Edexcel International A Level (IAL) Physics

Your notes

Nuclear Fusion & Fission

Contents

- Nuclear Binding Energy & Mass Deficit
- Atomic Mass Unit
- Binding Energy per Nucleon Graph
- Nuclear Fusion

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Nuclear Binding Energy

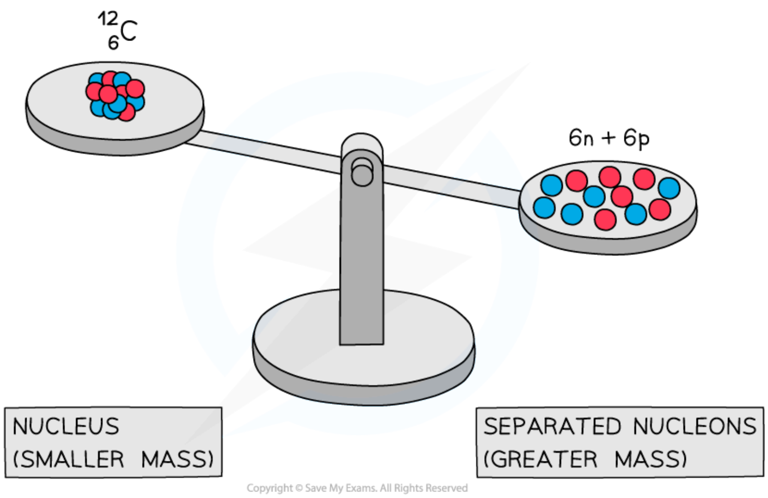
- Experiments into nuclear structure have found that the total mass of a nucleus is **less** than the sum of the masses of its constituent nucleons
  - This difference in mass is known as the **mass defect** or **mass deficit**
  - Mass defect is defined as:

    The difference between the measured mass of a nucleus and the sum total of the masses of its constituents

- The mass defect Δm of a nucleus can be calculated using:

$$\Delta m = Zm_p + (A - Z)m_n - m_{total}$$

- Where:
  - Z = proton number
  - A = nucleon number
  - m_p = mass of a proton (kg)
  - m_n = mass of a neutron (kg)
  - m_{total} = measured mass of the nucleus (kg)



A system of separated nucleons has a greater mass than a system of bound nucleons

- Due to mass-energy equivalence, this decrease in mass implies that energy is released
- Energy and mass are proportional, so, the total energy of a nucleus is **less than the sum of the energies** of its constituent nucleons

- Binding energy is defined as:

  The energy required to break a nucleus into its constituent protons and neutrons

- The formation of a nucleus from a system of isolated protons and neutrons therefore **releases energy**, making it an **exothermic** reaction
  - This can be calculated using the equation:

$$\Delta E = \Delta mc^2$$

Mass-Energy Equivalence

- Einstein showed in his Theory of Relativity that matter can be considered a form of energy and hence, he proposed:
  - Mass can be converted into energy
  - Energy can be converted into mass
- This is known as **mass-energy equivalence**, and can be summarised by the equation:

$$\Delta E = \Delta mc^2$$

- Where:
  - E = energy (J)
  - m = mass (kg)
  - c = the speed of light (m s^{-1})
- Some examples of mass-energy equivalence are:
  - The **fusion** of hydrogen into helium in the centre of the sun
  - The **fission** of uranium in nuclear power plants
  - Nuclear **weapons**
  - High-energy **particle collisions** in particle accelerators

Worked Example

The binding energy per nucleon is 7.98 MeV for an atom of Oxygen-16 (^{16}O).

Determine an approximate value for the energy required, in MeV, to completely separate the nucleons of this atom.

Answer:

Step 1: List the known quantities

- Binding energy per nucleon, E = 7.98 MeV

Step 2: State the number of nucleons

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- The number of nucleons is 8 protons and 8 neutrons, therefore 16 nucleons in total

Step 3: Find the total binding energy

- The binding energy for oxygen-16 is:

$$7.98 \times 16 = 127.7 \text{ MeV}$$

Step 4: State the final answer

- The approximate total energy needed to completely separate this nucleus is 127.7 MeV

Examiner Tips and Tricks

Binding energy is named in a confusing way, so be careful!

Avoid describing the binding energy as the energy stored in the nucleus – this is not correct – it is energy that must be put **into** the nucleus to **pull it apart.**

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Atomic Mass Unit

Atomic Mass Unit

- The unified atomic mass unit **(u or sometimes a.m.u)** is roughly equal to the mass of one proton or neutron:
  - **1 u = 1.66×10^{-27} kg**
  - This value is provided on the exam data sheet
- The a.m.u is commonly used in nuclear physics to express the mass of subatomic particles. It is defined as

The mass of exactly one-twelfth of an atom of carbon-12

- Therefore, one atom of carbon-12 has a mass of exactly 12 u
- Since mass and energy are interchangeable, the a.m.u can also be expressed in MeV
  - **1 u is equivalent to 931.5 MeV**

Table of common particles with mass in a.m.u

Particle	Mass / u
Proton	1
Neutron	1
Electron	0.0005
Alpha (α)	4

- The mass of an atom in a.m.u is roughly equal to the sum of its protons and neutrons (nucleon number)
  - For example, the mass of Uranium-235 is roughly equal to 235u
  - However, note that the actual mass is slightly lower than the expected mass, due to mass-energy equivalence
- a.m.u might be quoted in kg or MeV since mass and energy are equivalent via $\Delta E = \Delta mc^2$
  - MeV is a unit of energy whilst kg is a unit of mass

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Worked Example

Estimate the mass of the nucleus of the element copernicium-285 in kg. Give your answer to 2 decimal places.

Answer:

STEP 1

THE MASS OF AN ATOM IN A.M.U IS ROUGHLY ITS NUCLEON NUMBER

STEP 2

THE MASS OF COPERNICIUM-285 IN A.M.U:

285u

STEP 3

CONVERT TO kg:

$1\,u = 1.66 \times 10^{-27}\,kg$

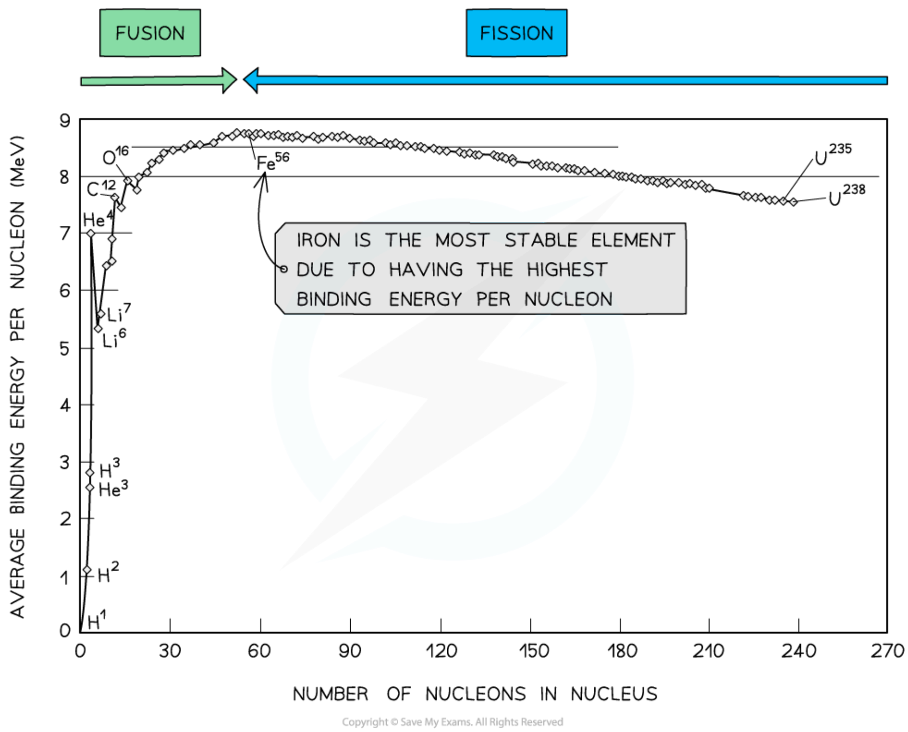
$285\,u = 285 \times 1.66 \times 10^{-27}\,kg = 4.73 \times 10^{-25}\,kg$ (2 d.p)

Binding Energy per Nucleon Graph

- When comparing the stability of different nuclei, it is useful to look at the **binding energy per nucleon**
- The binding energy per nucleon is defined as:

  The binding energy of a nucleus divided by the number of nucleons in the nucleus

- A higher binding energy per nucleon indicates a higher stability since it requires more energy to pull the nucleus apart
- Iron (A = 56) has the highest binding energy per nucleon, which makes it the most stable of all the elements



By plotting a graph of binding energy per nucleon against nucleon number, the stability of elements can be inferred

Key Features of the Graph

- At low values of A:
  - Nuclei tend to have a lower binding energy per nucleon, hence, they are generally less stable

- This means the lightest elements have weaker electrostatic forces and are the most likely to undergo **fusion**

- Helium (^4He), carbon (^{12}C) and oxygen (^{16}O) do not fit the trend
  - Helium-4 is a particularly stable nucleus hence it has a high binding energy per nucleon
  - Carbon-12 and oxygen-16 can be considered to be three and four helium nuclei, respectively, bound together
- At high values of A:
  - The general binding energy per nucleon is high and gradually decreases with A
  - This means the heaviest elements are the most unstable and likely to undergo **fission**

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Worked Example

Determine the binding energy per nucleon of Iron-56 ($^{56}_{26}Fe$) in MeV

Mass of a neutron = 1.675×10^{-27} kg

Mass of a proton = 1.673×10^{-27} kg

Mass of a $^{56}_{26}Fe$ nucleus = 9.288×10^{-26} kg

Answer:

Step 1: Calculate the mass defect

$$\text{Number of protons, } Z = 26$$

$$\text{Number of neutrons, } A - Z = 56 - 26 = 30$$

$$\text{Mass defect, } \Delta m = Zm_p + (A - Z)m_n - m_{total}$$

$$\Delta m = (26 \times 1.673 \times 10^{-27}) + (30 \times 1.675 \times 10^{-27}) - (9.288 \times 10^{-26})$$

$$\Delta m = 8.680 \times 10^{-28} \text{ kg}$$

Step 2: Calculate the binding energy of the nucleus

$$\text{Binding energy, } \Delta E = \Delta mc^2$$

$$E = (8.680 \times 10^{-28}) \times (3.00 \times 10^8)^2 = 7.812 \times 10^{-11} \text{ J}$$

Step 3: Calculate the binding energy per nucleon

$$\text{Binding energy per nucleon} = \frac{E}{A}$$

$$\frac{E}{H} = \frac{7.812 \times 10^{-11}}{56} = 1.395 \times 10^{-12}\ J$$

Step 4: Convert to MeV

J → eV: divide by 1.6×10^{-19}

eV → MeV: divide by 10^6

binding energy per nucleon =

$$\frac{1.395 \times 10^{-12}}{1.6 \times 10^{-19}} = 8\,718\,750\ eV = 8.7\ MeV\,(2\ s.f)$$

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Examiner Tips and Tricks

Checklist on what to include (and what not to include) in an exam question asking you to draw a graph of binding energy per nucleon against nucleon number:

- You will be expected to draw the best fit curve AND a cross to show the **anomaly that is Helium**
- Do not begin your curve at A = 0, this is not a nucleus!
- Make sure to correctly label both axes AND units for binding energy **per nucleon**
- You will be expected to include numbers on the axes, mainly at the peak to show the position of iron (^{56}Fe)

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Nuclear Fusion

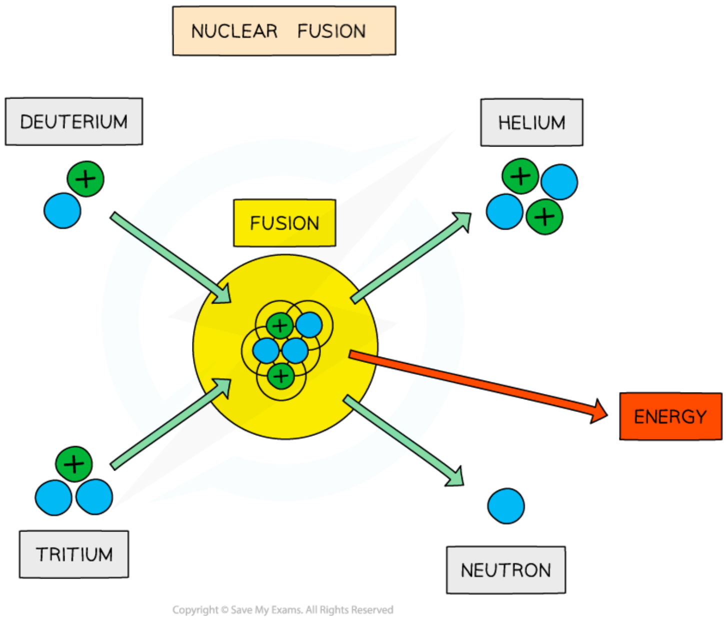
- Fusion is defined as:

Small nuclides that combine together to make larger nuclei, releasing energy

- Low mass nuclei, such as hydrogen and helium, can undergo fusion and release energy
- On Earth, research is focused on achieving the deuterium-tritium (D-T) reaction
- This involves fusing a deuterium nucleus and a tritium nucleus together to produce a helium nucleus and a neutron

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n$$

Deuterium-tritium fusion



The fusion of deuterium and tritium nuclei to form a helium nucleus and a neutron, with the release of energy

Conditions for nuclear fusion

- For two nuclei to fuse, both nuclei must have **high kinetic energy**
- This is because nuclei must be able to get close enough to fuse
- However, two forces acting within the nuclei make this difficult to achieve
  - **Electrostatic repulsion**

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    - Protons inside the nuclei are positively charged, which means that they **electrostatically** repel one another
  - **Strong nuclear force**
    - The strong nuclear force, which binds nucleons together, acts at very short distances within nuclei
    - Therefore, nuclei must get very close together for the strong nuclear force to take effect
- It takes a great deal of energy to overcome the electrostatic force, hence, fusion can only be achieved in an **extremely hot**, **dense** environment, such as the core of a star

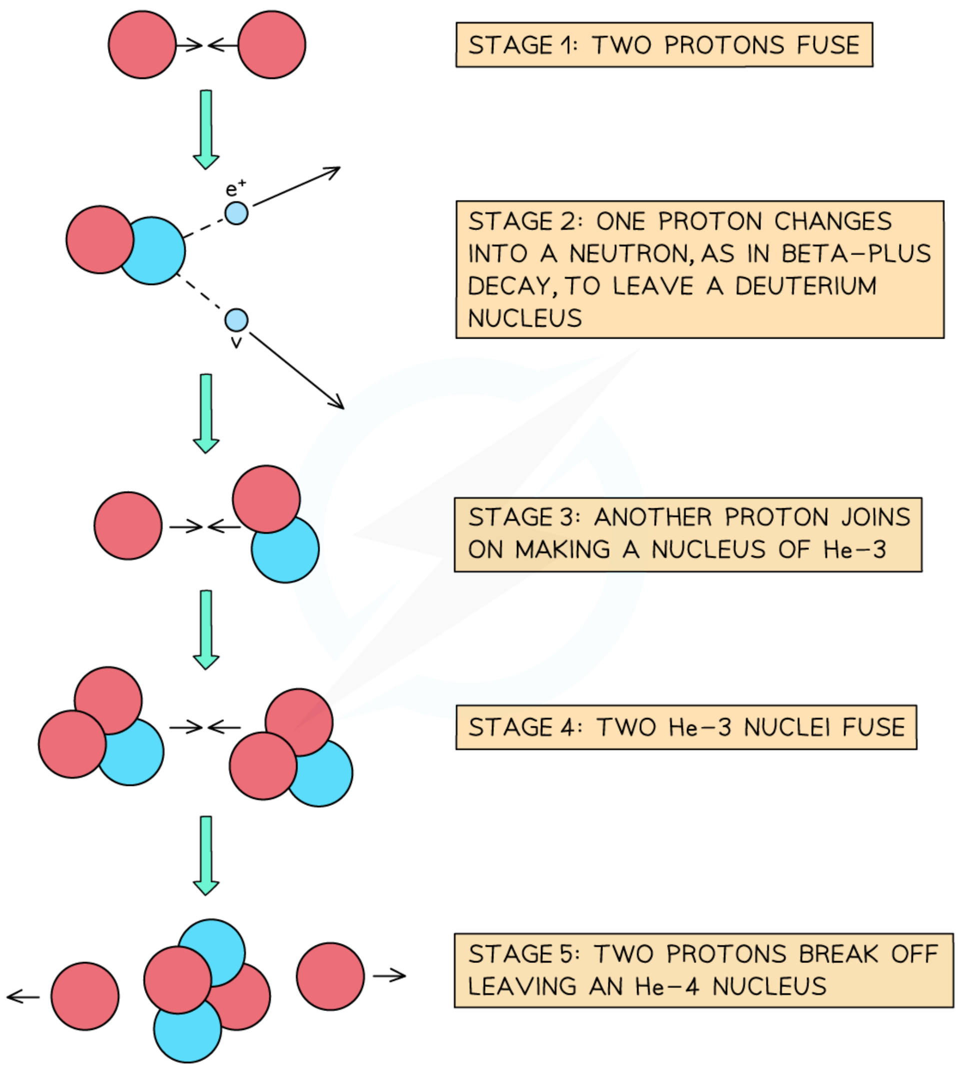
Fusion products

- When two hydrogen nuclei (protons) fuse, a deuterium nucleus is produced
  - A positron and an electron neutrino are also produced as one of the protons converts into a neutron through beta-plus decay

$$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + e^{+} + \nu_{e}$$

- In the centres of stars, four hydrogen nuclei $\left(^{1}_{1}H\right)$ fuse to produce a helium nucleus $\left(^{4}_{2}He\right)$, plus the release of energy
  - This provides fuel for the star to continue burning

The proton-proton chain



The proton-proton chain involves a series of nuclear reactions which produces a helium nucleus from the fusion of four protons

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Examiner Tips and Tricks

In the fusion process, the mass of the new, heavier nucleus is less than the mass of the constituent parts of the nuclei fused together, as some mass is converted into energy.

Not all of this energy is used as binding energy for the new, larger nucleus, so energy will be released from this reaction. The binding energy per nucleon afterwards is higher than at the start.