Edexcel International A Level (IAL) Physics

Your notes

Motion

Contents

Equations of Motion

- The kinematic equations of motion are a set of four equations which can describe any object moving with **constant** acceleration
- They are often referred to as the 'suvat' equations due to the variables they include
- These are:
  - *s* = **displacement** (m)
  - *u* = **initial velocity** (m s^{-1})
  - *v* = **final velocity** (m s^{-1})
  - *a* = **acceleration** (m s^{-2})
  - *t* = **time interval** (s)
- The four SUVAT equations are:

SUMMARY

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{(v + u)}{2}t$$

$$v^2 = u^2 + 2as$$

- These are all given on the data sheet
- All the variables, apart from **time t**, are vector quantities
  - This means they can either be positive or negative depending on their direction

Mastering the SUVAT Equations

- The SUVAT equations are **always** used when acceleration is constant, but not zero
- This is indicated in questions by
  - An explicit statement that an object is **'constantly accelerating'** or **'constantly decelerating'**, or words to that effect
  - A description of an object in freefall where either **'air resistance can be ignored'** or **'air resistance is negligible'**

- A statement that an accelerating or decelerating force is constant, since $F = ma$, constant force → constant acceleration

Your notes

- Questions may appear to be missing some information, this should be checked before starting to calculate
  - Objects in motion often start or end at rest, making either u or v = zero
  - Objects in freefall have a **constant** acceleration of 9.81 m s^{-2}
  - When an object is slowing down it has a **negative** value for acceleration, this must be included in the calculation
  - Objects thrown or shot upwards also have negative acceleration (since they are **slowing down** as they ascend) and reach a **final velocity of zero** at the top of their path

Worked Example

A pilot is landing a small aircraft. The airstrip is only 450 m long and the aircraft decelerates from 40 m s^{-1} at a constant rate of 2 m s^{-2}.

Determine if the plane will stop before it reaches the end of the runway.

Answer:

Step 1: List the known values

- Length of the runway, s_{max} = 450 m (this is the maximum value of displacement)
- Initial velocity, u = 40 m s^{-1}
- Final velocity, v = 0 m s^{-1} (since the plane will stop)
- Acceleration, a = – 2 m s^{-2} (the negative sign indicates slowing down)

Step 2: Identify the missing values (the unknown answer and the one not given)

- Known quantities: u, v and a
- Unknown quantity: displacement, s
- Time, t is not required in this question

Step 3: Choose the correct SUVAT equation and rearrange it

- The SUVAT equation which contains u, v, a and s and omits t is:

$$v^2 = u^2 + 2as$$

- Rearrange to make s the subject:

$$s = \frac{v^2 - u^2}{2a}$$

Step 4: Substitute the values into the equation and calculate s

$$s = \frac{0^2 - 40^2}{2 \times (-2)} = 400 \text{ m}$$

Step 5: Write the full answer to the question

- The plane stops after 400 m, which is 50 m before the end of the runway
- **Note:** In this solution, the negative value produced by $v^2 - u^2$ is cancelled out by the negative value for acceleration, giving a positive value for displacement

Your notes

Worked Example

A rocket is launched vertically, using motors which give it a constant vertical acceleration of 6.5 m s^{-2} for 30 s before switching off.

At the point when the motors switch off, calculate:

a) The velocity of the rocket

b) The height of the rocket above the launchpad

Answer:

Part (a)

Step 1: List the known values

- Initial velocity, u = 0 m s^{-1}
- Acceleration, a = 6.5 m s^{-2} (the positive sign indicates the speed is increasing due to the motor)
- Time, t = 30 s

Step 2: Identify the missing values (the unknown answer and the one not given)

- Known quantities: u, a, t
- Unknown quantity: final velocity, v
- Displacement, s, is not required in this question

Step 3: Choose the correct SUVAT equation

- The SUVAT equation which omits displacement, s, is:

$$v = u + at$$

Step 4: Substitute the values into the equation and calculate

$$v = 0 + (6.5 \times 30) = 195 \text{ m s}^{-1}$$

Step 5: Write the full answer to the question, taking care to check significant figures

- The final velocity, v, at 30 s is 200 m s^{-1} (2 s.f.)

Part (b)

Step 1: List the known values

- Initial velocity, u = 0 m s^{-1}
- Acceleration, a = 6.5 m s^{-2}
- Time, t = 30 s
- Final velocity, v = 195 m s^{-1}

Step 2: Identify the missing values (the unknown answer and the one not given)

- Known quantities: u, a, t, and v (from part a)

- Unknown quantity: final displacement, s
- All the SUVAT values are available in this case, but since v was calculated, first look for an equation that doesn't use it (in case of mistakes)

Step 3: Choose the correct SUVAT equation

- The SUVAT equation which omits velocity is:

$$s = ut + \frac{1}{2}at^2$$

Step 4: Substitute the values into the equation and calculate *s*

$$s = (0 \times 30) + \frac{1}{2}(6.5 \times 30^2) = 2925 \text{ m}$$

Step 5: Write the full answer to the question, taking care to check significant figures

- The height, s, above the launchpad is 2900 m (2 s.f)

Examiner Tips and Tricks

SUVAT questions are popular with examiners, who will find all sorts of ways to set these problems. The trick is practice.

Learn the four equations (even though you are given them) so that you can easily pick the one you need. With lots of practice you will see that often the problem can be simplified, especially when the initial or final velocity is zero.

Finally, never cut corners with these solutions! It's common for questions to have 5–6 marks available for a long calculation. Your job is to write down every step so that the examiner can award marks for as many elements of your answer as possible.

Your notes

Your notes

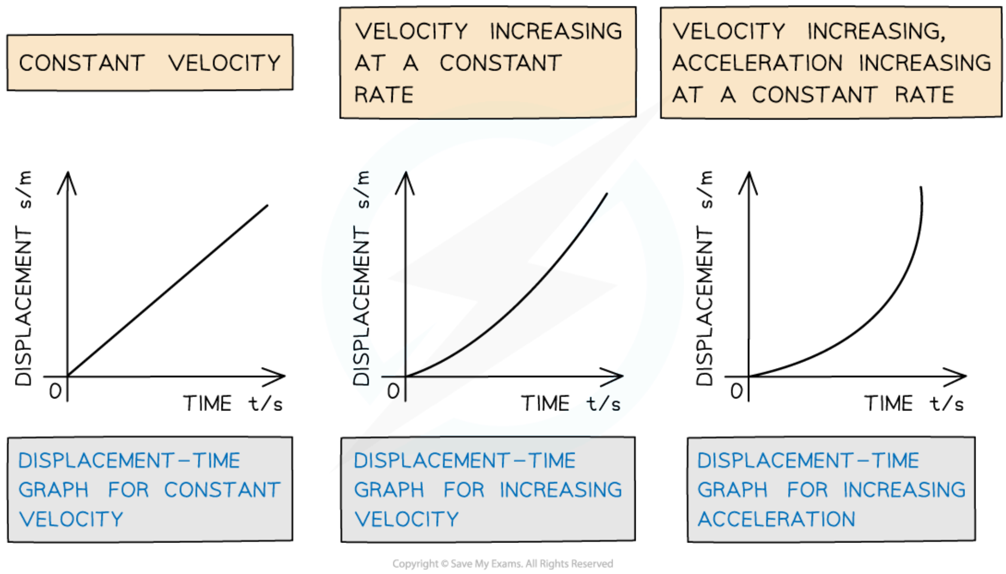
Motion Graphs

- Three types of graph that are used to represent motion are **displacement-time** graphs, **velocity-time** graphs and **acceleration-time** graphs
- Graphs are named 'y-axis - x-axis', so 'displacement-time' simply means 'displacement on the y-axis and time on the x-axis'
- Graphs of motion can be thought of as telling a story of something which has happened in the real world, such as
  - the time, direction and average speed of a journey to school
  - initial and final velocity and acceleration as a rocket takes off from the launchpad
  - movement and speed in various directions as an athlete takes part in an event
- **Interpreting graphs** means using the slope of the line, or the area under the line to find information about that 'story'
- **Drawing graphs** is a skill where careful planning, plotting and line drawing from some data yields further information from an investigation

Displacement-Time

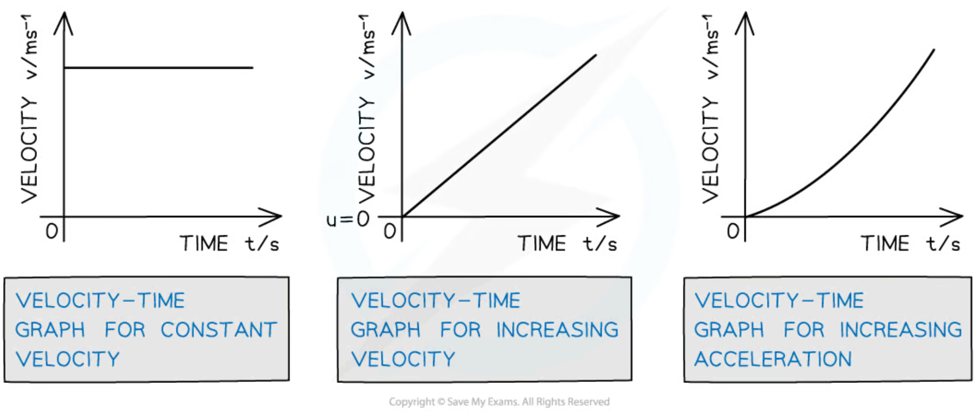
- On a **displacement-time graph;**
  - **Slope** equals **velocity**
    - A **straight** (diagonal) slope represents a **constant** velocity
    - A **curved** slope represents an **acceleration**
  - A **positive slope** represents motion in the **positive direction**
  - A **negative slope** represents motion in the **negative direction**
  - A **zero** slope (horizontal line) represents a state of **rest**
  - The area under the curve is meaningless

Your notes



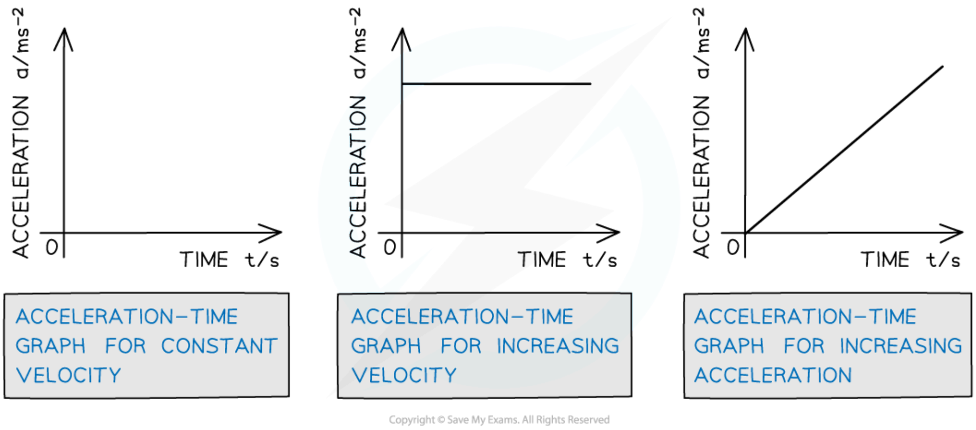
Velocity–Time

- On a **velocity–time graph…**
  - **Slope** equals **acceleration**
    - A **straight** line represents **uniform** acceleration
    - A **curved** line represents **non-uniform acceleration**
  - A **positive** slope represents an **increase** in **velocity** in the **positive direction**
  - A **negative** slope represents an **increase** in **velocity** in the **negative direction**
  - a **zero** slope (horizontal line) represents motion with **constant velocity**
  - The **area** under the curve equals the **displacement** or **distance** travelled



Acceleration–Time

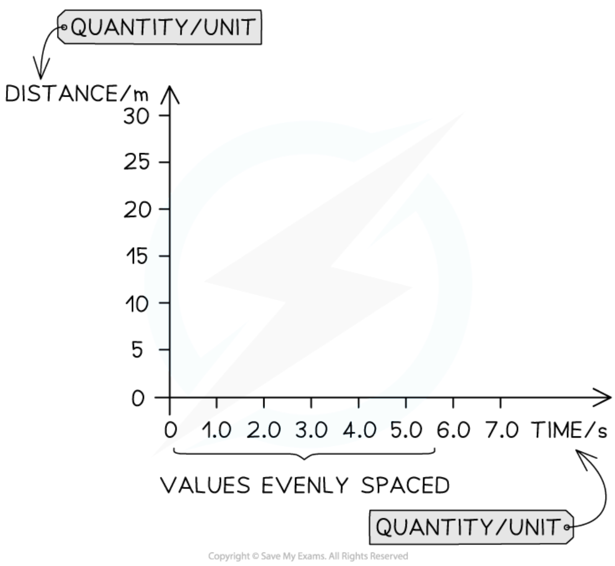
- On an **acceleration-time graph…**
- A zero slope (horizontal line) represents an object undergoing constant acceleration
- The area under the curve equals the change in velocity
- The steepness of the slope is meaningless



How displacement, velocity and acceleration graphs relate to each other

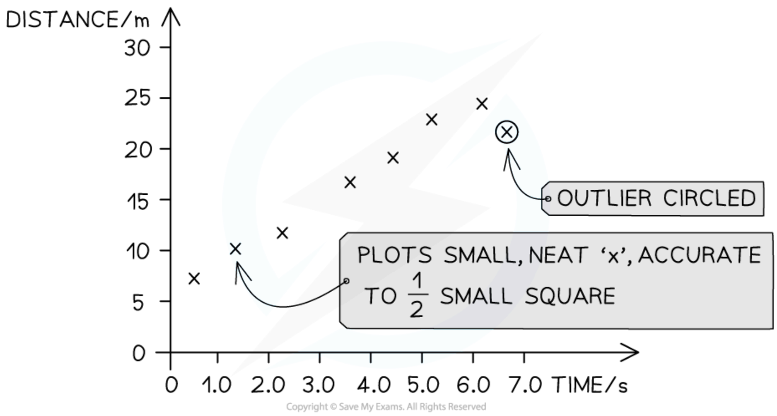
Drawing Graphs

- Drawing a good graph is an important skill
- Good graphs have the following:
  - Each axis has two labels; the name of the variable and the unit (and may include a multiplier, for example, m × 10^{-9})
  - Values on the axes are evenly spaced, with the same number of divisions for each amount

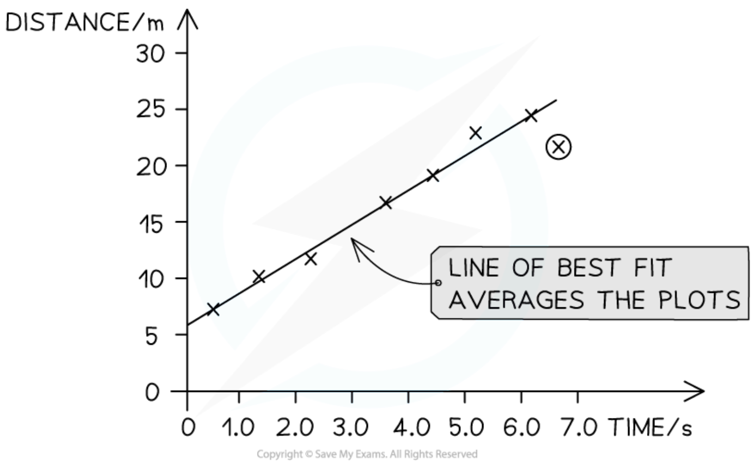


Your notes

- The axes start at the origin **only** when this fits the data or the relationship being graphed is directly proportional **or** when specified in the question
- Increments on the scale are in multiples of 2, 5 and 10 but **never** multiples of 3.
- The range of values being plotted takes up the length of the axis, so that the plotted line takes up most of the space on the graph
- Plots are accurate to within half a small square
- Plots and curves are drawn with sharp pencil so that they are narrow (don't take up more than half a small square)



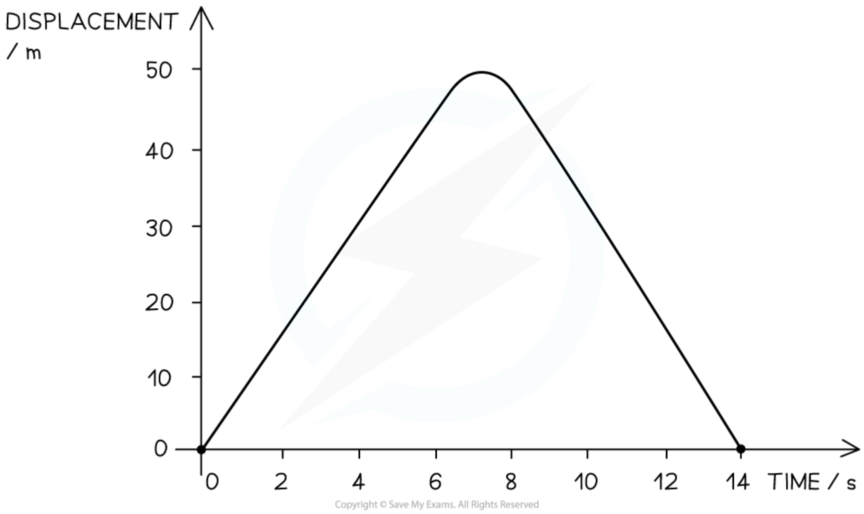
- Lines of best fit match the data, so that lines go through as many of the plots as possible (ignoring outliers) with the remainder evenly spaced above and below the line



Worked Example

Your notes

A runner is practicing short sprints between two markers. A displacement-time graph showing part of their training is shown.



(a) Find the distance between the markers

(b) Determine the speed of the runner at the fastest part of their sprint

Answer:

Part (a)

Step 1: Determine the direction of motion from the graph

- Initially displacement increases from 0 - 50 m
- This represents the sprinter running away from the starting position
- Secondly displacement decreases from 50 - 0 m
- This represents the sprinter returning towards the starting position

Step 2: Identify the sprinter's turning point on the graph

- The distance between the markers is from 0 → turning around point
- This is 50 m from the start point

Step 3: Write the correct answer

Distance between the markers = 50 m

Part (b)

Step 1: Identify the part of the graph that represents the velocity

- The graph shows displacement against time
- Velocity is equal to:

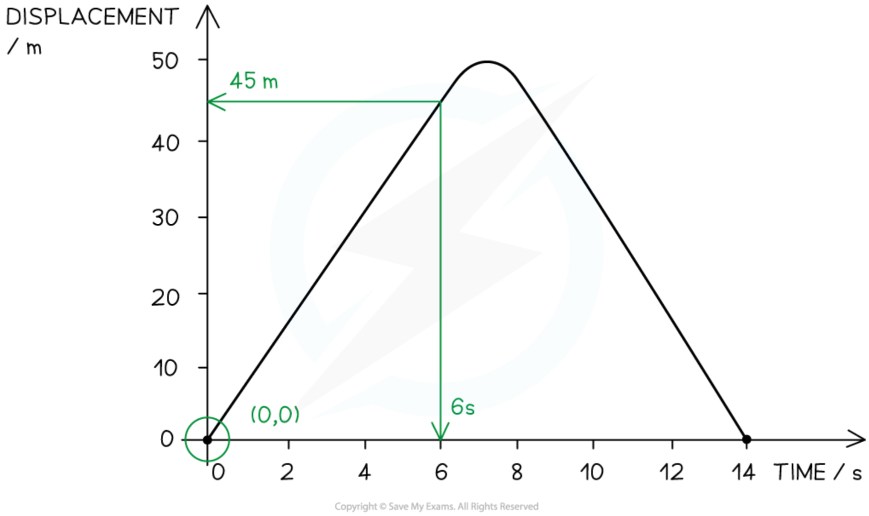
$$Velocity = \frac{displacement}{time}$$

- Therefore, the slope of the displacement-time graph gives velocity
- Displacement × time (area under the graph) is not a physical quantity

Step 2: Identify the steepest section of the graph

Your notes

- Select the section on the graph where the slope is the steepest
  - This represents the **fastest velocity**
  - Clearly mark two points on this line which are as far apart as possible



Step 3: Use the coordinates from the graph to calculate the slope

$$\text{slope} = \frac{\text{increase in } y}{\text{increase in } x} = \frac{45}{6} = 7.5 \text{ m s}^{-1}$$

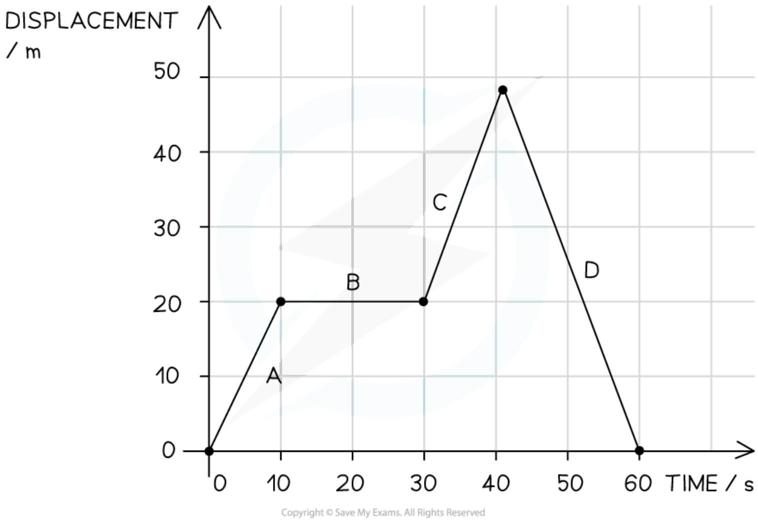
Step 4: Write down the answer to the correct number of significant figures

The sprinter's fastest speed = 7.5 m s^{-1}

Worked Example

Motion has been plotted on a displacement-time graph.

Your notes



Sketch a velocity-time graph for this motion. Include values and show your calculations.

Answer:

Step 1: Calculate each stage:

Part A

Step 1: Determine the time interval

Time from 0 - 10 s

Step 2: Calculate the velocity by finding the gradient

$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{20}{10} = 2 \text{ m s}^{-1}$$

Part B

Step 1: Determine the time interval

Time from 10 - 30 s

Step 2: Calculate the velocity by finding the gradient

Gradient = 0 (horizontal line), velocity = 0 m s^{-1}

Part C

Step 1: Determine the time interval

Time from 30 - 40 s

Step 2: Calculate the velocity by finding the gradient

$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{50 - 20}{40 - 30} = 3 \text{ m s}^{-1}$$

Your notes

Part D

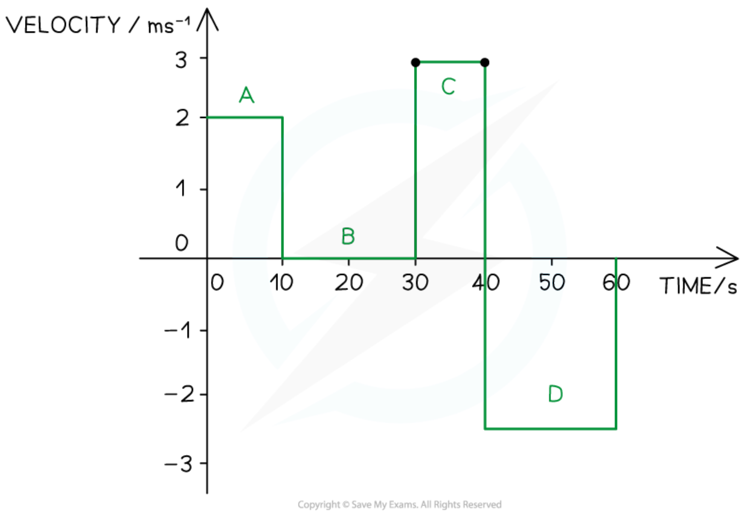
Step 1: Determine the time interval

Time from 40 - 60 s

Step 2: Calculate the velocity by finding the gradient

$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{0 - 50}{60 - 40} = -2.5 \text{ m s}^{-1}$$

Step 2: Use the calculated values to sketch a graph



Examiner Tips and Tricks

Interpreting Graphs

It's common for a question to ask you to solve something **'using a graphical method'**. This just means 'by finding your answer on a graph'.

In AS Physics most (but not all) graphs will be a straight line, meaning they follow the equation **y = mx + c**

This means that the 'graphical method' question is asking you to look at the **gradient**, or the **area under the line**. Once you are secure in finding these then graph questions will become much easier!

Drawing Graphs

A well-drawn graph will net you four or five quick and easy marks IF you have practiced the skill.

Common mistakes are starting at the origin when there is no need (so the line takes up less than half of the page) and trying to work with a scale that goes in multiples of three (making it hard to get the plotting right).

Your notes

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Slope & Area of Motion Graphs

Slope & Area of Motion Graphs

- Three types of graph that are used to represent motion are **displacement-time** graphs, **velocity-time** graphs and **acceleration-time** graphs
- To interpret these find either the slope or the area
- **Slope** is found using $slope = \frac{change\ in\ y}{change\ in\ x}$, which can be related to any similar algebraic relationship, for example, $v = \frac{d}{t}$
  - The slope of a distance-time graph will equal velocity
- **Area** is found using area = change in y × change in x, which can be related to any similar algebraic relationship, for example, velocity × time = distance
  - The area under a velocity-time graph will equal distance

Finding Slope of a Straight Line

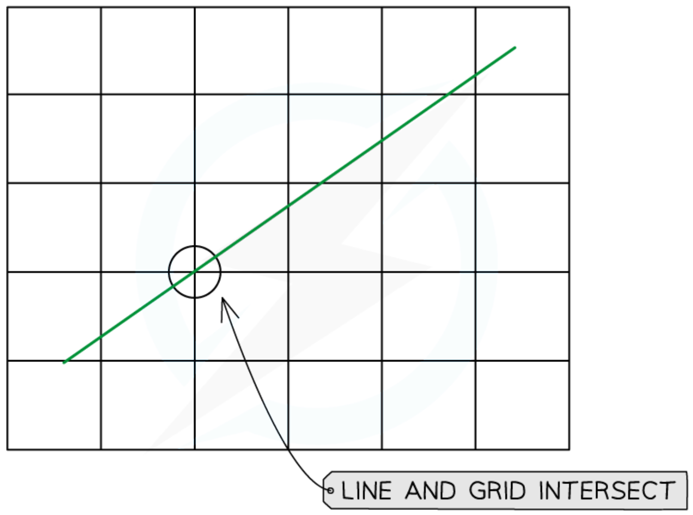
- The slope of a line is found using the formula:

$$\text{slope} = \frac{rise}{run}$$

- This is also written:

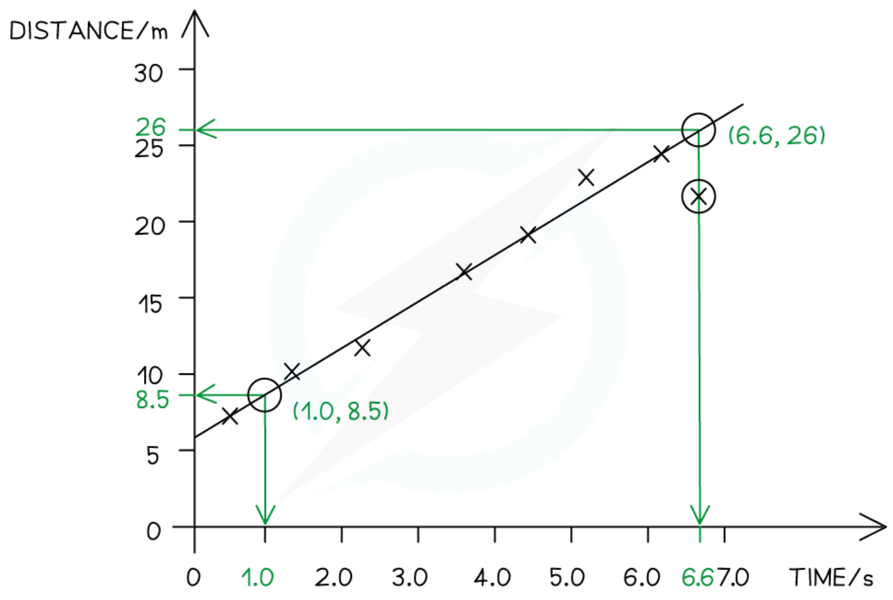
$$\text{slope} = \frac{increase\ in\ y}{increase\ in\ x}$$

- To find the slope;
  - Identify two points **as far apart as possible** which intercept with the grid of the graph paper



- The points must be on the line itself, **not** the furthest apart plots
  - Clearly mark the points chosen
  - Find the co-ordinates of the two points
  - Calculate to find the slope

$$\frac{increase\ in\ y}{increase\ in\ x} = \frac{(26 - 8.5)}{(6.6 - 1.0)} = 3.125$$



- Use the values on the axes of the graph (or in the data) to identify the correct significant figures

- Use the units to find the correct units, which also equal

$$\text{units of slope} = \frac{units\ of\ y}{units\ of\ x}$$

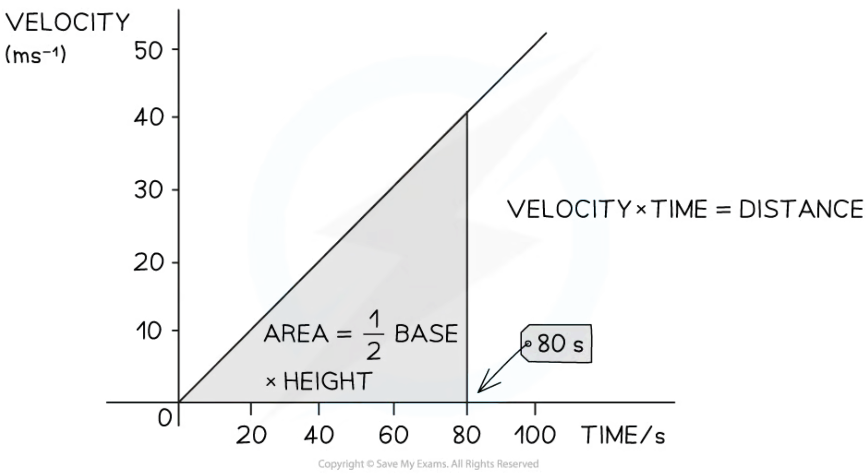
- On this graph all values are to 2 significant figures,
- Therefore slope = 3.1 m s^{-1}

Finding the Area Under the Line

- The area underneath a line represents the x axis multiplied by the y axis.
- For example, if velocity is plotted on the y-axis and time is on the x-axis, then
  - area = velocity × time = distance
- When the area is a rectangle or a triangle it is easy to find by calculating

area of rectangle = base × height

area of a triangle = ½ base × height

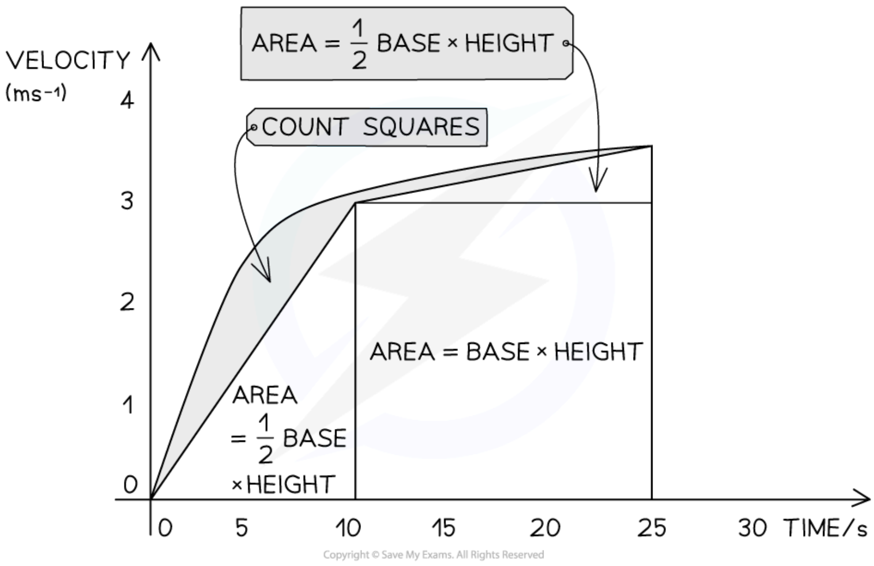


To find the area under a straight-line graph treat is as any rectangle or triangle

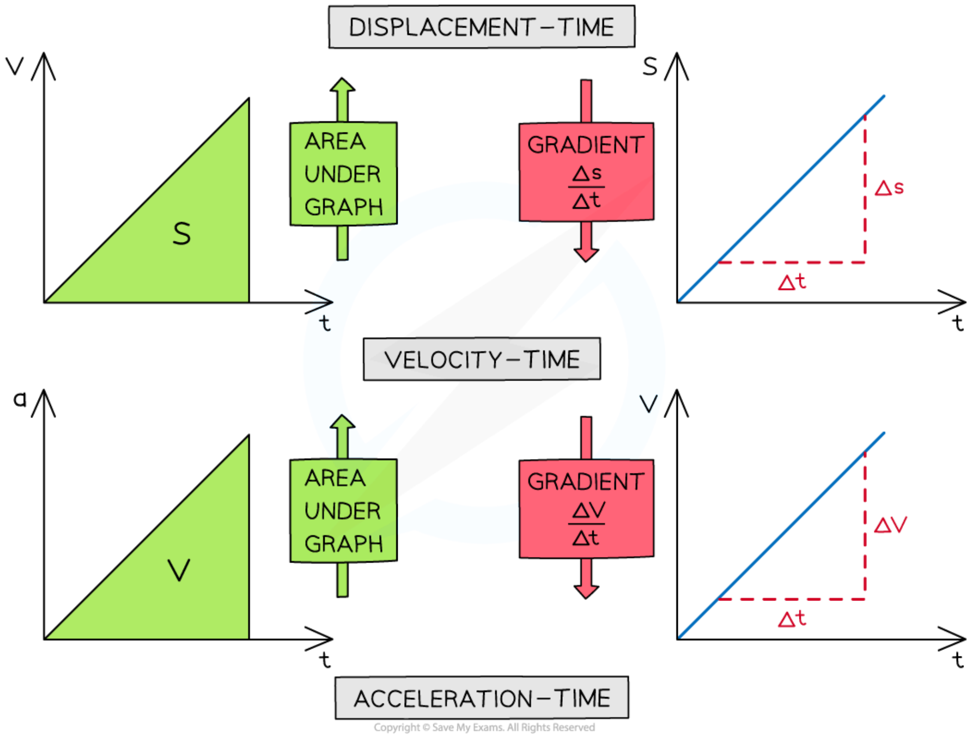
- When the line is curved, area is found though the following steps
  - Divide the shape into rectangles and triangles as shown
  - Find the area for each by calculating
  - Count the remaining squares
  - Add the totals together

Your notes

Your notes



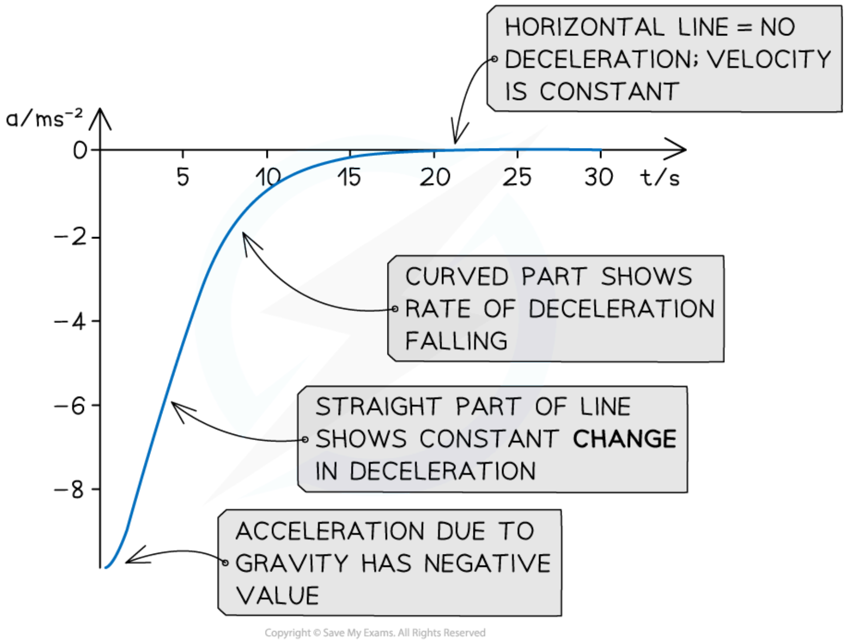
Find area under a curve by dividing it into sections which reduce the number of squares to be counted to the minimum



Non-uniform Acceleration

- In certain cases, acceleration changes throughout a period of motion
- An example of this is a skydiver falling due to their weight, against air resistance
  - As speed increases, air resistance increases

- The increasing resultant force against the direction of motion acts to reduce acceleration
- Eventually the weight and air resistance balance and acceleration become zero
- Acceleration for a falling object is conventionally shown as a negative value

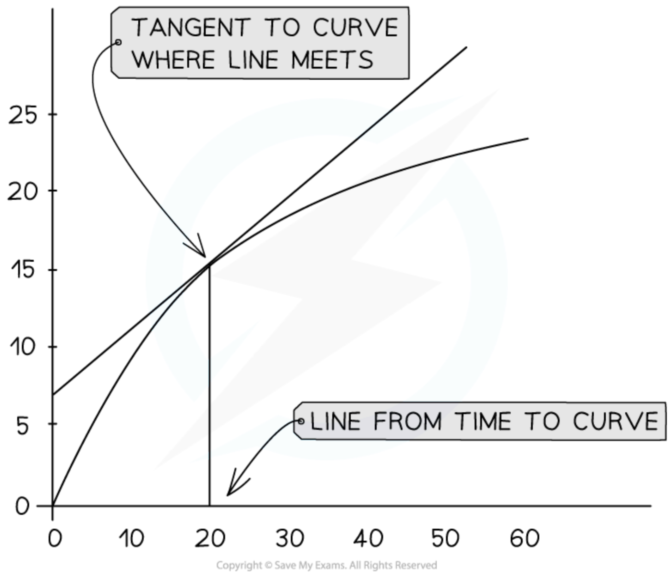


- Non-uniform acceleration will produce a curved velocity-time graph

Finding the Slope of a Curved Line

- Some questions ask for velocity or acceleration at a particular point or time
- This is called 'instantaneous' velocity or acceleration
- It is found by taking the slope of the curved line
- Identify the point on the x-axis which corresponds with the question
    - Draw a line vertically to meet the curve
- Take a tangent to the curve at this point
    - Do this carefully, trying out more than one attempt before choosing the correct tangent

Your notes



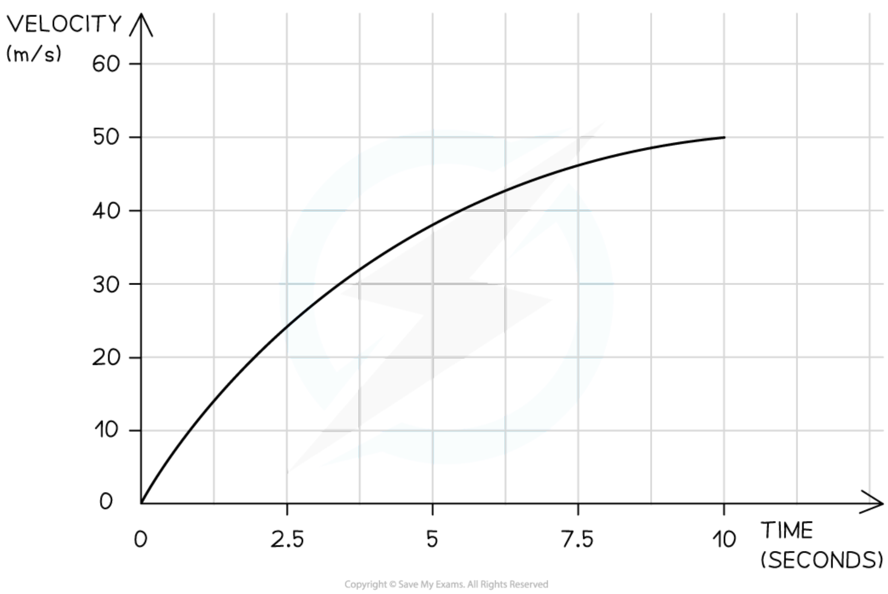
Tangents are a useful way to find the slope of a curved line

- Draw the tangent in place, making it as long as will fit onto the graph
    - Follow the steps above to find the slope of the tangent

Worked Example

A skydiver jumps from a plane and reaches terminal velocity after 15 seconds. A graph of her motion is shown.

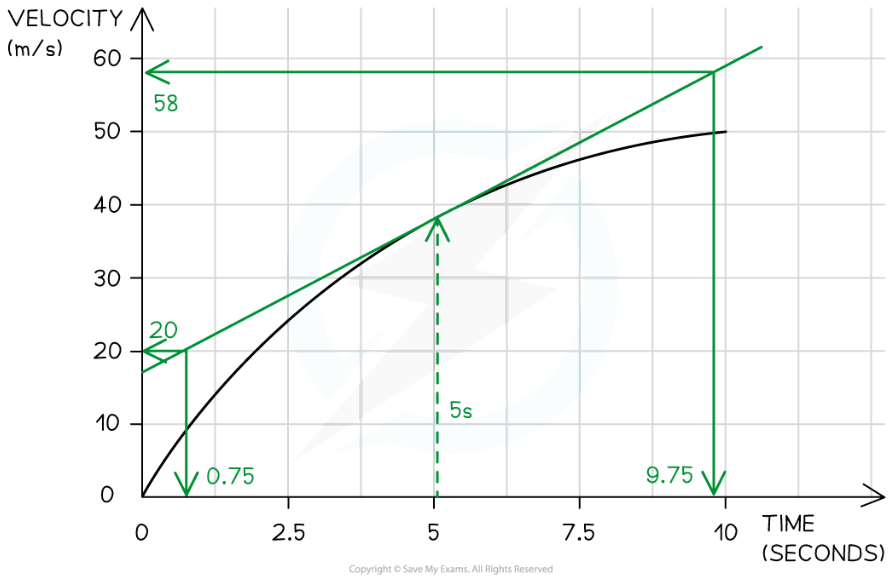
Use the graph to find the acceleration at 5 seconds.



Your notes

Answer:

Step 1: Draw a tangent to the curve at the point where t = 5 s



Step 2: Select points on the graph which are very far apart, and determine the corresponding values of x and y, write these down

Increase in y = (58 -20) = 38 m s^{-1}

Increase in x = (9.75 - 0.75) = 9.0 s

Step 3: Calculate the slope

$$\text{slope} = \frac{increase\ in\ y}{inrease\ in\ x} = \frac{38}{9} = 4.222\ \text{m s}^{-2}$$

Step 4: Write the answer to the correct significant figures

The acceleration at 5.0 s = 4.2 m s^{-2} (2 sf)

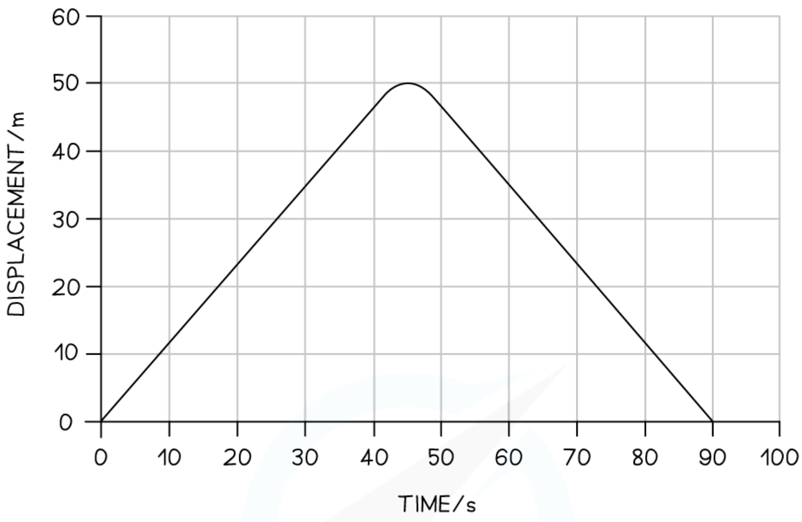
Examiner Tips and Tricks

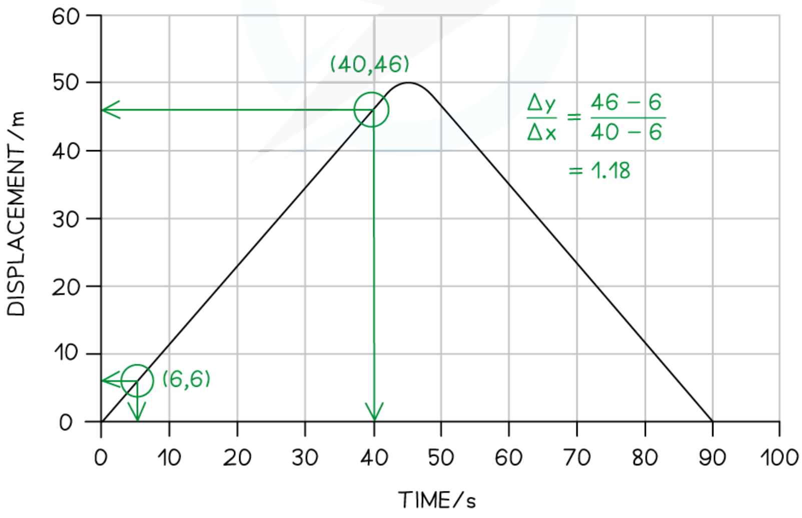
When working with graphs follow the advice, 'show don't tell'. The examiner will want to see that you used the graph to find your answer, as this skill is part of the toolkit of a good scientist.

Mark very clearly on the graph any points you use to calculate and indicate with clear lines or large triangles where you chose your data from.

And always remember to use the largest section of the graph that you can to find the slope.

You will be given an exam paper with pristine graphs. Don't leave them that way! By the time you finish with the paper, it should be clearly annotated with your method, so the examiner can see exactly where you have earned marks.





Your notes

Scalars & Vectors

- All quantities can be one of two types:
  - a **scalar**
  - a **vector**

Scalars

- Scalars are quantities that have **magnitude** but not direction
  - For example, **mass** is a **scalar** quantity because it has **magnitude** but no direction

Vectors

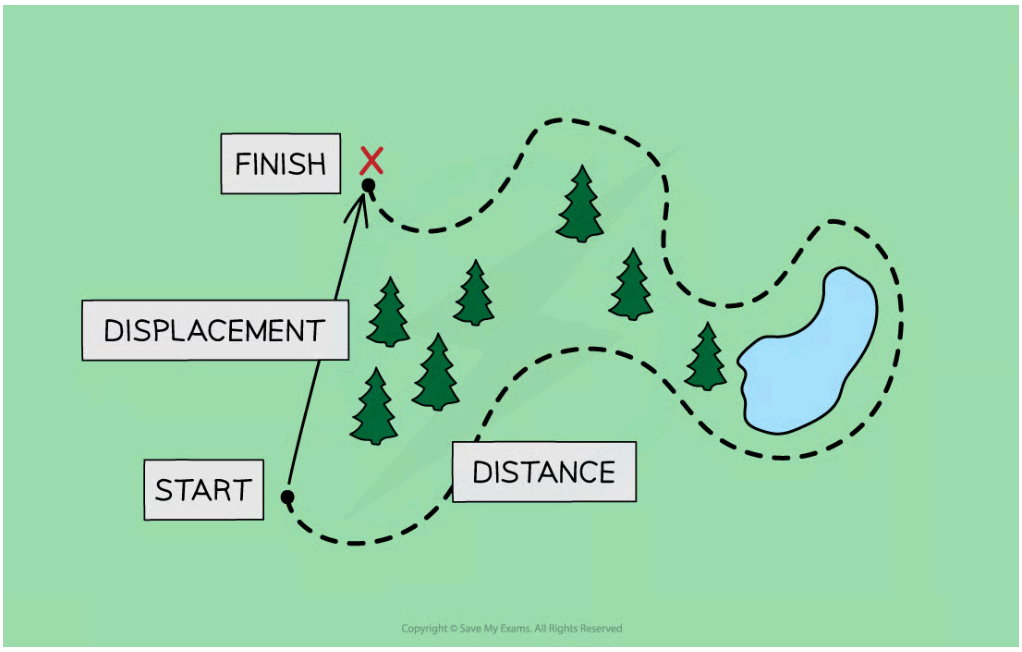
- Vectors are quantities that have both **magnitude** and **direction**
  - For example, **weight** is a **vector** quantity because it is a force and has **both** magnitude and direction

Distance and displacement

- **Distance** is a measure of how far an object has travelled, **regardless of direction**
  - Distance is the total length of the path taken
  - Distance, therefore, has a **magnitude** but **no direction**
  - So, distance is a **scalar** quantity
- **Displacement** is a measure of how far it is between two points in space, **including the direction**
  - Displacement is the **length** and **direction** of a **straight line** drawn from the starting point to the finishing point
  - Displacement, therefore, has a **magnitude** and a **direction**
  - So, displacement is a **vector** quantity

What is the difference between distance and displacement?

Your notes



Displacement is a vector quantity while distance is a scalar quantity

- When a student travels to school, there will probably be a **difference** in the distance they travel and their displacement
  - The **overall distance** they travel includes the total lengths of all the roads, including any twists and turns
  - The **overall displacement** of the student would be a straight line between their home and school, regardless of any obstacles, such as buildings, lakes or motorways, along the way

Speed and velocity

- **Speed** is a measure of the **distance** travelled by an object per unit time, **regardless of the direction**
  - The speed of an object describes how fast it is moving, but **not** the direction it is travelling in
  - Speed, therefore, has **magnitude** but **no direction**
  - So, speed is a **scalar** quantity
- **Velocity** is a measure of the **displacement** of an object per unit time, **including the direction**
  - The velocity of an object describes how fast it is moving **and** which direction it is travelling in
  - An object can have a **constant speed** but a **changing velocity** if the object is **changing direction**
  - Velocity, therefore, has **magnitude and direction**

- So, velocity is a **vector** quantity

Examples of scalars & vectors

- The table below lists some common examples of scalar and vector quantities

Table of scalars and vectors

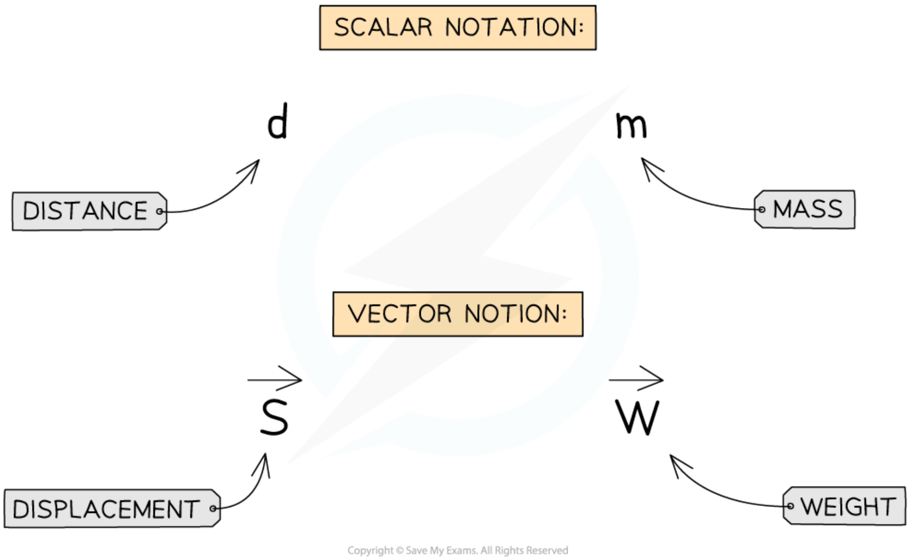
Scalars	Vectors
distance	displacement
speed	velocity
mass	acceleration
time	force
energy	momentum
volume	
density	
pressure	
electric charge	
temperature	

Vector notation

- Sometimes, vector quantities are written in a certain way to distinguish them from scalar quantities
- Some examples of scalar and vector notation are shown below:

Your notes

Your notes



The arrow on vector notation does not indicate an actual direction, just that the quantity has a direction

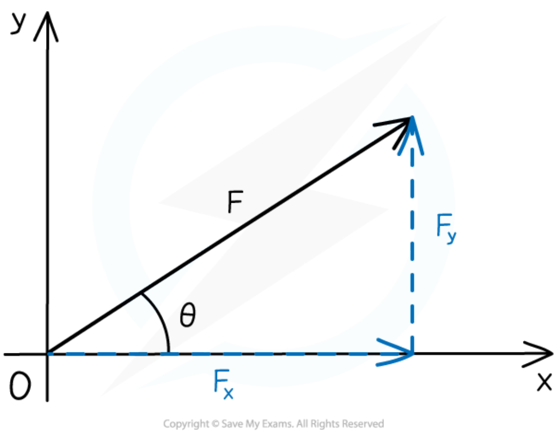
- This means writing the quantity to make it clear that it is a vector
- In textbooks, vectors are often shown as bold and italic, for example, ***F*** or ***s***
- Another form of notation, and easier to do in handwriting, is putting an arrow over the top of the quantity, for example, or $\vec{s}$
  - The arrow does **not** indicate the actual direction of the vector, only that it **has** a direction

Resolving Vectors

- **Vectors** are represented by an arrow
  - The **arrowhead** indicates the **direction** of the vector
  - The **length** of the arrow represents the **magnitude**
  - **Component** vectors are sometimes drawn with a dotted line and a **subscript** indicating horizontal or vertical for example F_v is the vertical component of the force F

Resolving Vectors

- A single resultant vector can be resolved
  - This means it can be represented by **two** vectors, which in combination have the same effect as the original one
  - Resolving vectors makes calculating much easier when working with projectiles or any motion or force at an angle
- When a single resultant vector is broken down into its **parts**, those parts are called **components**
- A vector of **magnitude** *F* at **angle theta** to the horizontal is shown



Resolving Vectors by Scale Drawing

- To resolve vectors by scale drawing means carefully producing a scale drawing with all lengths and angles correct
  - This should be done using a sharp **pencil**, **ruler** and **protractor**
- Follow these steps

- Choose a scale which fits to the page. For example, if resolving a resultant displacement of 100 m, use 1 cm = 10 m so that the resultant drawing is around 10 cm high
- Clearly mark the starting point and a line to represent either the vertical or horizontal direction. This will depend on the angle given in the question, for example if the direction is '30° east of north' then start with a vertical line representing north
- Carefully measure the angle and start by drawing the resultant vector
- Draw a triangle using two of the sides of this rectangle

Your notes

Worked Example

A hiker walks a distance of 6 km due east and 10 km due north.

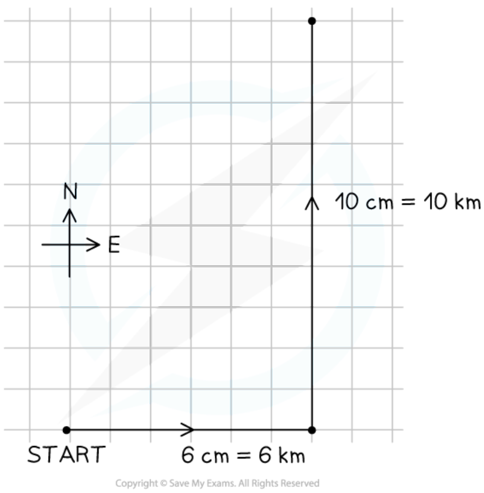
By making a scale drawing of their route, find the magnitude of their displacement and its direction from the horizontal.

Answer:

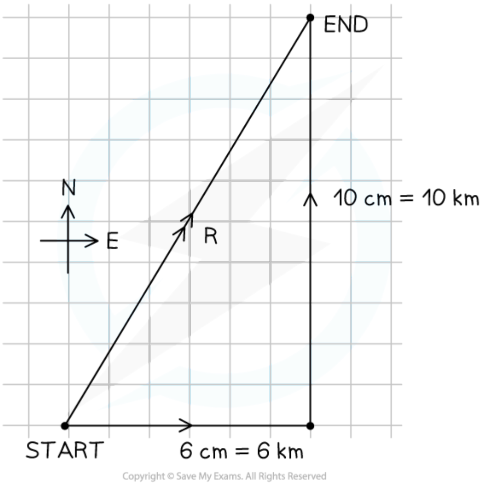
Step 1: Choose a sensible scale

The distances are 6 and 10 km, so a scale of 1 cm = 1 km will fit easily on the page, but be large enough for an accurate scale drawing

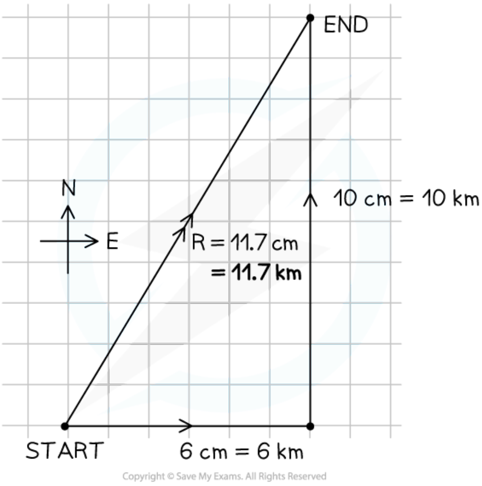
Step 2: Draw the two components using a ruler and making the measurements accurate to 1 mm



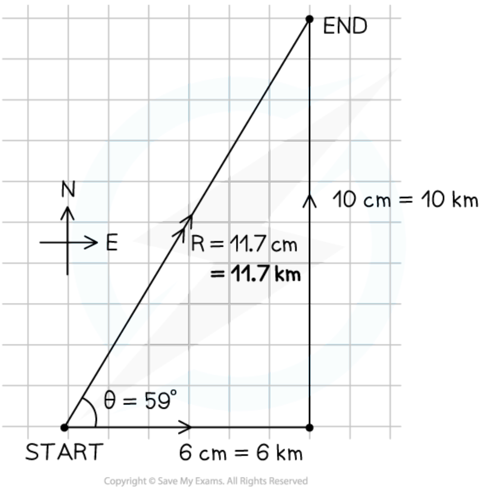
Step 3: Add the resultant vector, remembering the start and finish points of the journey



Step 4: Carefully measure the length of the resultant and convert using the scale



Step 5: Measure the angle between the vector and the horizontal line



Your notes

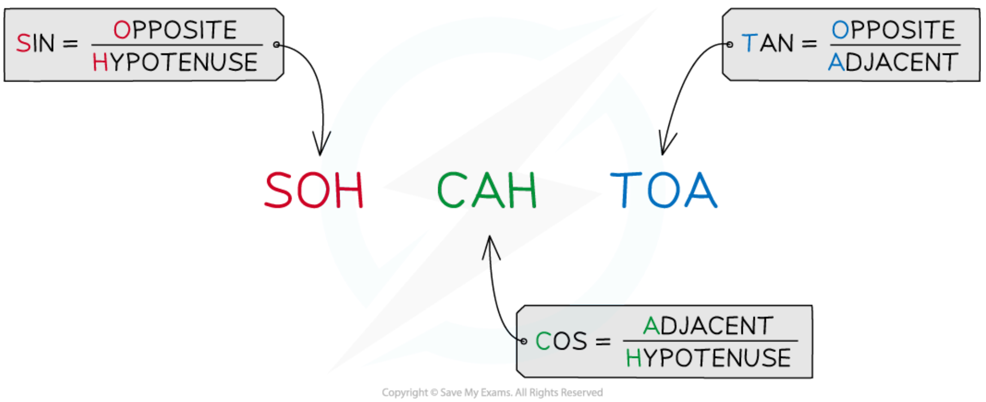
Step 6: Write the complete answer, giving both magnitude and direction

$$R = 11.7 = 12 \text{ km}$$

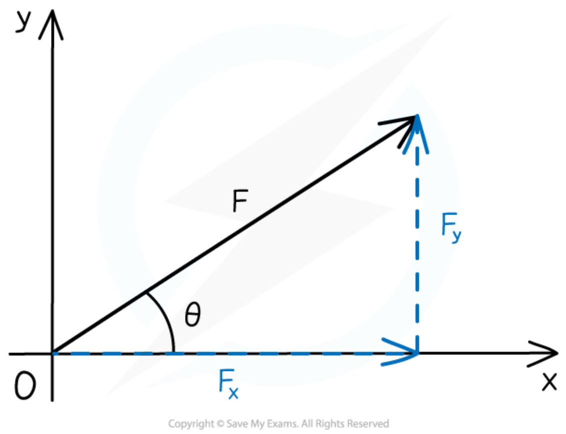
$$\theta = 59° \text{ east and upwards from the horizontal}$$

Resolving Vectors by Calculation

- In this method a diagram is still essential but it does **not** need to be exactly to scale
- The diagram can take the form of a sketch, as long as the resultant, component and sides are clearly labelled
- Use **trigonometry** to resolve the two sides
- The mnemonic '**soh-cah-toa**' is used to remember how to apply sines and cosines to resolving sides of a triangle



- It is possible to **resolve** this vector into its **horizontal** and **vertical** components using trigonometry



The resultant force F can be split into its horizontal and vertical components

- For the **horizontal** component, $F_x = F\cos\theta$

- For the **vertical** component, $F_y = F \sin \theta$

Your notes

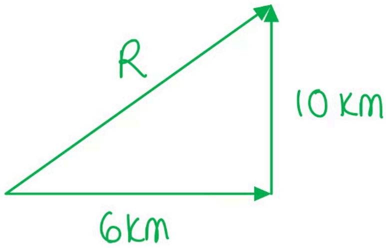
Worked Example

A hiker walks a distance of 6 km due east and 10 km due north.

Calculate the magnitude of their displacement and its direction from the horizontal.

Answer:

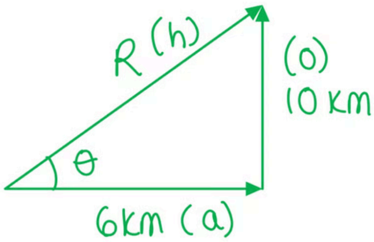
Step 1: Draw a vector diagram



Step 2: Calculate the magnitude of the resultant vector using Pythagoras' Theorem

$$R = \sqrt{6^2 + 10^2} = 2\sqrt{34}$$

Step 3: Calculate the direction of the resultant vector using trigonometry



$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{6}$$

$$\theta = \tan^{-1}\left(\frac{10}{6}\right) = 59°$$

Step 4: State the final answer complete with direction

$$R = 2\sqrt{34} = 11.66 = 12 \text{ km}$$

$$\theta = 59° \text{ east and upwards from the horizontal}$$

Your notes

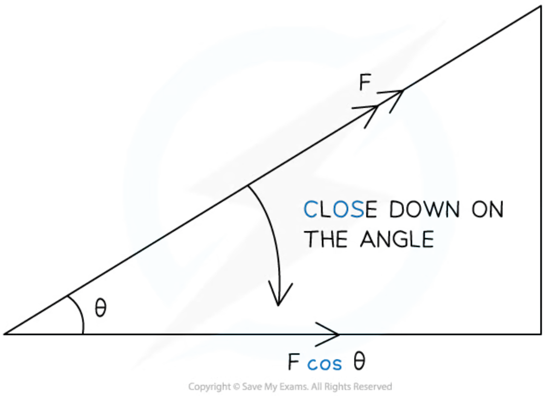
Examiner Tips and Tricks

Did you notice that the two worked examples above were the same question, being solved in two different ways?

If the question specifies calculation or scale drawing you must solve the problem as asked. However, if the choice is left up to you then any correct method will lead to the correct answer.

Scale drawing sometimes feels easier than calculating, but once you are confident with trigonometry and Pythagoras you will find calculating quicker and more accurate.

A quick tip to remember whether to use sin or cos is that when the resultant is closing down onto the angle, use cos.



Adding Vectors

- Vectors can be combined by **adding** or **subtracting** them to produce the resultant vector
  - The resultant vector is sometimes known as the 'net' vector (eg. the net force)
- There are two methods that can be used to add vectors
  - **Calculation** – if the vectors are perpendicular
  - **Scale drawing** – if the vectors are not perpendicular

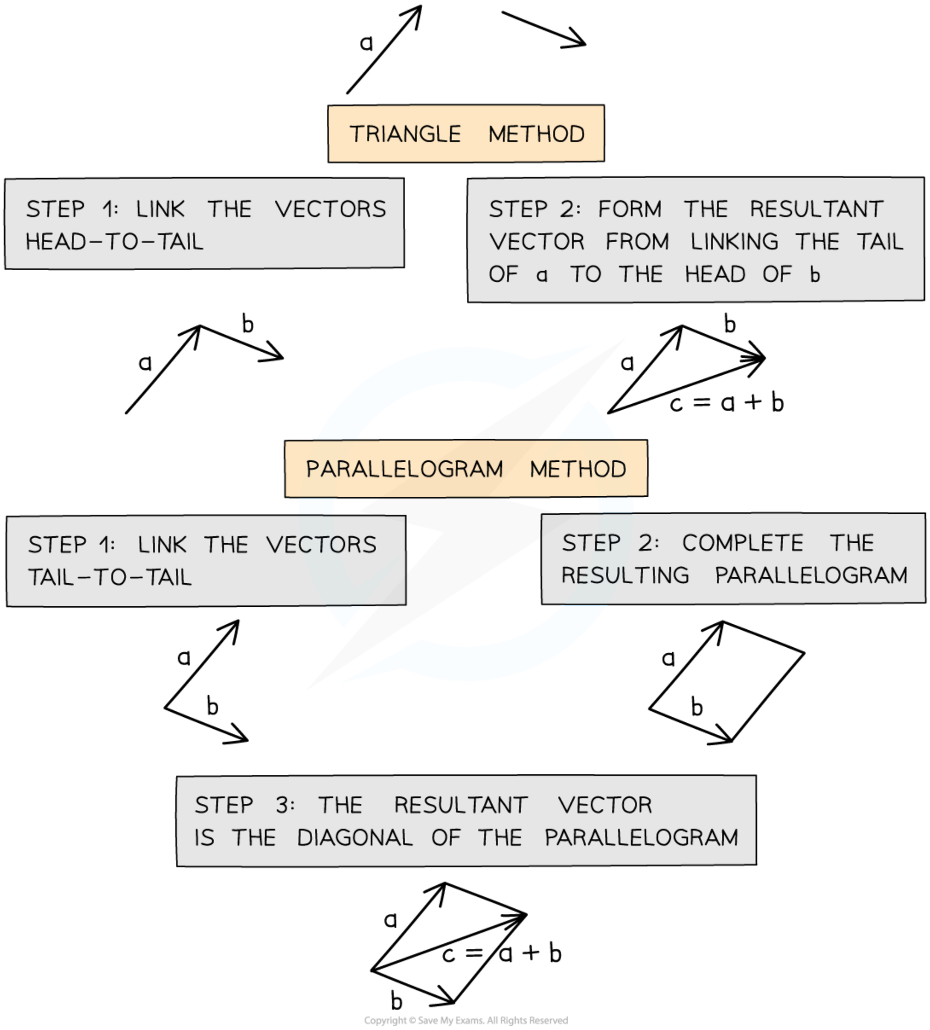
Combining Vectors Using a Scale Diagram

- There are two methods that can be used to combine vectors using a scale diagram: the **triangle method** and the **parallelogram method**
- To combine vectors using the triangle method:
  - **Step 1:** link the vectors head-to-tail
  - **Step 2:** the resultant vector is formed by connecting the tail of the first vector to the head of the second vector
- To combine vectors using the parallelogram method:
  - **Step 1:** link the vectors tail-to-tail
  - **Step 2:** complete the resulting parallelogram
  - **Step 3:** the resultant vector is the diagonal of the parallelogram

Worked Example

Draw the vector c = a + b

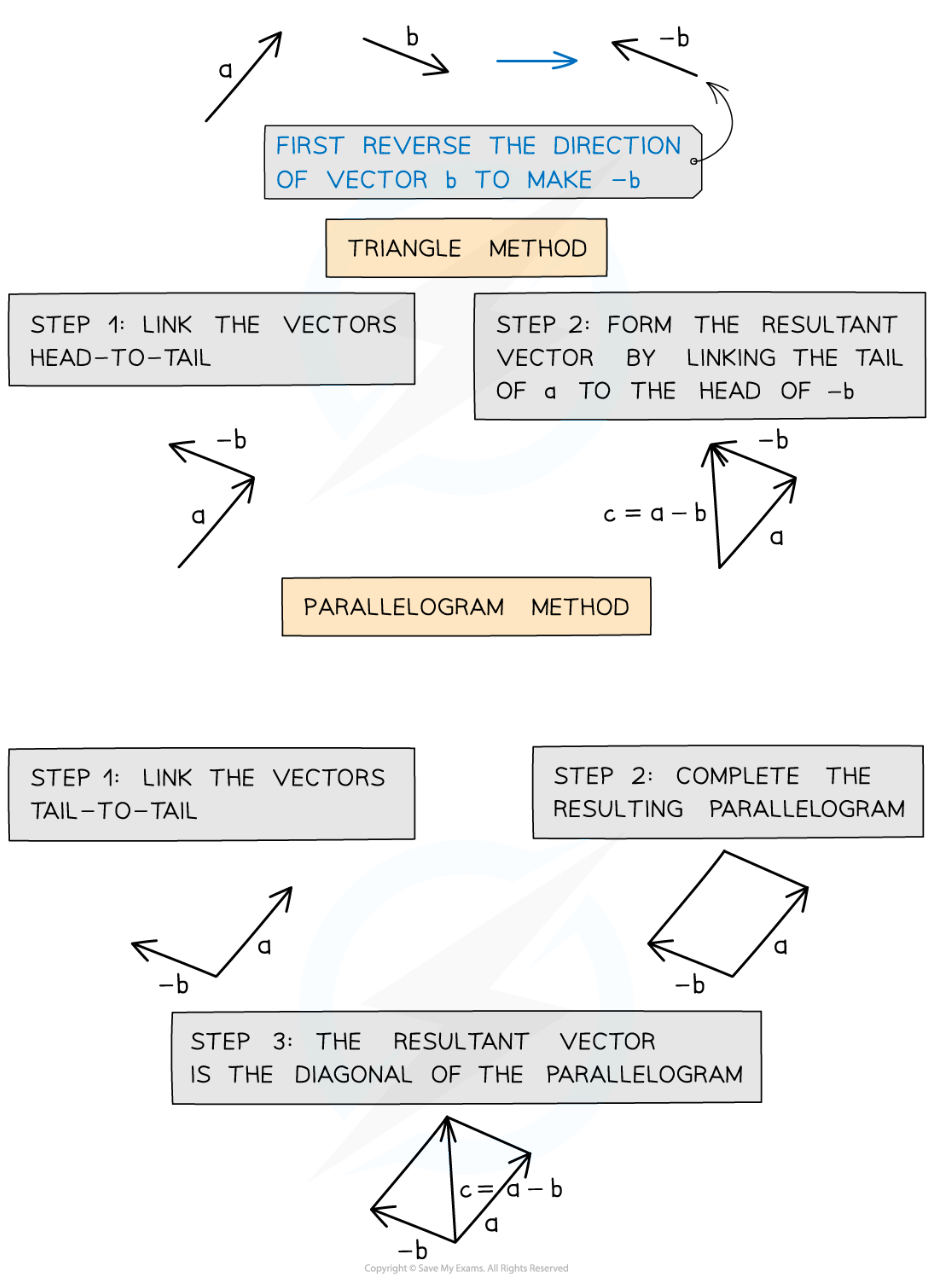
Answer:



Worked Example

Draw the vector c = a – b

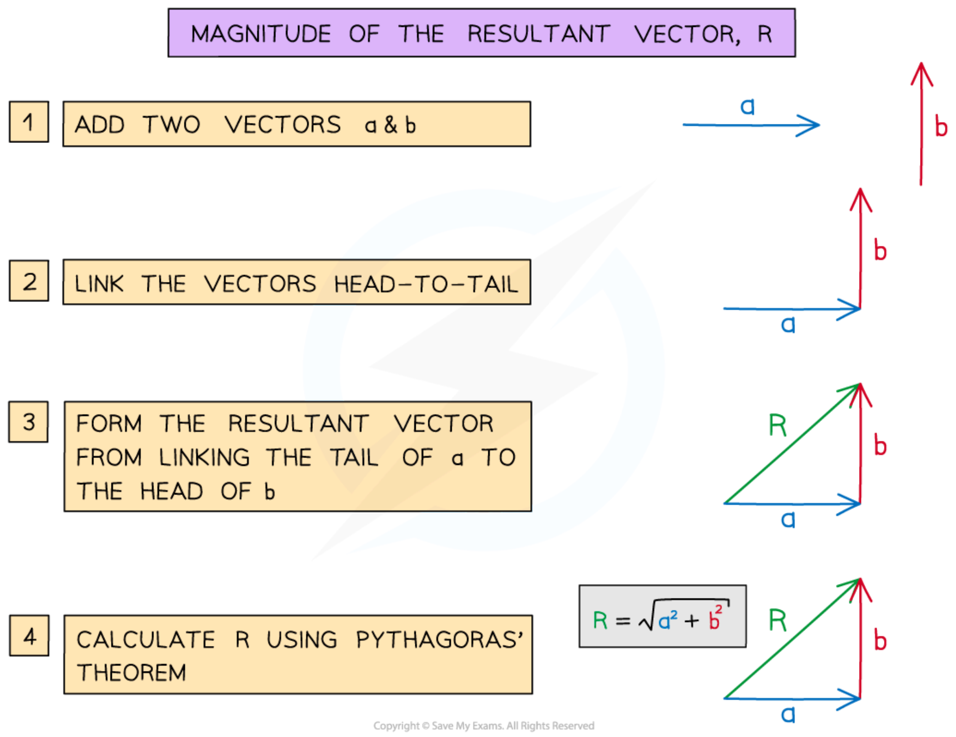
Answer:



Combining Vectors by Calculating

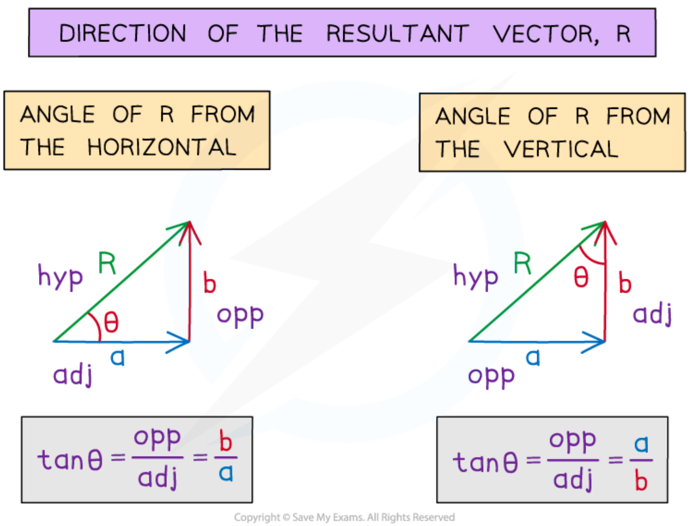
- Combining vectors by calculation is a two-step process
  - Finding the direction of the resultant using trigonometry
  - Finding the magnitude of the resultant using Pythagoras

Your notes



The magnitude of the resultant vector is found by using Pythagoras' Theorem

- The direction of the resultant vector is found from the angle it makes with the horizontal or vertical
    - The question should imply which angle it is referring to (ie. Calculate the angle from the x-axis)
- Calculating the angle of this resultant vector from the horizontal or vertical can be done using **trigonometry**
    - Either the sine, cosine or tangent formula can be used depending on which vector magnitudes are calculated



The direction of vectors is found by using trigonometry

Your notes

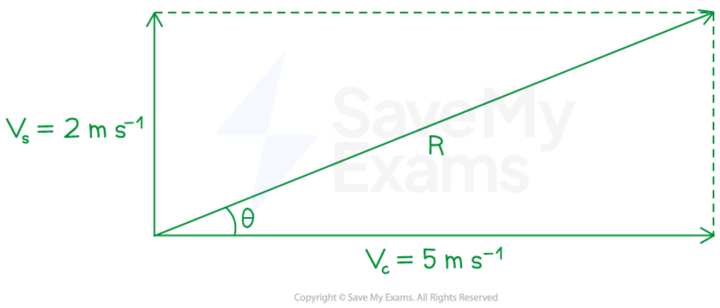
Worked Example

A swimmer is crossing a river by swimming due north at 2 m s^{-1}. The current flows east at 5 m s^{-1}.

Determine the resultant velocity of the swimmer's motion.

Answer:

Step 1: Sketch a diagram, including all known values



Step 2: Calculate the magnitude of the resultant using Pythagoras

$$R = \sqrt{V_s^{\,2} + V_c^{\,2}} = \sqrt{4 + 25} = 5.4$$

Step 3: Calculate the angle using trigonometry

$$tan\theta = \frac{2}{5} = 0.4$$

$$\theta = \tan^{-1}(0.4)$$

$$\theta = 21.8$$

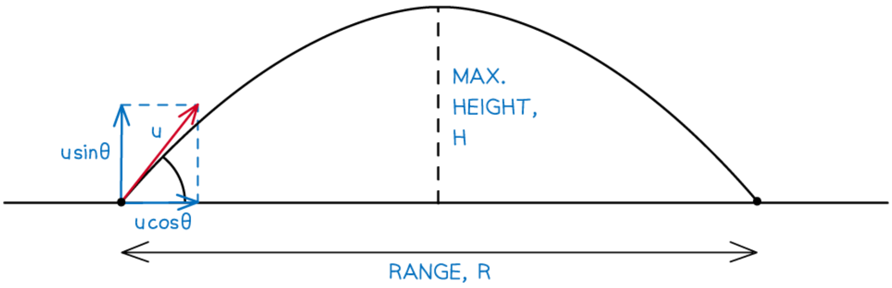
Step 4: Write the answer in full giving both magnitude and direction of the velocity and all units

The swimmer's velocity is 5.4 ms^{-1} at 22° to the horizontal direction

Components of Velocity

- The trajectory of an object undergoing projectile motion consists of a **vertical** component and a **horizontal** component
  - These need to be evaluated separately
- Some key terms to know, and how to calculate them, are:
  - **Time of flight:** how long the projectile is in the air
  - **Maximum height attained:** the height at which the projectile is momentarily at rest
  - **Range:** the horizontal distance traveled by the projectile

Your notes



VERTICAL MOTION (↑)

INITIAL SPEED, $u = u\sin\theta$

ACCELERATION, $a = 9.81\ \text{ms}^{-2}$

DISPLACEMENT = 0

TIME OF FLIGHT

$u = u\sin\theta \quad v = 0 \quad a = -g \quad t = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v = u + at$

$0 = u\sin\theta - gt$

$t = \dfrac{u\sin\theta}{g}$

$2t = \dfrac{2u\sin\theta}{g}$

IF THE TIME TO MAXIMUM HEIGHT IS t, THEN THE TIME OF FLIGHT IS 2t

MAXIMUM HEIGHT ATTAINED

$u = u\sin\theta \quad v = 0 \quad a = -g \quad H = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2gH$

$2gH = (u\sin\theta)^2$

$H = \dfrac{(u\sin\theta)^2}{2g}$

HORIZONTAL MOTION (→)

INITIAL SPEED, $u = u\cos\theta$

ACCELERATION, $a = 0$

DISPLACEMENT = R

RANGE (R)

$u = u\cos\theta \quad t = \dfrac{2u\sin\theta}{g} \quad a = 0 \quad R = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

DISTANCE = SPEED × TIME

$R = (u\cos\theta)t$

$R = \dfrac{2u^2\sin\theta\cos\theta}{g}$

USING THE TRIG IDENTITY:

$2\sin\theta\cos\theta = \sin 2\theta$

$R = \dfrac{u^2\sin 2\theta}{g}$

How to find the time of flight, maximum height and range

Problems Involving Projectile Motion

- There are two main considerations for solving problems involving two-dimensional motion of a projectile
  - Constant velocity in the horizontal direction
  - Constant acceleration in a perpendicular direction
- The only force acting on the projectile, after it has been released, is **gravity**
- There are three possible scenarios for projectile motion:
  - **Vertical** projection
  - **Horizontal** projection
  - **Projection** at an **angle**

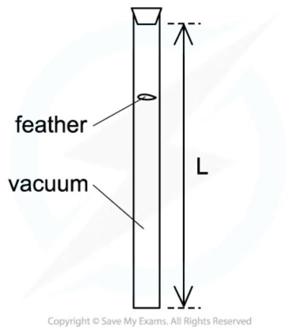
Your notes

Worked Example

To Calculate Vertical Projection (Free Fall)

A science museum designed an experiment to show the fall of a feather in a vertical glass vacuum tube.

The time of fall from rest is 0.5 s.



What is the length of the tube, L?

Answer:

IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION. FIRST WE MUST LIST THE KNOWN VARIABLES.

$a = 9.81 \text{ ms}^{-2}$ $\quad u = 0$ $\quad t = 0.5 \text{ s}$ $\quad L = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$L = \frac{1}{2}gt^2$$

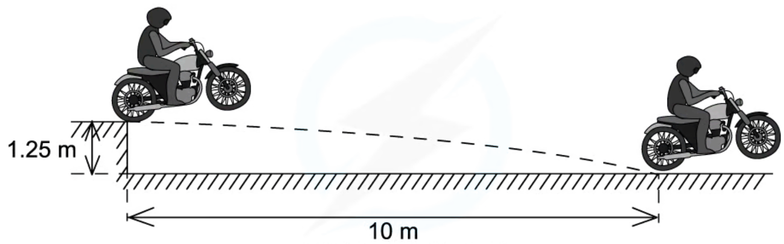
$$L = \frac{1}{2} \times 9.81 \times 0.5^2 = 1.2 \text{ m}$$

Worked Example

To Calculate Horizontal Projection

A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown.

What was the speed at take-off? (ignoring air resistance)



Answer:

IN THIS PROBLEM, WE NEED TO CONSIDER BOTH VERTICAL AND HORIZONTAL MOTION. LET'S CONSIDER THE VERTICAL MOTION FIRST. THE KNOWN VARIABLES ARE

$s = 1.25$ m $\quad a = 9.81$ ms^{-2} $\quad u = 0 \quad t = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 1.25}{9.81}} = 0.5\text{s}$$

NEXT LET'S CONSIDER THE HORIZONTAL MOTION. THE KNOWN VARIABLES ARE

$s = 10$ m $\quad a = 0 \quad t = 0.5$ s $\quad u = ?$

SINCE THE ACCELERATION IS ZERO, WE CAN USE

$$\text{VELOCITY} = \frac{\text{DISPLACEMENT}}{\text{TIME}}$$

$$v = \frac{10}{0.5} = 20 \text{ ms}^{-1}$$

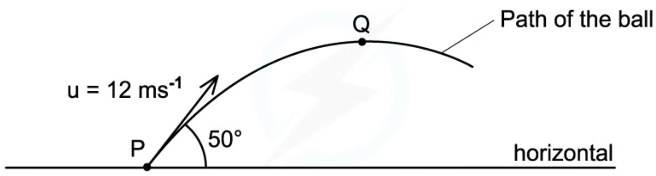
Your notes

Worked Example

To calculate projection at an angle

A ball is thrown from a point P with an initial velocity u of 12 m s^{-1} at 50° to the horizontal.

What is the value of the maximum height at Q? (ignoring air resistance)



Answer:

IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION UP TO THE POINT Q. FIRST WE MUST LIST THE KNOWN VARIABLES

$u = 12\sin(50)$ $a = -9.81\ \text{ms}^{-2}$ $v = 0$ $H = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

$$s = \frac{(v^2 - u^2)}{2a}$$

$$H = \frac{0 - (12\sin 50)^2}{2 \times (-9.81)}$$

$$H = \frac{(12\sin 50)^2}{19.62} = 4.3\ \text{m}$$

Your notes

Examiner Tips and Tricks

In almost every question using the SUVAT equations one component of velocity will have constant acceleration (for example, the vertical component of projectile motion) and the other will have no acceleration (for example the horizontal component of projectile motion when we ignore air resistance).

The trick which you nearly always have to remember is find the common value - time - first, then substitute it into the second part of your calculation.

As long as your working is clear, even if you forget, you are likely to still achieve half marks. But writing good clear Maths and remembering this tip should get you to the end of your calculation - and full marks!

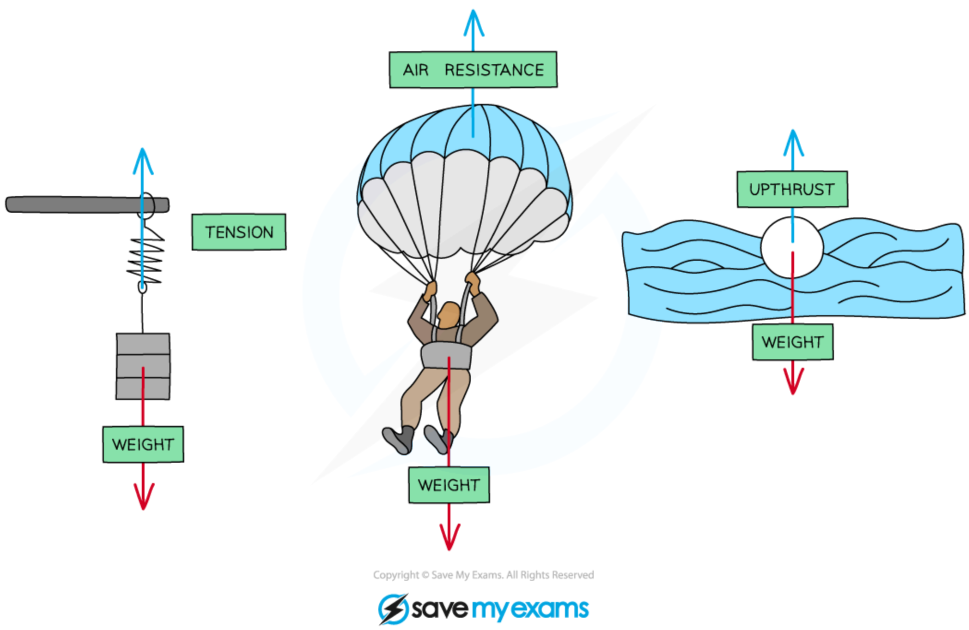
And finally.... don't forget that deceleration is **negative** as the object rises.

Free-body Force Diagrams

- Free body diagrams are useful for modeling the forces that are acting on an object
- Each force is represented as a **vector** arrow, where each arrow:
  - Is scaled to the **magnitude** of the force it represents
  - Points in the **direction** that the force acts
  - Is **labelled** with the name of the force it represents or an appropriate symbol
- Free body diagrams can be used:
  - To identify which forces act in which plane
  - To resolve the net force in a particular direction

Simple Free Body Diagrams

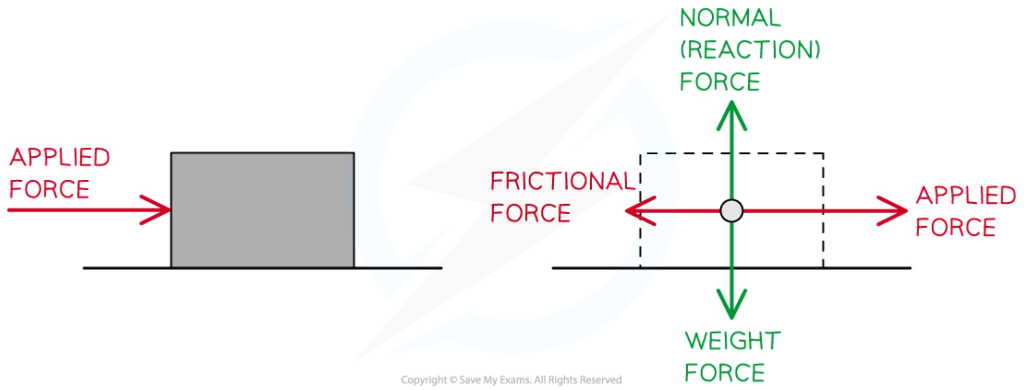
- The rules for drawing a free-body diagram are the following:
  - **Rule 1:** Draw a point in the centre of mass of the body
  - **Rule 2:** Draw the body free from contact with any other object
  - **Rule 3:** Draw the forces acting on that body using vectors with correct direction and proportional length



Free body diagrams can be used to show the various forces acting on objects

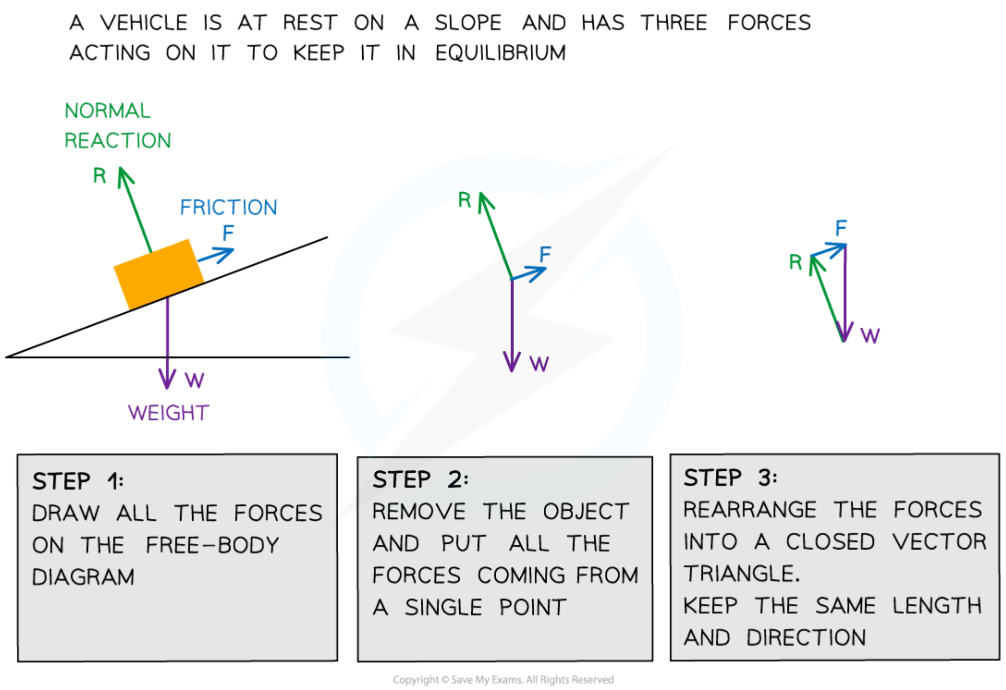
Forces on a Particle or an Extended but Rigid Body

- Free body diagrams simplify problems by reducing complex shapes into simple one
- Any object can be thought of as a particle
  - Even something as large as the Earth can be modelled as a point mass for most calculations
- Objects can also be thought of as being **extended but rigid bodies**
  - This simply means that all parts stay in the same position relative to each other when the object moves



Point particle representation of the forces acting on a moving object

- The below example shows an object sitting on a slope in equilibrium



Three forces on an object in equilibrium form a closed vector triangle

Worked Example

Draw free-body diagrams for the following scenarios:

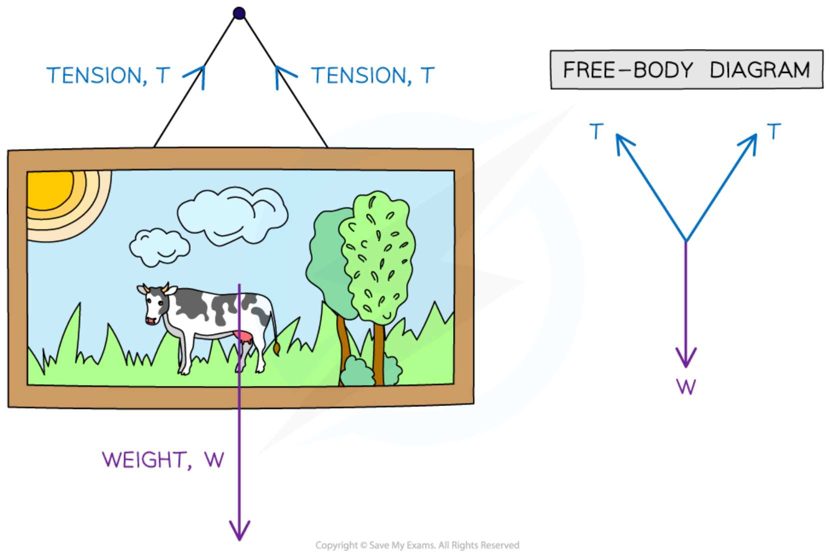
a) A picture frame hanging from a nail

b) A box sliding down a slope

Answer:

Part (a)

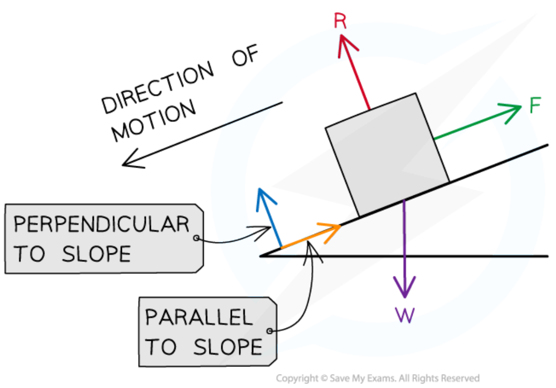
A picture frame hanging from a nail:



- The size of the arrows should be such that the 3 forces would make a closed triangle as they are **balanced**

Part (b)

A box sliding down a slope:



- There are three forces acting on the box:
  - The **normal contact force**, *R*, acts perpendicular to the slope
  - **Friction**, *F*, acts parallel to the slope and in the opposite direction to the direction of motion
  - **Weight**, *W*, acts down towards the Earth

Your notes

Examiner Tips and Tricks

When labeling force vectors, it is important to use conventional and appropriate naming or symbols such as:

- ***w*** or Weight force or ***mg***
- ***N*** *or* ***R*** for normal reaction force (depending on your local context either of these could be acceptable)

Getting used to this notation will make labelling free-body diagrams automatic - leaving you time to focus on the Physics rather than the process. This is crucial as a good diagram will show you exactly what to include in your calculations.