Edexcel International A Level (IAL) Physics

Your notes

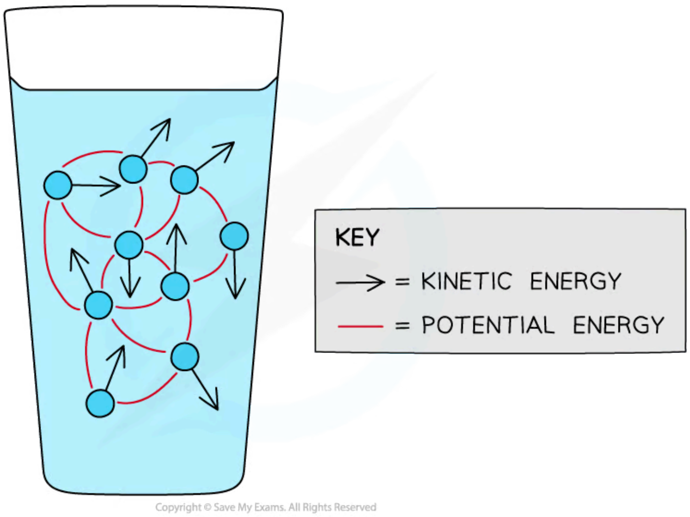
Kinetic Theory & Ideal Gases

Contents

- Internal Energy
- Temperature & Absolute Zero
- Ideal Gas Equation
- Core Practical 14: Investigating Gas Pressure & Volume
- Average Molecular Kinetic Energy

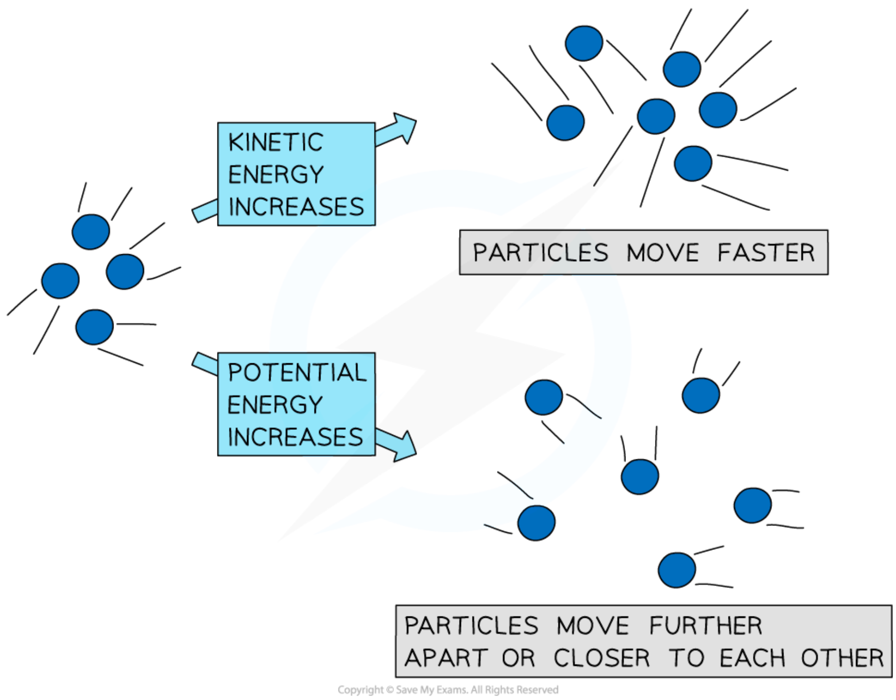
Internal Energy

- Energy can be classified into two forms: kinetic or potential energy
- The molecules of all substances contain both kinetic and potential energies
  - Kinetic energy is due to the speed of the molecules and gives the material its temperature
  - Potential energy is due to the separation between the molecules and their position within the structure



- The amount of kinetic and potential energy a substance contains depends on its phase of matter (solid, liquid or gas)
  - This is known as the **internal energy**
- The internal energy of a substance is defined as:

  The sum of the kinetic and potential energies of all the molecules within a given mass of a substance



- The symbol for internal energy is *U*, with units of **Joules** (J)
- Particles are randomly distributed, meaning they all have different speeds and separations
- The internal energy of a system is determined by:
  - **Temperature** (higher temperature, higher kinetic energy and vice versa)
  - The **random motion** of molecules
  - The **phase of matter**: gases have the highest internal energy, solids have the lowest
  - **Intermolecular forces** between the particles (greater intermolecular forces, higher potential energy and vice versa) - this is linked to the phase (solid, liquid, gas) that the matter is in
- The internal energy of a system can increase by:
  - **Doing work** on it
  - **Adding heat** to it
- The internal energy of a system can decrease by:
  - **Losing heat** to its surroundings
  - **Changing state** from a gas to a liquid or a liquid to a solid

Examiner Tips and Tricks

Your notes

Always remember internal energy is made up of both the kinetic and potential energy of the particles in a substance.

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Absolute Zero

- On the thermodynamic (Kelvin) temperature scale, absolute zero is defined as:

  The lowest temperature possible. Equal to 0 K or –273.15 °C

- It is not possible to have a temperature lower than 0 K
  - This means a temperature in Kelvin will **never** be a negative value
- Absolute zero is defined as:

  The temperature at which the molecules in a substance have zero kinetic energy

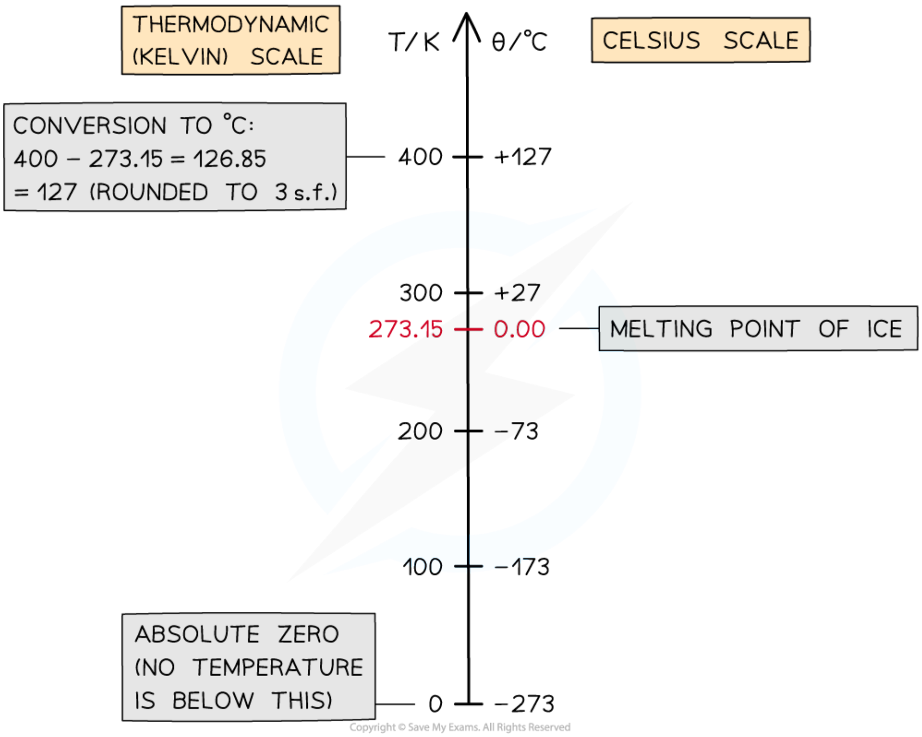
- This means for a system at 0 K, it is not possible to remove any more energy from it
- Even in space, the temperature is roughly 2.7 K, just above absolute zero

Using the Kelvin Scale

- To convert between temperatures θ in the Celsius scale, and T in the Kelvin scale, use the following conversion:

$$\theta / ^{\circ}\text{C} = \text{T / K} - 273.15$$

$$\text{T / K} = \theta / ^{\circ}\text{C} + 273.15$$



Conversion chart relating the temperature on the Kelvin and Celsius scales

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- The divisions on both scales are equal. This means:

A change in a temperature of 1 K is equal to a change in temperature of 1 °C

- This is why when using the specific heat capacity equation

$$\Delta E = mc\Delta\theta$$

- $\Delta\theta$ does not require the temperature to be in either unit
  - This is because the difference in temperature between two values whether in Kelvin or Celsius will be exactly the same

Worked Example

In many ideal gas problems, room temperature is considered to be 300 K. What is this temperature in Celsius?

Answer:

Step 1: Kelvin to Celsius equation

$$\theta / ^{\circ}\text{C} = T / \text{K} - 273.15$$

Step 2: Substitute in value of 300 K

$$300\ \text{K} - 273.15 = 26.85\ ^{\circ}\text{C}$$

Kinetic Energy & Temperature

- Heating an object causes its temperature to rise
- According to **kinetic theory** when energy is supplied to an object the molecules in that object receive the energy as kinetic energy and move faster
  - In solids this is in the form of **vibrations**
  - In a gas, molecules move quickly around their container
  - This kinetic energy determines the temperature of the substance.

Solid, Liquid & Gas Summary Table

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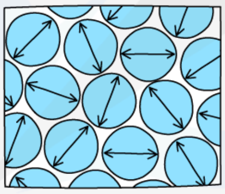
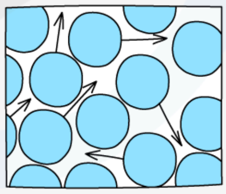
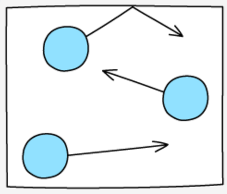
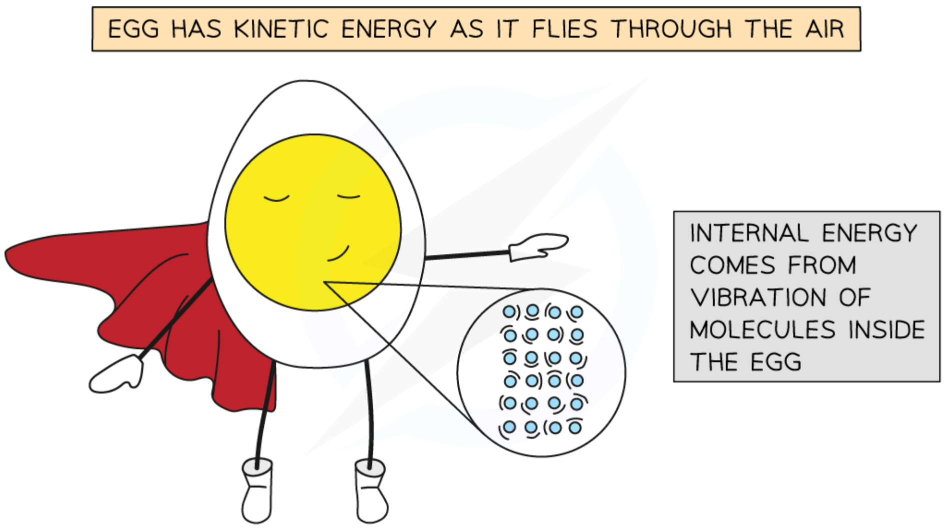
State	Solid	Liquid	Gas
Density	High	Medium	Low
Arrangement of particles	Regular pattern	Randomly arranged	Randomly arranged
Movement of particles	Vibrate around a fixed position	Move around each other	Move quickly in all directions
Energy of particles	Low energy	Greater energy	Highest energy
2D diagram			

- The internal kinetic energy on the atomic scale is separate from the overall movement of an object
- Molecules vibrate within the material and do not cause it to move
  - For example, an egg being thrown through the air has kinetic energy because of the movement of the whole egg and not because of the movement of the individual molecules inside it.



- Taking energy away from the molecules of a substance causes its temperature to become lower
  - In a situation where kinetic energy is continuously being taken away from a collection of molecules, theoretically there would be a moment where all the kinetic

energy has been removed from the substance

- This is when the substance reaches absolute zero, which remains theoretical point, having never been achieved by scientists in a laboratory

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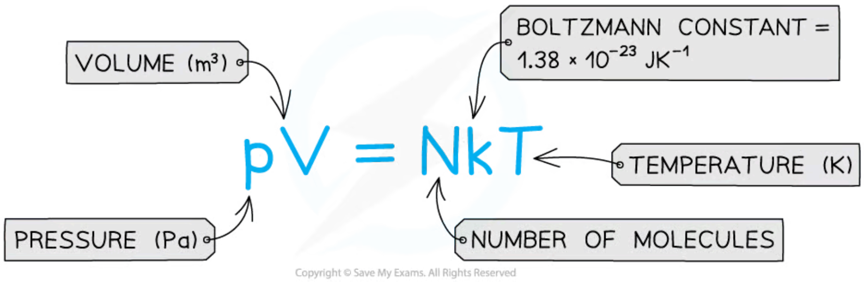
Examiner Tips and Tricks

If you forget in the exam whether it's +273.15 or –273.15, just remember that 0 ˚C = 273.15 K. This way, when you know that you need to +273.15 to a temperature in degrees to get a temperature in Kelvin. For example: 0 ˚C + 273.15 = 273.15 K.

Also remember that C is nearer the beginning of the alphabet to K. So to go back from K to C we must subtract.

Ideal Gas Equation

- When calculating for gases, assume that the gas is an ideal gas
- The **three gas laws** (explained below) can be combined to create one equation in terms of pressure, volume, temperature and amount of gas.



The Boltzmann Constant, k

- The Boltzmann constant *k* is used in the ideal gas equation and is defined as:

$$k = \frac{R}{N_A}$$

- Where:
  - R = molar gas constant
  - N_A = Avogadro's constant
- Boltzmann's constant therefore has a value of

$$k = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23}\ J\,K^{-1}$$

- The Boltzmann constant relates the properties of microscopic particles (e.g. kinetic energy of gas molecules) to their macroscopic properties (e.g. temperature)
  - This is why the units are $J\,K^{-1}$
- Its value is very small because the increase in kinetic energy of a molecule is very small for every incremental increase in temperature

The Gas Laws

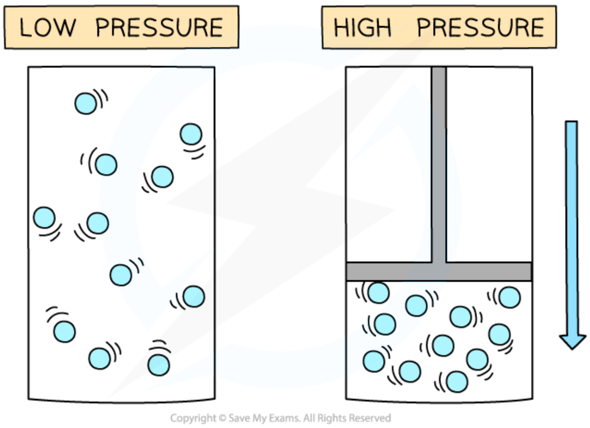
- The ideal gas laws are the experimental relationships between pressure (*P*), volume (*V*) and temperature (*T*) of an ideal gas
- The mass and the number of molecules of the gas is assumed to be constant for each of these quantities

Boyle's Law

- If the temperature *T* of an ideal gas is constant, then **Boyle's Law** is given by:

$$P \propto \frac{1}{V}$$

- This means the pressure is **inversely proportional** to the volume of a gas



Pressure increases when a gas is compressed

- The relationship between the pressure and volume for a fixed mass of gas at constant temperature can also be written as:

$$P_1V_1 = P_2V_2$$

- Where:
  - P_1 = initial pressure (Pa)
  - P_2 = final pressure (Pa)
  - V_1 = initial volume (m^3)
  - V_2 = final volume (m^3)

Charles's Law

- If the pressure *P* of an ideal gas is constant, then **Charles's law** is given by:

$$V \propto T$$

- This means the volume is **proportional** to the temperature of a gas
  - The relationship between the volume and thermodynamic temperature for a fixed mass of gas at constant pressure can also be written as:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

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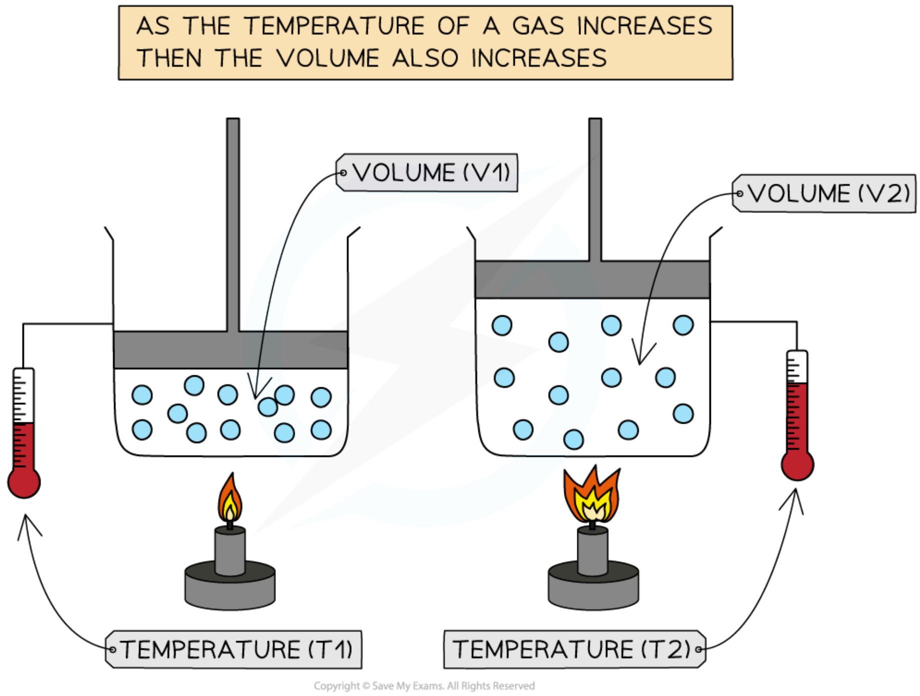
- Where:
  - V_1 = initial volume (m^3)
  - V_2 = final volume (m^3)
  - T_1 = initial temperature (K)
  - T_2 = final temperature (K)



Pressure Law

- If the volume V of an ideal gas is constant, the **Pressure law** is given by:

$$P \propto T$$

- This means the pressure is **proportional** to the temperature
  - The relationship between the pressure and thermodynamic temperature for a fixed mass of gas at constant volume can also be written as:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

- Where:
  - P_1 = initial pressure (Pa)
  - P_2 = final pressure (Pa)
  - T_1 = initial temperature (K)

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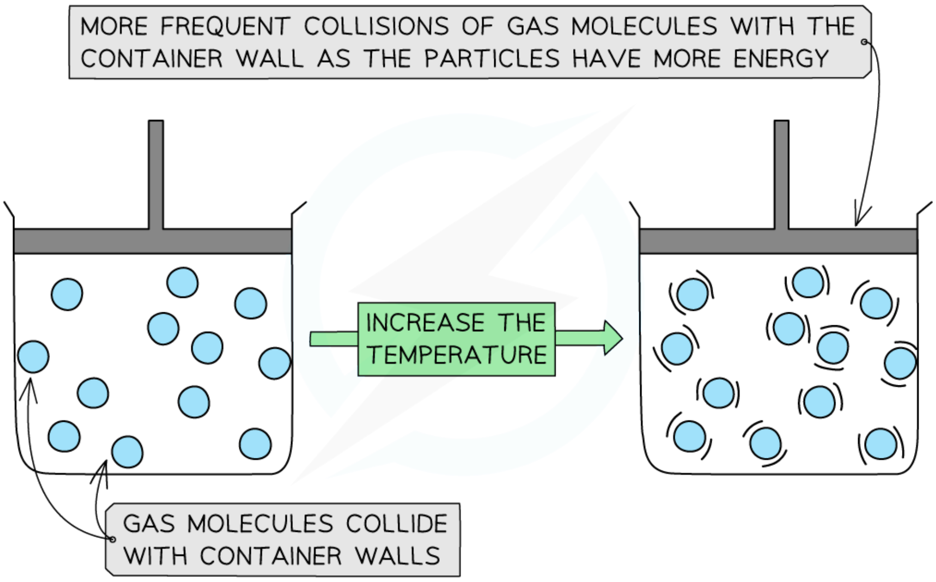
- T_2 = final temperature (K)



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Worked Example

A storage cylinder of an ideal gas has a volume of 8.3 × 10^3 cm^3. The gas is at a temperature of 15°C and a pressure of 4.5 × 10^7 Pa. Calculate the number of molecules of gas in the cylinder.

Answer:

Step 1: Write down the ideal gas equation

$$pV = NkT$$

Step 2: Rearrange the equation for the number of molecules, *N*

$$N = \frac{pV}{kT}$$

Step 3: Substitute in values

$$V = 8.3 \times 10^3\ cm^3 = (8.3 \times 10^3) \times 10^{-6} = 8.3 \times 10^{-3}\ m^3$$

$$T = 15\ ^oC + 273.15 = 288.15\ K$$

$$N = \frac{(4.5 \times 10^7) \times (8.3 \times 10^{-3})}{(1.38 \times 10^{-23}) \times 288.15} = 9.4 \times 10^{25}\ \text{molecules}\ (2\ sf)$$

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Examiner Tips and Tricks

After you solve a problem using any of the gas laws (or all of them combined), always check whether your final result makes physical sense - e.g. if you are asked to calculate the final pressure of a fixed mass of gas being heated at constant volume, your result must be greater than the initial pressure given in the problem (since Gay-Lussac's law states that pressure and absolute temperature are directly proportional at constant volume).

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Investigating Gas Pressure and Volume

- Boyle's Law relates pressure and volume at constant temperature
  - This investigation is one valid example of how this required practical might be tackled, others do exist

Variables

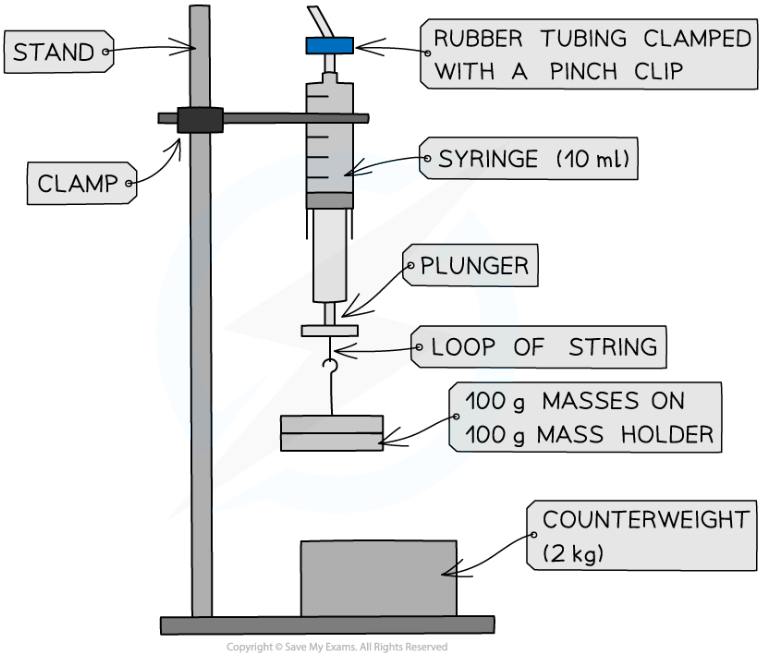
- Independent variable = Mass, *m* (kg)
- Dependent variable = Volume, *V* (m^3)
- Control variables:
  - Temperature
  - Cross-sectional area of the syringe

Equipment List

Apparatus	Purpose
Clamp Stand	To hold the equipment
Syringe (10 ml)	To measure the volume of the air inside the tube
Rubber Tubing	To stop air getting into the syringe
Pinch Clip	To hold the rubber tubing in place
String	To hang the masses
100g masses with 100g holder	Used to change the volume of the air
Counterweight (or G clamp)	To keep the stand and clamp secure and rigid on the flat surface
Vernier Caliper	To calculate the diameter of the syringe

- Resolution of measuring equipment:
  - Pressure gauge = 0.02×10^5 Pa
  - Volume = 0.2 cm^3
  - Vernier Caliper = 0.02 mm

Method



Apparatus setup for Boyle's Law

1. With the plunger removed from the syringe, measure the inside diameter, *d* of the syringe using a Vernier caliper.
    - Take 3 readings and find an average
2. The plunger should be replaced and the rubber tubing should be fit over the nozzle and clamped with a pinch clip as close to the nozzle as possible
3. Set up the apparatus as shown in the diagram
4. Push the syringe upwards until it reads the lowest volume of air visible. Record this volume
5. Add the 100 g mass holder to the loop of string at the bottom of the plunger. Wait a few seconds before taking the reading (this allows temperature to equilibrate after work is done against the plunger when the volume increases)
6. Record the value of the new volume from the syringe scale
7. Repeat the experiment by adding 100 g masses and recording the readings up to 10 readings.

- An example table of results might look like this:

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READ FROM THE SYRINGE SCALE

NO MASS HOLDER

100g MASS HOLDER AND A 100g MASS

MASS / kg	VOLUME / cm^3
0.0	
0.2	
0.4	
0.6	
0.8	
1.0	
1.2	

DIAMETER OF THE SYRINGE	DIAMETER 1/mm	DIAMETER 2/mm	DIAMETER 3/mm	AVERAGE DIAMETER/mm

Analysing the Results

- Boyle's Law can be represented by the equation:

$$pV = \text{constant}$$

- This means the pressure must be calculated from the experiment
- The exerted pressure of the masses is calculated by:

$$p = \frac{F}{A}$$

- Where:
  - F = weight of the masses, mg (N)
  - A = cross-sectional area of the syringe (m^2)
- The cross-sectional area is found from the equation for the area of a circle:

$$A = \frac{\pi d^2}{4}$$

- To calculate the pressure of the gas:

Pressure of the gas = Atmospheric pressure – Exerted pressure from the masses

- Where:
  - Atmospheric pressure = 101 kPa
- The table of results may need to be modified to fit these extra calculations. Here is an example of how this might look:

$F = mg$

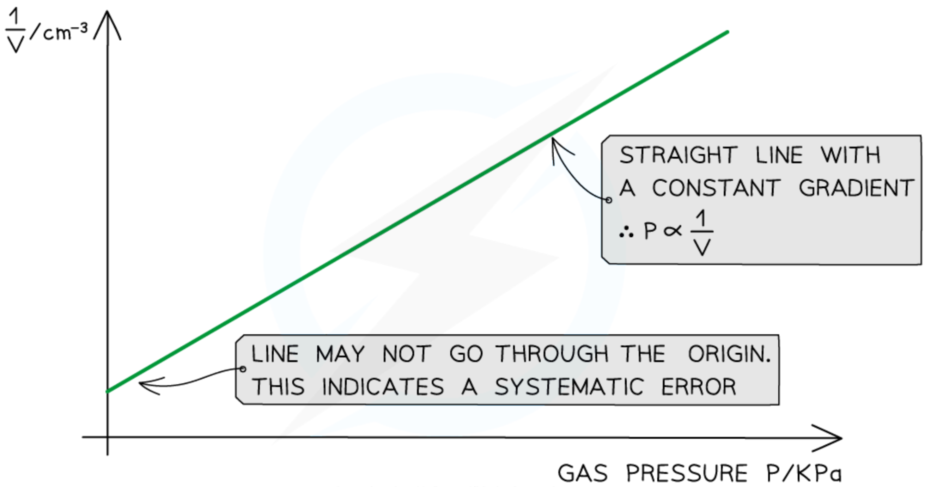
$P = \frac{F}{A}$

ATMOSPHERIC PRESSURE − EXERTED PRESSURE

MASS / kg	FORCE / N	EXERTED PRESSURE / Pa	GAS PRESSURE P/KPa	VOLUME V/cm^3	$\frac{1}{V}$ /cm^{-3}
0.0					
0.2					
0.4					
0.6					
0.8					
1.0					
1.2					

- Once these values are calculated:

1. Plot a graph of *p* against 1 / *V* and draw a line of best fit
2. If this plot is a straight line graph, this means that the pressure is proportional to the inverse of the volume, hence confirming Boyle's Law (pV = constant)



Evaluating the Experiment

Systematic Errors:

- There may be friction in the syringe which causes a systematic error
  - Use a syringe that has very little friction or lubricated it, so the only force is from the weights pulling the syringe downwards

Random Errors:

- The reading of the volume should be taken a few seconds after the mass has been added to the holder to allow temperature changes to equilibrate
- Room temperature must be kept constant

Safety Considerations

- A counterweight or G-clamp must be used to avoid the stand toppling over and causing injury, especially if the surface is not completely flat

Your notes

Average Molecular Kinetic Energy

- An important property of molecules in a gas is their **average kinetic energy**
- This can be deduced from the ideal gas equations relating pressure, volume, temperature and speed
- Recall the **ideal gas equation** in terms of **number of molecules**:

$$pV = NkT$$

- Also, recall the equation linking **pressure and mean square speed** of the molecules:

$$pV = \frac{1}{3}Nm(c_{rms})^2$$

- The left-hand side of both equations are equal to pV
- This means the right-hand sides of both equations are also equal:

$$\frac{1}{3}Nm(c_{rms})^2 = NkT$$

- N will cancel out on both sides and multiplying by 3 on both sides too obtains the equation:

$$m(c_{rms})^2 = 3kT$$

- Recall the familiar kinetic energy equation from mechanics:

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

- Instead of v^2 for the velocity of one particle, $(c_{rms})^2$ is the average speed of all molecules
- Multiplying both sides of the equation by ½ obtains the **average molecular kinetic energy** of the molecules of an ideal gas:

$$E_k = \frac{1}{2}m(c_{rms})^2 = \frac{3}{2}kT$$

- Where:
  - E_k = kinetic energy of a molecule (J)

- m = mass of one molecule (kg)
- $(c_{rms})^2$ = mean square speed of a molecule (m^2 s^{-2})
- k = Boltzmann constant
- T = temperature of the gas (K)

- **Note:** this is the average kinetic energy for only **one** molecule of the gas
- To find the average kinetic energy for many molecules of the gas, multiply both sides of the equation by the number of molecules N to obtain:

$$E_k = \frac{1}{2}Nm(c)^2 = \frac{3}{2}NkT$$

- A key feature of this equation is that the **mean kinetic energy** of an ideal gas molecule is **proportional** to its thermodynamic **temperature**

$$E_k \propto T$$

- The **Boltzmann constant** k can be replaced with

$$k = \frac{R}{N_A}$$

- Substituting this into the average molecular kinetic energy equation means it can also be written as:

$$E_k = \frac{1}{2}m(c_{rms})^2 = \frac{3}{2}kT = \frac{3RT}{2N_A}$$

Worked Example

Helium can be treated as an ideal gas. Helium molecules have a root-mean-square (r.m.s.) speed of 720 m s^{-1} at a temperature of 45 °C. Calculate the r.m.s. speed of the molecules at a temperature of 80 °C.

Answer:

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Step 1: Write down the equation for the average kinetic energy

$$E_k = \frac{1}{2}m(c_{rms})^2 = \frac{3}{2}kT$$

Step 2: Determine the relation between c_{rms} and the temperature T

Since m and k are constant, $(c_{rms})^2$ is directly proportional to T

$$(c_{rms})^2 \propto T$$

Therefore

$$c_{rms} \propto \sqrt{T}$$

Step 3: Change the proportionality into an equation

$$c_{rms} = a\sqrt{T}$$

where a is the constant of proportionality

Step 4: Calculate the constant of proportionality

Substitute the values given already for a temperature and corresponding c_{rms} for helium

c_{rms} = 720 m s^{-1} at a temperature of 45 °C

$$T = 45\ °C + 273.15 = 318.15\ K$$

$$a = \frac{c_{rms}}{\sqrt{T}} = \frac{720}{\sqrt{318.15}}$$

Step 5: Calculate c_{rms} at T = 80 °C by substituting the value of a

$$T = 80\ °C + 273.15 = 353.15\ K$$

$$c_{rms} = \frac{720}{\sqrt{318.15}} \times \sqrt{353.15} = 758.57 = \mathbf{760\ m\ s^{-1}\ (2\ s.f.)}$$

Examiner Tips and Tricks

You can remember the equation through the rhyme 'Average K.E is three-halves kT'.

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