat energy transferred
2- The mass of the object
3- The specific heat capacity of the material from which the object is made

Specific heat capacity: the amount of energy needed to raise the temperature of 1 kg of a particular substance by 1K.

Different materials have different specific heat capacities because their molecular structures are different and so their molecules will be affected to different degrees by additional heat energy. For a certain amount of energy, ΔE, transferred to a material, the change in temperature $\Delta\theta$ is related to the mass of the material, m, and the specific capacity, c, by the expression:

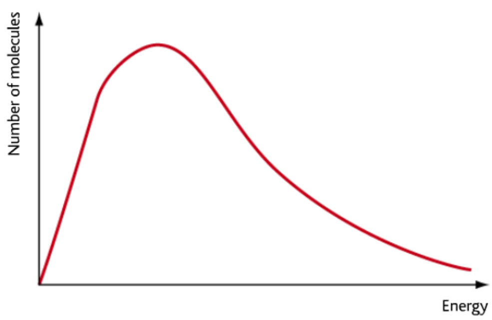
$$\Delta E = mc\Delta\theta$$

3 INTERNAL ENERGY

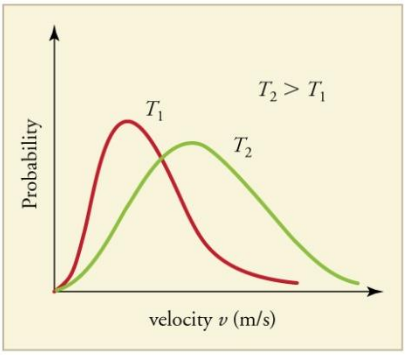
The average kinetic energy of the molecules in a material give it its temperature. However, as well as having kinetic energy, each molecule will have some potential energy virtue of its position within the structure of the material, or in relation to other molecules in the substance. This potential energy is due to the bonds between molecules. **If we sum the kinetic and potential energies of all molecules within a given mass of a substance, we have measured its internal energy**.

It is important to note that the molecules do not all have the same amount of kinetic and potential energies. The internal energy is randomly distributed across all the molecules according to the Maxwell-Boltzmann distribution. If we identify the individual velocity of each molecule in a particular sample, the values will range from a few moving slowly to a few moving very fast, with the majority moving at close to the average speed.

Maxwell-Boltzmann distribution: plot of the kinetic energy against the number of molecules that have that energy.



A Maxwell-Boltzmann distribution graph is for one specific temperature. As the temperature changes, so the graph changes. The peak on the graph moves towards higher energies (and therefore higher speeds) as the temperature increases.



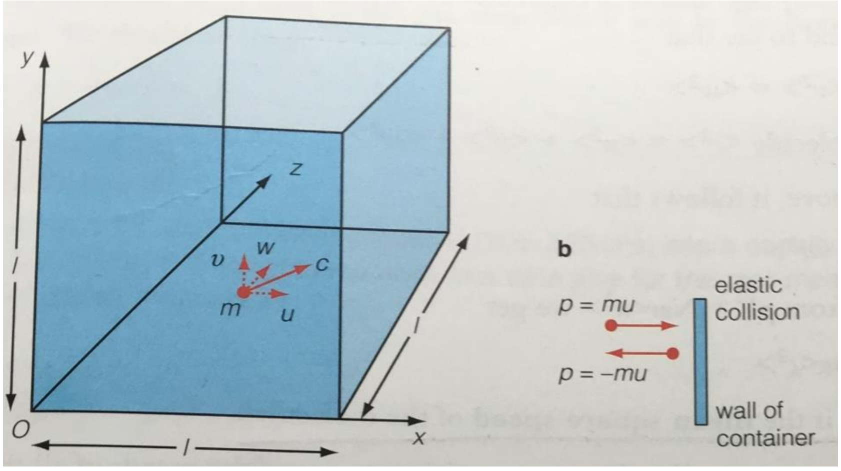
4 THE KINETIC THEORY

Evidence for the existence of molecules as well as the kinetic theory comes from Brownian motion. Robert Brown noticed that tiny grains of pollen, when suspended in water and viewed under a microscope, continually moved backwards and forwards with small, random, jerky paths. This happens because of the unequal bombardment of the very fine grains of pollen by the invisible water molecules, which themselves must therefore be in continuous motion.

Brownian motion provides a strong evidence for particles of matter being in continuous motion. Kinetic theory relates the macroscopic behaviour of an ideal gas, in terms of its pressure, volume and temperature, to the microscopic properties of its molecules.

Assumption	Experimental evidence
A gas consists of a very large number of molecules.	Brownian motion.
These molecules are in continuous, rapid, random motion.	
Collisions between molecules and between molecules and the walls of a container are perfectly elastic (i.e. no kinetic energy is lost)	If not, molecules would gradually slow down – this cannot be the case as Brownian motion is observed to be continuous. It would also mean that the gas would gradually cool down.
The volume occupied by the molecules themselves is negligible compared with the volume of the container.	It is easy to compress a gas by a large amount.
Intermolecular forces are negligible except during a collision.	From the above it follows that on average the molecules are very far apart relative to their size and so the intermolecular forces become very small.
The duration of collisions is negligible compared with the time spent in between collisions.	This also follows from the fact that the molecules, on average, are very much further apart than their size.

5 KINETIC MODEL OF TEMPERATURE



The molecule will have momentum mu in the $0x$ direction. On making an *elastic collision* with the wall of the container it will rebound with momentum $-mu$. The *change* in its momentum will therefore be $-2mu$.

If the speed of the molecule in the $0x$ direction is u, then it will take a time $t = l/u$ to cross the container. It will therefore make u/l crossings per second. As half of these crossings will be in each direction, it will make $\frac{1}{2}u/l$ collisions with each opposite wall per second.

As force is equal to the rate of change of momentum, the *force acting on the molecule* will be given by the number of collisions per second multiplied by the change in momentum on each collision, i.e.

$$F = \frac{1}{2}\frac{u}{l}(-2mu) = -\frac{mu^2}{l}$$

By Newton's third law, the *force of the molecule on the wall* is equal and opposite, namely

$$F = \frac{mu^2}{l}$$

As pressure is force per unit area, the pressure p exerted on the wall (which has area l^2) is

$$p = \frac{\text{force}}{\text{area}} = \frac{mu^2/l}{l^2} = \frac{mu^2}{l^3} = \frac{mu^2}{V} \text{ where } V = l^3 = \text{volume of container}$$

Hence: $pV = mu^2$

Now for the tricky bit! So far we have just considered *one* component (in the $0x$ direction) of *one* molecule. Let us assume that we have N such molecules with components $u_1, u_2, u_3 \ldots u_N$. We then have :

$$pV = m(u_1^2, u_2^2, u_3^2 \ldots u_N^2)$$

If we let the symbol $<u^2>$ represent the average or *mean* value of all the squares of the components in the $0x$ direction, we have

$$<u^2> = \frac{u_1^{\,2} + u_2^{\,2} + u_3^{\,2} + \ldots + u_N^{\,2}}{N} \Rightarrow N<u^2> = u_1^{\,2} + u_2^{\,2} + u_3^{\,2} + \ldots + u_N^{\,2}$$

$$pV = mN<u^2>$$

If we have a *very large number* of molecules of varying speed in *random* motion, then it is valid to say that

$$<u^2> = <v^2> = <w^2>$$

For each molecule $<c^2> = <u^2> + <v^2> + <w^2>$

From the above, it follows that

$$<u^2> = \frac{1}{3}<c^2>$$

Therefore, from $pV = Nm<u^2>$ we get

$$pV = \frac{1}{3}Nm<c^2>$$

where $<c^2>$ is the **mean square speed** of the molecules.

The 'mean square speed' is the average of the squares of the speeds of all the individual molecules of the gas. This means we have to square the speed of each molecule, add up all these squared speeds and divide by the total number of molecules to give the average. The 'root mean square speed' is then the square root of the mean square speed, i.e. $\sqrt{<c^2>}$.

The equation $pV = \frac{1}{3}Nm<c^2>$ that we have just derived can be rearranged into a more practical form by dividing each side of the equation by V. We then have:

$$p = \frac{1}{3}\frac{Nm}{V}<c^2>$$

As Nm is the total mass of the gas, then Nm/V will be the density ρ of the gas. We therefore have:

$$p = \frac{1}{3}\rho<c^2>$$

Let us now combine the equation $pV = \frac{1}{3}Nm<c^2>$, which we derived by using Newton's laws in conjunction with some simple kinetic theory assumptions, with the ideal gas equation $pV = NkT$, which is derived from experimental observation (see Section 12.3):

$$\frac{1}{3}Nm<c^2> = NkT \Rightarrow \frac{1}{3}m<c^2> = kT$$

If we now multiply both sides by $\frac{3}{2}$ we get

$$\frac{1}{2}m<c^2> = \frac{3}{2}kT$$

The expression $\frac{1}{2}m\langle c^2\rangle$ is the **average kinetic energy** of the randomly moving molecules of the gas. As k is a constant, it follows that

the average kinetic energy of the molecules of a gas is *proportional to the absolute temperature* of the gas.

It also follows from the above equation that, in this simple, non-quantum model, the average random kinetic energy of the molecules will be zero at absolute zero, i.e. when $T = 0$.

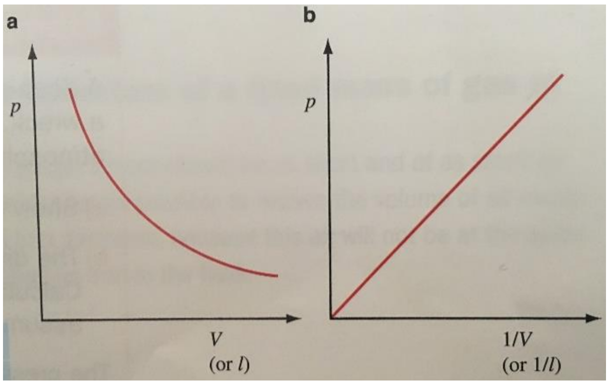
6 KINETIC THEORY OF GASES

A- Boyle's law

Boyle's law states that:

For a constant mass of gas at a constant temperature, the pressure exerted by the gas is inversely proportional to the volume it occupies.

$$p \alpha \frac{1}{V}$$



The graph of p against V is called an isothermal. An isothermal is a curve that shows the relationship between the pressure and volume of a gas at a particular temperature. As this is a straight line through the origin we can deduce that:

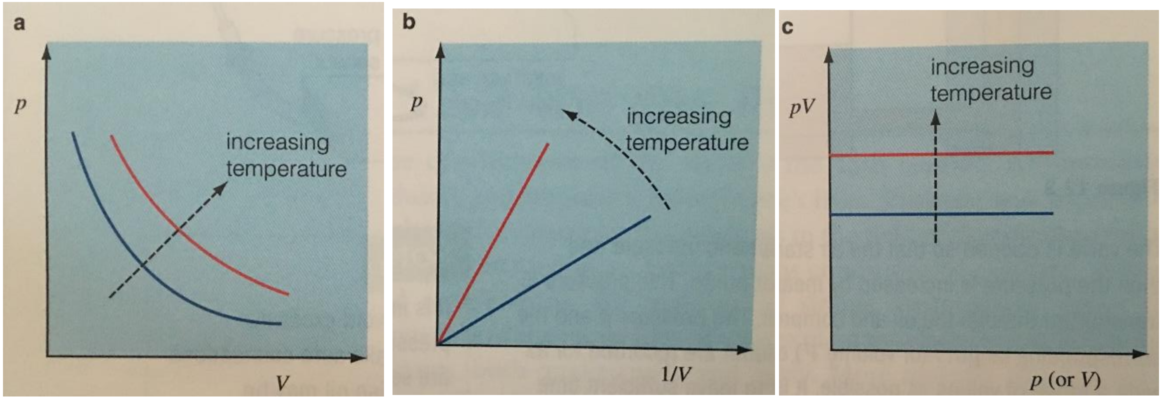
$$p = \frac{1}{V} \text{ or } p = constant \times \frac{1}{V}$$

Giving pV = constant

A convenient way of remembering Boyle's law for calculations is:

$$p_1V_1 = p_2V_2$$

Three different graphs may be drawn which illustrate Boyle's law and these are shown below along with how each graph would be altered with an increase in temperature.

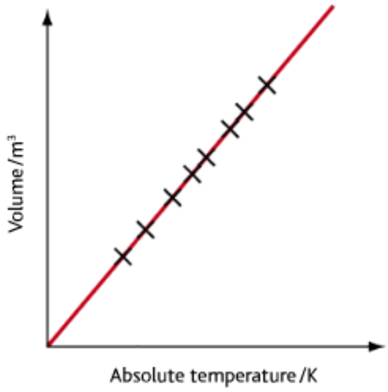


B- Charles's law

Charles's law states that:

For a constant mass of gas at a constant pressure, the volume occupies by the gas is proportional to its absolute temperature.

$$V \propto T$$



C- The pressure law

The pressure law states that:

For a constant mass of gas at a constant volume, the pressure exerted by the gas is proportional to its absolute temperature.

The pressure law can be shown in symbols as:

$$p \propto T$$

As the graph is a straight line through the origin, it shows that

$$p \propto T \text{ or } \frac{p}{T} = constant$$

for a fixed mass of gas at constant volume.

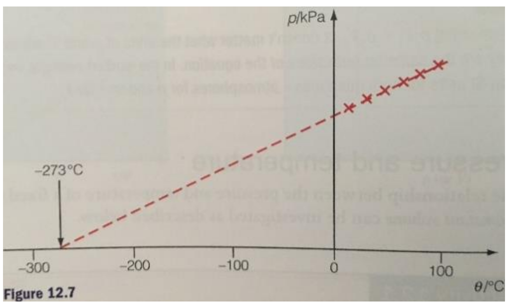


Figure 12.7

7 IDEAL GAS LAW

Assuming an ideal gas, we can combine the three gas laws to produce a single equation relating the pressure, volume, temperature and amount of gas:

$$pV = NkT$$

Where N is the number of molecules of the gas and k is the Boltzmann constant. The temperature must be absolute temperature in Kelvin.

This is known as the equation of state for an ideal gas, expressed in terms of the number of molecules present.

To make this more useful in practice, where we are dealing with very large numbers of molecules, we replace the Boltzmann constant and the number N to obtain:

$$pV = nRT$$

Where n is the number of moles of the gas; R is the Universal gas constant, R = 8.31 J kg^{-1} mol^{-1}.

Nuclear decay

1 BACKGROUND RADIATION

Background radiation: low levels of radiation always present in the environment, largely from natural sources.

The main natural sources of background radiation are:

- **Radioactive gases** (mainly radon) emitted from the ground, which can be trapped in buildings and build up to potentially dangerous level – high levels of radon can greatly increase the risk of lung cancer.
- **Radioactive elements** in the Earth's crust – mainly uranium and the isotopes it forms when it decays – these give rise to gamma radiation, which is emitted from the ground and rocks and building materials.
- **Cosmic rays** from outer space which bombard the Earth's atmosphere producing showers of lower-energy particles such as muons, neutrons and electrons also gamma rays.
- Naturally occurring radioactive isotopes present in our **food and drink**, and in the air we breathe, including carbon-14 and potassium-40.

Background radiation also comes from artificial sources such as **medical sources**.

2 NUCLEAR RADIATION

When a nuclear decay occurs, the radiation particle emitted will leave the nucleus with a certain amount of kinetic energy. As the particle travels, it will ionise particles in its path, losing a small amount of that kinetic energy at each ionisation. When all the kinetic energy is transferred, the radiation particle stops and is absorbed by the substance it is in that moment.

Alpha particles

Alpha particles are composed of two protons and two neutrons, the same as a helium nucleus. This is a relatively large particle with a significant positive change (+2e), so it is highly ionising. As it ionises so much, it quickly loses its kinetic energy and is easily absorbed. **A few centimetres** travel in air is enough to absorb alpha particles, and they are completely blocked by paper and skin.

Beta particles

A beta particle is an electron emitted at high speed from the nucleus when a neutron decays into a proton. With its single negative charge and much smaller size, the beta particle is much less ionising than an alpha particle, and thus penetrate much further. Several metres of air, or a **thin sheet of aluminium**, are needed to absorb beta particles.

Gamma rays

Gamma rays are high energy, high frequency, electromagnetic radiation. These photons have no charge and no mass and so will rarely interact with particles in their path, which means they are the least ionising nuclear radiation. They are never completely absorbed, although their energy can be significantly reduced by several centimetres of lead, or several meters of concrete. If the energy is reduced to a safe level, gamma rays are often said to have been absorbed.

3 DANGERS FROM NUCLEAR RADIATIONS

Ionising radiations can interact with the particles which make up human cells. There may be so much ionisation that the cells die as a result. Where there is less ionisation, the molecules of DNA in the cells may change slightly. These DNA mutations can cause cells to have an increased tendency to become cancerous. As the different types of nuclear radiation ionise to different extents, the hazard to humans is different for each type.

4 RATE OF RADIOACTIVE DECAY

Nuclear decay is spontaneous and random in nature. **Any radioactive nucleus may decay at any moment**. For each second that it exists, there is a certain probability that the nucleus will decay. This probability is called the decay constant, λ. The likelihood that a particular nucleus will decay is not affected by factors outside the nucleus, such as temperature and pressure, or by the behaviour of the neighbouring nuclei – each nucleus acts entirely independently.

If we have a large sample of the nuclei, the probability of decay will determine the fraction of these nuclei that will decay each second. Naturally, if the sample is larger, then the number that decay in a second will be greater. So the number decaying per second, called the activity A, is proportional to the number of nuclei in the sample, N. Mathematically, this is expressed as:

$$A = -\lambda N$$
$$\frac{dN}{dt} = -\lambda N$$

The minus sign in the formula occurs because the number of nuclei in the sample, N, decreases with time.

The formula for the rate decay of nuclei in a sample is a differential equation. It can be solved to give a formula for the number of nuclei remaining in a sample, N, after a fixed time, t:

$$N = N_0 e^{-\lambda t}$$

Half-life

The activity of a radioactive sample decreases over time as the radioactive nuclei decay. While activity of a sample depends on the nuclei present, the rate at which the activity decreases depends only on the particular isotope. A measure of this rate of decrease of activity is the half-life. This is the time taken for half of the atoms of that nuclide within a sample to decay.

Mathematically, the half-life can be found by putting $N = ½ N_0$ into the decay equation:

$$N = N_0 e^{-\lambda t}$$
$$\frac{N_0}{2} = N_0 e^{-\lambda t_{½}}$$
$$½ = e^{-\lambda t_{½}}$$
$$\ln(½) = -\lambda t_{½}$$
$$-\ln 2 = -\lambda t_{½}$$
$$t_{½} = \frac{\ln 2}{\lambda}$$

Rearranging, this also gives us:

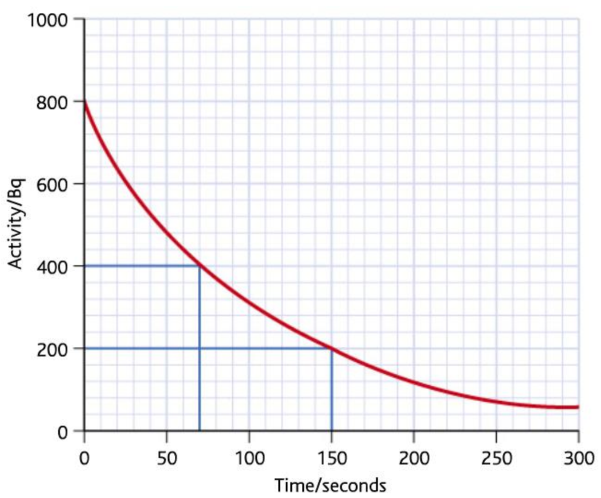
$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Half-life graphs

An experiment to determine the half-life of a substance will usually measure its activity over time. As activity is proportional to the number of nuclei present, when the activity is plotted against time, the shape of the curve is exponential decay. The activity, A, follows the equation:

$$A = A_0 e^{-\lambda t}$$

We can use the graph of activity against time to determine the half-life of the substance by **finding the time taken for the activity to halve**.



Notice that the time interval is not identical each time. This is due to random nature of radioactive decay, plus experimental and graphing errors. The best-fit curve will be a matter of the drawer's judgement. Thus, to get the best answer for the half-life, we must undertake the analysis on the graph several times in different parts of the graph and average the results.

5 NUCLEAR BINDING ENERGY

Energy-mass equivalence

Particle	Mass / atomic mass units (u)	Mass / SI units (kg)
Proton	1.007 276	1.672623×10^{-27}
Neutron	1.008 665	1.674929×10^{-27}
Electron	0.000 548 58	9.109390×10^{-31}

We might expect that if we known the constituent parts of any nucleus, we can calculate its mass by finding the total mass of its nucleons. However, in practice we find that the actual, measured mass of a nucleus is always less than the total mass of its constituent nucleons. This difference is called the **mass deficit**, or sometimes the mass defect.

$$1 \text{ u} = 1.66 \times 10^{-27}$$

Nuclear binding energy

The mass deficit comes about because a small amount of the mass of the nucleons is converted into the energy needed to hold the nucleus together. This is called binding energy. It is calculated using Einstein's mass-energy relationship:

$$\Delta E = c^2 m$$

,where c is the speed of light.

There are two common systems of units for calculating binding energy. If you have calculated the mass deficit in kilograms (SI units) then using c = 3.00×10^8 m s^{-1} will give the binding energy in joules. Alternatively, if you have calculated the mass deficit in atomic mass units, then you convert this into binding energy in mega-electronvolts (MeV) using:

1 u = 931.5 MeV

6 Nuclear binding energy per nucleon

Nuclear binding energy per nucleon which is the energy that would be needed to remove one nucleon from a nucleus can be calculated by dividing the nuclear binding energy by the number of nucleons. Drawing a graph of binding energy per nucleon against mass number for the nuclei gives us a useful means of comparing how tightly different nuclides are bound together.

The graph shows us that small nuclides can combine together to make larger nuclei (up to Fe-56) with a greater binding energy per nucleon. This process is called nuclear fusion. Similarly, larger nuclei can break up into smaller pieces which have a greater binding energy per nucleon. Reactions like this are called nuclear fission.

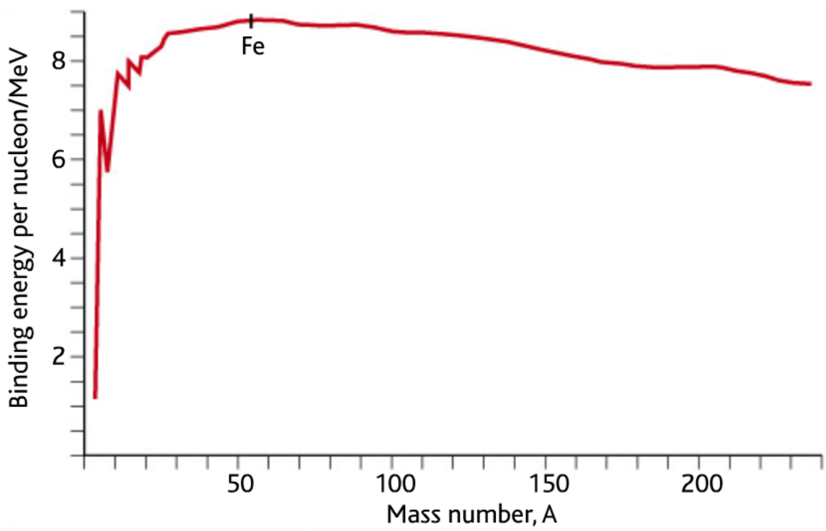


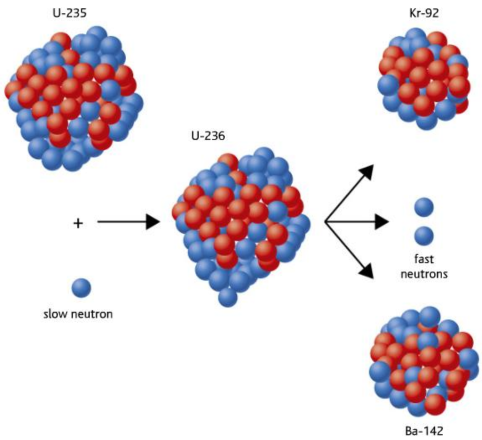
fig. 5.2.2 Graph of binding energy per nucleon against mass number, A.

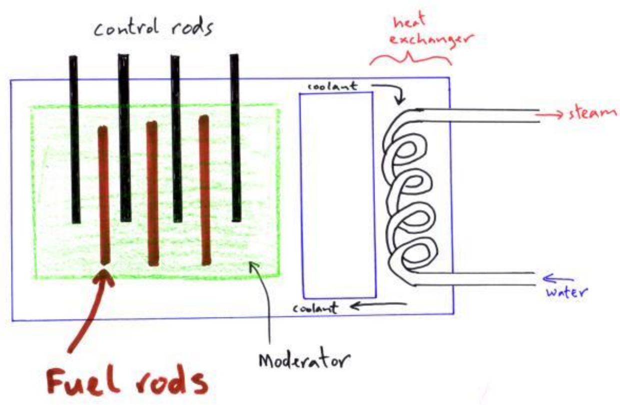
7 NUCLEAR FISSION

Nuclear fission is the process by which a large nucleus breaks up into smaller daughter nuclei, with the release of some neutrons and energy.

Process of fission of U-235

- U-235 absorbs a slow-moving neutron.
- U-235 splits and daughter elements are formed.
- This releases energy and more neutrons.
- Once fission has taken place, the neutrons can be absorbed by other nuclei and further reactions can take place. This is called a chain reaction.

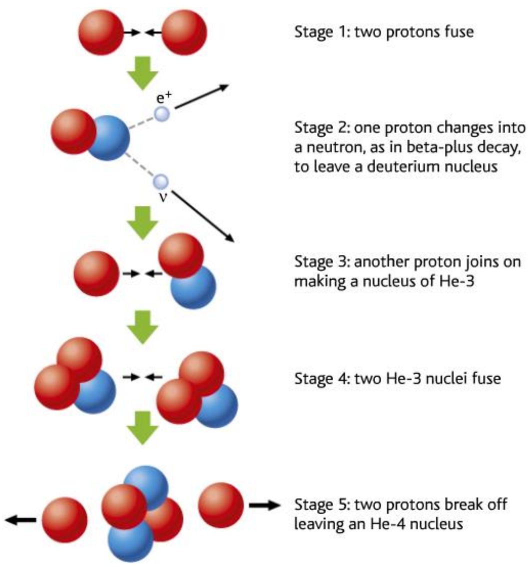




Nuclear power stations use fission reactions to generate the heat needed to produce steam. The nuclear reactor controls the chain reaction so that energy is steadily released. Fission occurs in the fuel rods and causes them to become very hot. The coolant is a fluid pumped through the reactor. Control rods, made of boron, absorb neutrons, preventing the chain reaction getting out of control. Moving the control rods in and out of the reactor core changes the amount of fission that takes place.

8 NUCLEAR FUSION

If we take some light nuclei and force them to join together, the mass of the new, heavier, nucleus will be less than the mass of the constituent parts, as some mass is converted into energy. However, not all of this energy is used as binding energy for the new larger nucleus, so energy will be released from this reaction. The binding energy per nucleon afterwards is higher than at the start. This is the process of nuclear fusion and is what provides the energy to make stars shine.



Problems about nuclear fusion

Scientists have not yet successfully maintained a controlled nuclear fusion reaction.

- Very high temperatures are needed
- To overcome electrostatic repulsion / forces
- Nuclei come close enough to fuse / for strong (nuclear) force to act
- Very high densities are needed
- (Together with high nuclei speeds) this gives a sufficient collision rate
- (Very high) temperatures lead to confinement problems
- Contact with container causes temperature to fall (and fusion to cease)

Oscillations

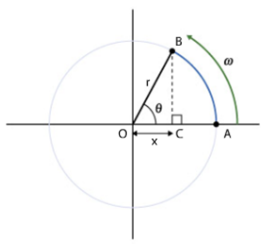
1 Simple harmonic motion

Simple harmonic motion is periodic motion about an equilibrium position and all SHMs share two common characteristics:

- The resultant force acting on the oscillating body, and therefore its acceleration, is proportional to the displacement of the body from the equilibrium position.
- The resultant force, and therefore the acceleration, always acts in a direction towards the equilibrium position.

These conditions are combined into a simple equation:

$$F = -kx \text{ or } a = -A\omega^2$$



X is the horizontal distance from the centre. **The motion is a projection of a circular motion** so the equations for angular velocity, displacement, frequency and time are equally valid for simple harmonic motion.

When the object is at position A, this projected distance x is equal to the radius of the circle, r, but at position B this distance is shown by OC. This distance can be calculated from:

$$x = r\cos\theta$$

$$\text{Or: } x = r\cos(\omega t)$$

In this experiment a spring is used therefore r is replaced with A which denotes the displacement. The equation for SHM is then:

$$F = -kA\cos(\omega t)$$

Equations for displacement, velocity and acceleration can be derived from this single equation.

$$x = \frac{-F}{k} \rightarrow \frac{-F}{-k} = A\cos(\omega t)$$

Therefore,

$$x = A\cos(\omega t)$$

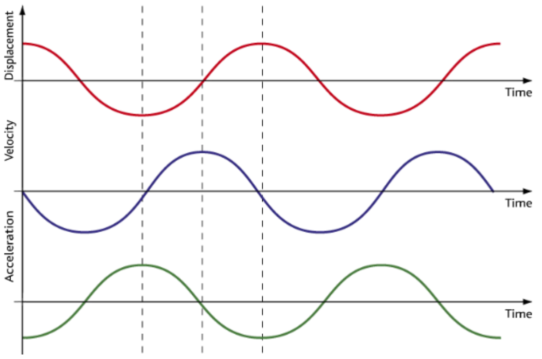
$$\frac{dx}{dt} = velocity\ (v) \text{ and } \frac{d^2x}{dt^2} = acceleration\ (a)$$

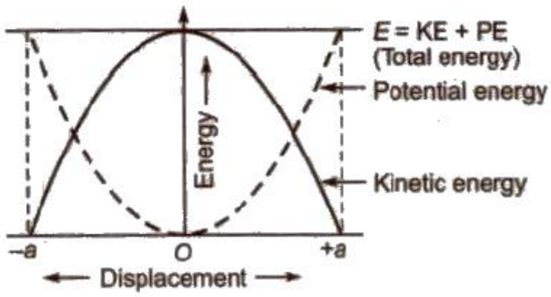
$$v = -A\omega\sin(\omega t)$$

$$a = -A\omega^2\cos(\omega t)$$

$T = \frac{1}{f} = \frac{2\pi}{\omega}$ so, $\omega = 2\pi f$

2 GRAPHS OF SIMPLE HARMONIC MOTION





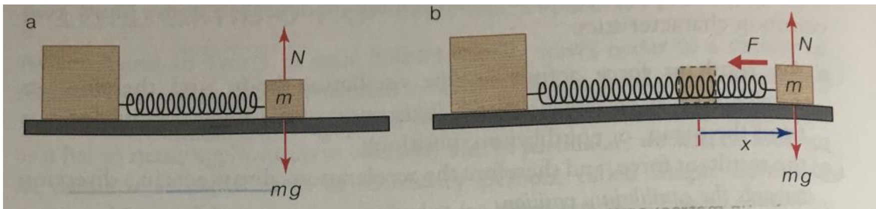
There are two key points in the motion:

- At each end of the oscillation, when the velocity is momentarily zero, the displacement curve is a maximum, positive or negative (A or –A), and its gradient = 0.
- At the centre of the motion, corresponding to x = 0, the velocity has its maximum value – you can see from the graph for x that the gradient, and hence the velocity, is a maximum at each point where the curve crosses the t-axis; furthermore, the gradient alternates between being positive and negative as the body moves first one way, and then back again, through the mid-point of its oscillations.

As a pendulum swings to and fro there is a continuous **interchange of kinetic and gravitational potential energy**. At one end of its swing, the pendulum momentarily comes to rest and so its kinetic energy is zero. At this point it will have maximum potential energy because the bob is at its highest point. As the bob swings down, it loses gravitational potential energy and gains kinetic energy. At the bottom of the swing, the midpoint of motion, the bob will have maximum velocity and thus maximum kinetic energy. As this is the lowest point of its motion, its gravitational potential energy will have its minimum value. This cyclic interchange of energy is repeated twice every oscillation.

3 SPRING

The equation for a spring that obeys Hooke's law is F = kx where k is the spring constant, or stiffness of the spring, and x is the extension.



From $F = ma \quad \Rightarrow a = \frac{F}{m}$, where $F = -kx$

$$\Rightarrow a = -\frac{kx}{m} = -\frac{k}{m}x$$

Note the introduction of the minus sign is to show the difference in direction of F and x.

This equation is of the form

$$a = -\omega^2 x$$

where $\omega = \sqrt{\frac{m}{k}}$.

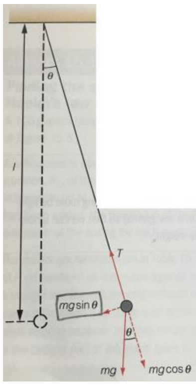
As shown later, $\omega = 2\pi f$, and so $T = 2\pi/\omega$. Therefore a mass m oscillating horizontally on a spring that obeys Hooke's law will execute s.h.m. with an oscillation period given by the equation

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Analysis of a spring oscillating *vertically* is more complex (see Exam Practice Question 15). However, the result turns out to be the same! A mass oscillating on a **light vertical** spring that obeys Hooke's law also execute s.h.m with period given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

4 SIMPLE PENDULUM



In this context simple means

- A small, dense, pendulum bob and
- A light, inextensible string

In the case of the simple pendulum, the force causing the oscillation is provided by a component of the weight of the pendulum bob.

The required force is the component mgsin θ. If θ is small then sin θ is close to θ and so the force is proportional to displacement. It can then be shown, for oscillations of small amplitude that the period, to a good approximation is given by

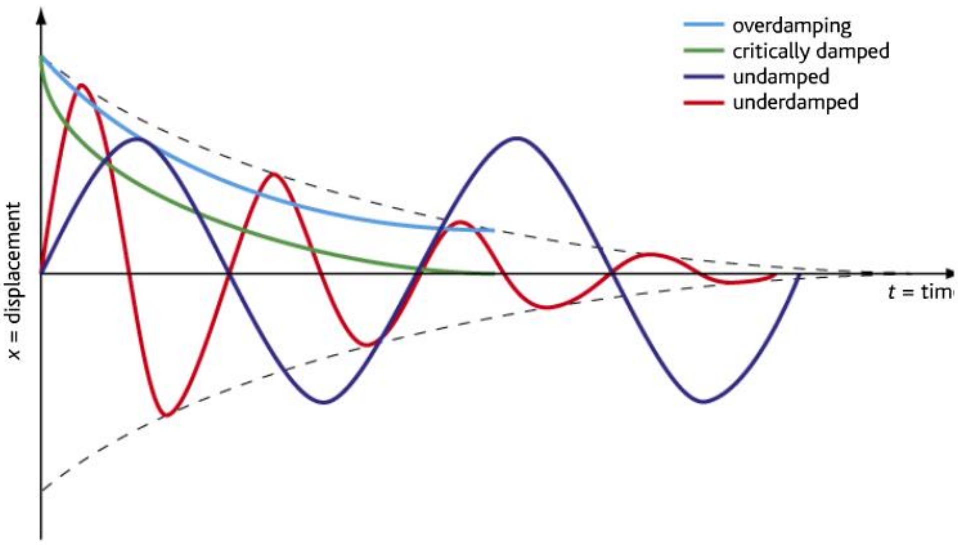
$$T = 2\pi\sqrt{\frac{l}{g}}$$

5 FREE, DAMPED AND FORCED OSCILLATIONS

A **free oscillation** is one in which no external force acts on the oscillation system except the force that gives rise to the oscillation.

A **damped oscillation** is one in which energy is being transferred to the surroundings, resulting in oscillations of reduced amplitude and energy. An oscillation system does work against external forces acting on it, such as air resistance, and so uses up some energy. This transfer of energy from the oscillating system to internal energy of the surrounding air causes the oscillations to slow down and eventually die away.

Forced oscillations occur if a force is repeatedly or continually applied to keep the oscillation going so that the system is make to vibrate at the frequency of the vibrating source and not at its own natural frequency of vibration.



6 RESONANCE

If a system or object is forced to vibrate at its natural frequency, it will the maximum energy and amplitude of the oscillations will reach its maximum.

The effects of resonance can be reduced by means of damping. If there is damping, the resonant frequency at which the amplitude is a maximum is lower than the natural frequency, and that this difference increases as the degree of damping increases. As the amount of damping increases, the resonant peak is much lower, and the resonance curve broadens out. Damping can be achieved by using a material which absorbs energy from the oscillation (energy is transferred into the internal energy of the object) or it can be achieved by the plastic deformation of ductile materials.

Astrophysics and cosmology

1 GRAVITATIONAL FIELDS

A particle that has mass will feel a force when it is in a gravitational field. Like electric fields created by charged particles, a massive particle will generate a radial gravitational field around itself. Unlike electric fields, gravity is always attractive.

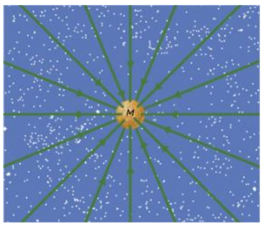
The force that a body will feel is the strength of the gravitational field (g) multiplied by the amount of mass (m), as given by the equation:

$$F = mg$$

Newton was the first scientist to determine the equation that gives us the gravitational force between two masses, m_1 and m_2, which are separated by a distance, r, between their centres of gravity.

$$F = \frac{-Gm_1m_2}{r^2}$$

G has the value 6.67×10^{-11} N m^2 kg^{-2}



Radial fields

Any mass will generate a gravitational field that will exert a force on any mass within the field. As gravity is always attractive, the field produced by a point mass will be radial in shape and the field lines will always point towards the mass.

Gravitational field strength

The radial field produced by a point mass naturally has its field lines closer together nearer the mass. This means that the strength of the field decreases with increasing distance from the mass causing it.

$$F = \frac{-GMm}{r^2} \qquad F = mg$$

These two expressions are calculating the same force, so must themselves be equal:

$$F = \frac{-GMm}{r^2} = mg$$

Therefore:

$$g = \frac{-GM}{r^2}$$

i.e. the field strength is independent of the object being acted upon.

Similarities and differences between gravitational and electric fields

	Gravitational field	Electric field
Force	$F = \frac{-Gm_1m_2}{r^2}$	$F = \frac{kq_1q_2}{r^2}$
Force is between	Objects with mass	Objects with charge
Field strength	$g = \frac{-Gm}{r^2}$	$E = \frac{kq}{r^2}$
Constant of proportionality	G	k
Force in a radial field	Always attractive	Can be attractive or repulsive
Potential in a radial field	J kg^{-1} Scalar Always less than zero	J C^{-1} Scalar Sign depends on charges

2 LUMINOSITY AND FLUX

Luminosity is used to describe the total output power of a star, unit W. For example, the luminosity of the Sun is L_0 = 3.90 x 10^{26} W.

The electromagnetic wave energy per second per unit area from a star reaching us on Earth is called the **radiation flux** from the star, symbol F, unit W m^{-2} (i.e. J s^{-1} m^{-2}).

$$F = \frac{L}{4\pi d^2}$$

3 STANDARD CANDLES

- Standard candles are stellar objects of known luminosity.
- Standard candle's brightness (radiation flux) on earth is measured or known.
- The inverse square law $F = \frac{L}{4\pi d^2}$ is used.
- Distance to standard candle is calculated.

Cepheid stars are an example for standard candles. These stars have a luminosity which varies periodically. The process of finding the distance to a Cepheid star is:

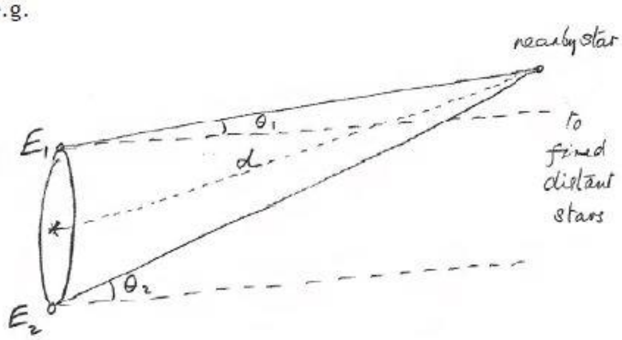
- Locate the Cepheid variable star.
- Measure its period T.
- Find the star's luminosity L.
- Measure the radiation flux (brightness) F from the star at Earth
- Calculate d using $= \sqrt{\frac{L}{4\pi F}}$.

4 TRIGONOMETRIC PARALLAX

To measure the distance to relatively close stars, astronomers use a method that is commonly used in surveying, known as trigonometric parallax. As the Earth moves around the Sun, a relatively close star will appear to move across the background of more distant stars. This optical illusion is used to determine the distance of the star. The star itself does not move significantly during the course of the observations. To determine the trigonometric parallax you measure the angle to a star, and observe how that changes as the position of the Earth changes. We know that in six months the Earth will be exactly on the opposite side of its orbit, and therefore will be two astronomical units from its location today.

Using observations of the star to be measured against a background of much more distant stars, we can measure the angle between the star and the Earth in these two different positions in space, six months apart. As we know the size of the Earth's orbit, geometry allows calculation of the distance to the star.

e.g.



- The star is viewed from two positions at 6 months intervals.
- The change in angular position of the star against background of fixed stars is measured.
- Trigonometry is used to calculate the distance to the star.
- The diameter/radius of the Earth's orbit about the Sun must be known.

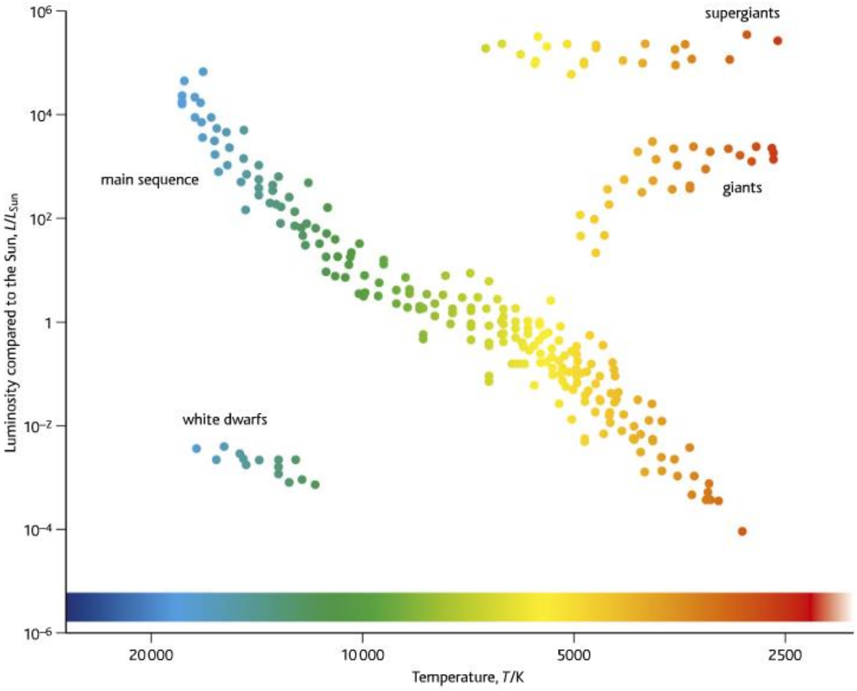
This method of distance measurement is only suitable for the stars and star clusters closest to the Earth.

- If the stars are too far away the angular displacement is too small to be determined and the uncertainty is too large.

5 THE HERTZSPRUNG-RUSSELL DIAGRAM

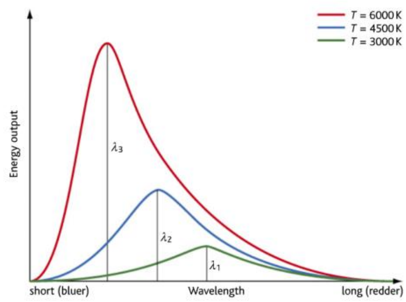
The H-R diagram is a **plot of stellar luminosity against surface temperature**. Note it is a diagram not a graph – each dot represents a single star. To understand the diagram you need to remember what the two axes are telling you.

- The vertical axis is luminosity, scaled in Sun-powers, i.e. multiples of L_0. The positions of the stars go up and down from 1 in powers of 10, i.e. the scale is logarithmic. The Sun's luminosity $L_0 = 3.90 \times 10^{26}$ W.
- The horizontal axis is the surface temperature T of the star in Kelvin. This scale is also logarithmic and goes from high temperature on the left to low temperatures on the right.



6 WIEN'S LAW

To calculate luminosity, we need to know the temperature of the star. When we examine the range of wavelengths emitted by a star, known as its spectrum, we find that some wavelengths are given off with more intensity than others. This law assumes stars are black-body radiators meaning they emit all wavelengths, yet with different intensities. Very hot stars are blue and cool stars are red. At higher temperatures the curve has a more pronounced peak, and the wavelength of the peak output gets shorter as the temperature rises, the relationship between the peak wavelength and temperature is described by Wien's law:



$$\lambda_{max} T = 2.898 \times 10^{-3}\,\mathrm{m\,K}$$

7 THE STEFAN-BOLTZMANN LAW

The electromagnetic spectrum goes from the smaller γ-rays and X-rays to the longer microwaves and radio waves. A star emits a continuous spectrum for which the total power output, its luminosity L, is proportional not to its temperature T, but to T^4.

That this does not match up with the L-T relationship shown for main sequence stars on an H-R diagram is because the high-luminosity stars on the H-R diagram are larger than stars like the Sun, and the Sun is larger than stars at the bottom right of the H-R diagram. The full relationship is called the Stefan-Boltzmann law:

$$L = \sigma T^4 \times \text{surface area}$$

where σ is the Stefan–Boltzmann constant,
$\sigma = 5.67 \times 10^{-8} \mathrm{W\,m^2\,K^{-4}}$.

So:

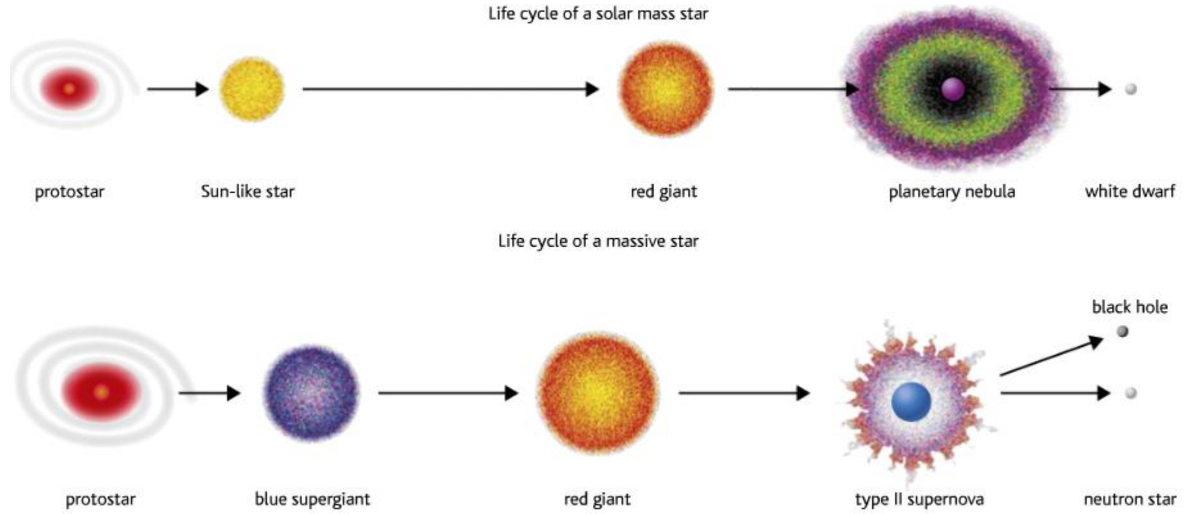
$$L = \sigma A T^4$$

which for a sphere would become:

$$L = 4\pi r^2 \sigma T^4$$

Working on the assumption that a star acts like a black body emitter, which is a very good approximation, this equation describes the luminosity of a star.

8 THE LIFE CYCLE OF A STAR



The majority of material in the Universe is hydrogen or helium, and it is from these elements that stars are initially formed. From an accreting collection of these gases, called a protostar, the life cycle of a star follows a number of stages, with the star ending its life as a white dwarf, neutron star or black hole.

- As the star undergoes nuclear fusion, the binding energy differences of the nuclei before and after fusion mean that the process releases energy, often as electromagnetic radiation, to heat the star. **The pressure from the vibration of its particles and the electromagnetic radiation trying to escape hold up the structure of the star against gravitational collapse.** It is this constant battle between outward pressure and gravity that drives the evolution of a star throughout its lifetime. The initial mass of the star is a critical factor in determining how the battle wages, and thus which of the possible life cycles a star will follow.

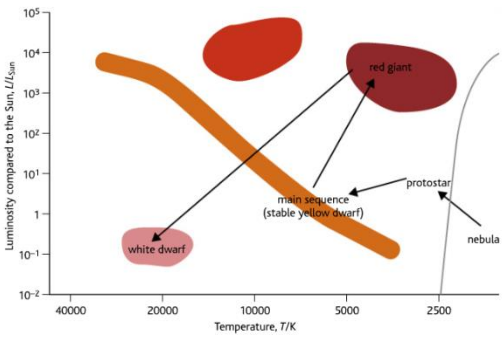
Low-mass stars (Life cycle of our Sun)

Stage 1: The Sun was formed or born from a cloud of hydrogen and helium.

Stage 2 (current stage): The Sun joined the main sequence stars as it undergoes nuclear fusion of hydrogen, converting into helium.

Stage 3: Eventually, the Sun will run low on hydrogen fuel, but will have produced so much energy that it will expand slightly. This expansion causes the temperature to fall and the star becomes a red giant. Once most of the hydrogen fuel is used, the star will start fusing helium nuclei. This complex process can cause an explosion that throws some material from the star out into space, forming a planetary nebula. As the fuel to produce energy to support the star runs out, the outward pressure from fusion drops and gravity takes hold, causing the star to contract to a much smaller size. This heats significantly and it becomes a white dwarf.

Stage 4 (theoretical): As time continues, the star will slowly run out of energy and die, passing through the red dwarf stage to become a black dwarf.



Main sequence stars:

- Fusion of hydrogen atoms into helium nuclei
- Constant size

Red giants:

- Fusion of helium nuclei and formation of heavier elements
- Loss of material into space and formation of a planetary nebula
- Decrease in size and increase in temperature leading to formation of white dwarfs

White dwarfs:

- White dwarf stars are the core remnant of a red giant star
- There is no fusion going on in the white dwarf
- They have small surface area so they are not very luminous
- They are very hot, and appear white (because they emit all visible wavelengths)

Massive stars

If a protostar has more than four times the mass of our Sun, the star begins life as a blue supergiant. As with low mass stars, nuclear fusion begins and the star enters a stable stage of life in which heat pressure and gravity are in equilibrium. However, the fusion processes happen at much higher temperatures than in lower mass stars. This means that it burns very quickly, and the conditions make it possible for further fusion of some of the larger atoms it produces to occur. The fusion of helium can produce a variety of the larger elements.

When the material of such a star has been fused to the point where it is mostly iron, it can no longer undergo nuclear fusion and it stops producing energy. With the enormous gravitational forces produced by the large mass, the star undergoes an incredible collapse. This sudden increase in density produces a sudden huge burst of energy, effectively bouncing the collapse back out. This explosion is a supernova and it is the most immense burst of energy ever witnessed.

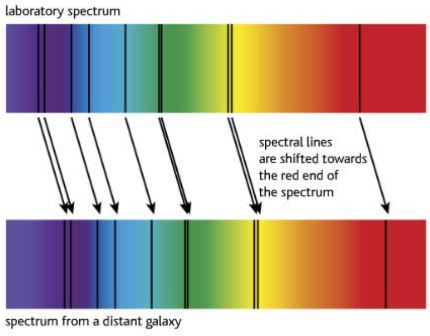
Within a supernova explosion there is so much energy that nuclear reactions occur which produce the elements above iron in the periodic table. The natural occurrence of these elements is evidence that supernovae must have occurred in the past, as the binding energies of these heavy elements are such that they cannot be created in other natural processes in the Universe.

After a high-mass star has exploded as a supernova, the entire star may be completely shattered. If there remains a central core of stellar material, this will either be a neutron star or a black hole. A neutron star consists almost entirely of neutrons, packed as densely together as the nucleons within the nucleus of an atom. Black holes are even smaller and hold even more matter than neutron stars. This means that their gravitational pull is immense – so strong that even things travelling at the speed of light cannot escape.

9 THE AGE OF THE UNIVERSE AND HUBBLE'S LAW

Doppler red shift

The amount of red shift a galaxy exhibits, z, allows us to calculate how fast it is moving. This can be done using measurements of either wavelength or frequency changes.



$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$$

Hubble's law

The recession velocity of a galaxy is directly proportional to its distance away from us. The constant of proportionality, the Hubble constant, can be found from the gradient of the graph. The best modern value is considered accurate to within 5%. H_0 = 71 km s^{-1} Mpc^{-1}. With an accurate value for H_0, astronomers can now use Hubble's law to determine distances to newly observed objects.

$$v = H_0 d$$

Age of the universe

As all distant objects show a red shift, they are all moving away from us. This implies that the Universe as a whole is expanding. If we imagine time running backwards from the present, then the Universe would contract until a moment where everything was in the same place. This would be the time of the Big Bang, when everything first exploded outwards from that single point. Thus, if we can find the Hubble constant, it will tell us how quickly the Universe is expanding.

For an object to travel a distance d_0 from the beginning of time, at a speed of v_0, the time taken, T_0, can be calculated from the basic equation for speed:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$v_0 = \frac{d_0}{T_0}$$

$$T_0 = \frac{d_0}{v_0}$$

If we consider the gradient of the Hubble graph, $H_0 = \frac{v_0}{d_0}$:

$$T_0 = \frac{1}{H_0}$$

Note that in this calculation you should use the same units for the distance and for the length component of the units for recession velocity. Usually, the Hubble constant is quoted in units of $\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

$$H_0 = 71\,\mathrm{km\,s^{-1}\,Mpc^{-1}} = 71\,000\,\mathrm{m\,s^{-1}\,Mpc^{-1}}$$

$$1\,\mathrm{pc} = 3.08 \times 10^{16}\,\mathrm{m}$$

$$1\,\mathrm{Mpc} = 3.08 \times 10^{22}\,\mathrm{m}$$

$$\therefore\ H_0 = \frac{71\,000}{3.08 \times 10^{22}}$$

$$= 2.31 \times 10^{-18}\,\mathrm{m\,s^{-1}\,m^{-1}}$$

$$H_0 = 2.31 \times 10^{-18}\,\mathrm{s^{-1}}$$

So, $$T_0 = \frac{1}{H_0}$$

$$= \frac{1}{2.31 \times 10^{-18}}$$

$$\therefore\ T_0 = 4.33 \times 10^{17}\,\mathrm{s}$$

This value for T_0 gives the age of the Universe as 13.7 billion years.

10 THE FATE OF THE UNIVERSE

The 5% of the universe

- Nearby galaxies do not have enough visible gravitationally attracting (hadronic) matter to keep the outermost stars moving in circles around the galaxy's centre.
- Distant galaxies appear to be moving away from the Solar system at an accelerating rate, rather than slowing down as predicted by Newton's law of gravitation.

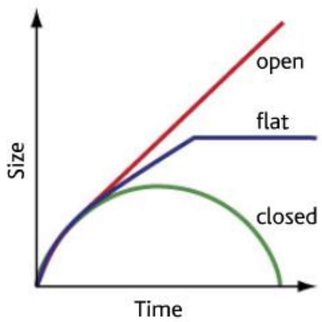
The first problem suggests 24% of the mass of nearby galaxies consist of **dark matter**. Dark matter cannot be detected via the EM-interaction but it has mass and exerts gravitational force.

The second problem suggests that about 71% of our universe consists of a mysterious antigravity material known as **dark energy**.

Therefore, 95% of the universe is unknown while the 5% of it is ordinary atoms.

The future of the universe

What the fate of the universe will be depends on the average density of matter it now contains. With a high density the universe implodes into a Big Crunch; with a small density it will go on expanding forever. When cosmologists try to estimate the average density of matter in the universe, they discover problems. Some of these problems are:



- Difficulty in making accurate measurement of distance to galaxies
- Hubble constant has a large uncertainty or age = 1/H may not be valid as gravity is changing the expansion rate
- Because of the existence of dark matter
- Values of the average density/mass of the universe have a large uncertainty
- Dark energy may mean we don't understand gravity as well as we thought we did (so it's hard to predict how gravity will determine the ultimate fate.

Open universe: If the density of the universe is lower than the critical density, the universe will continue expanding forever.

Closed universe: If the density of the universe is greater than the critical density, the universe will reach a maximum size and will then reduce in size until imploding in a Big Crunch.