Your notes

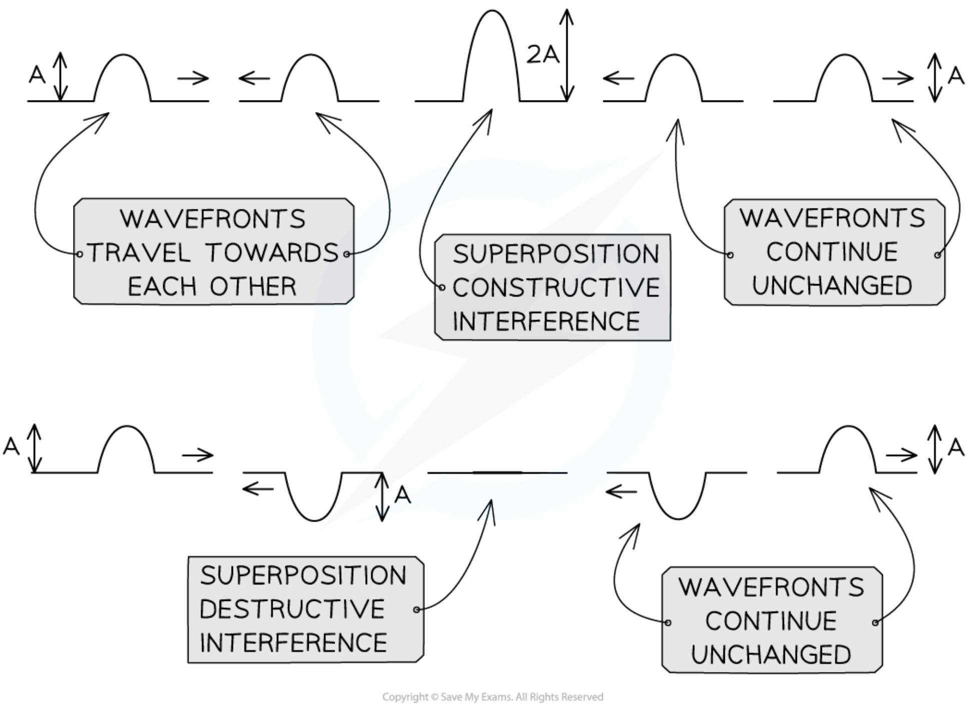
Edexcel International A Level (IAL) Physics

Interference & Stationary Waves

Contents

Interference & Superposition of Waves

- **Interference** occurs whenever two or more waves combine to produce a resultant wave with a new amplitude
- **Superposition** literally means to be **positioned over** something
  - When waves interfere and combine, they do so according to the **principle of superposition**
- If two wavefronts are travelling towards each other they will combine by superposition and then pass through
- The wavefronts will emerge unchanged on the other side

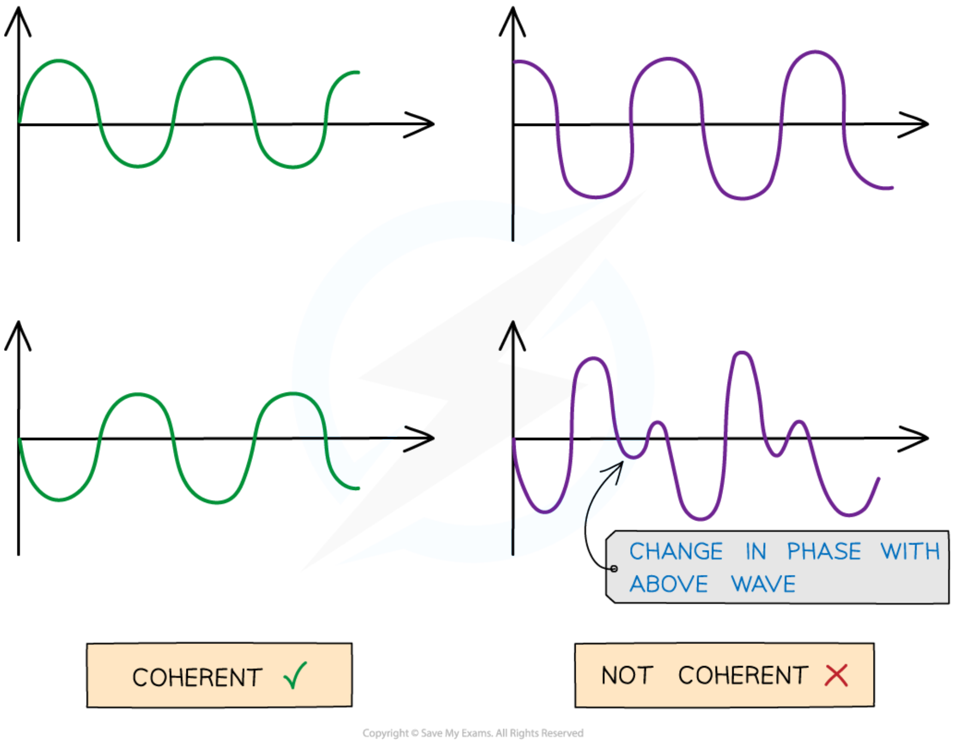


- Interference due to superposition can be constructive or destructive
  - **Constructive interference** happens when the resultant wave has a larger amplitude than any of the individual waves
  - **Destructive interference** happens when the resultant wave has a smaller amplitude than the individual waves

Coherence

- Interference is only observable if produced by a **coherent** source

- Waves are said to be coherent if they have:
  - A **constant phase difference**
  - The **same frequency**



Coherent waves (on the left) and non-coherent waves (on the right). The abrupt change in phase creates an inconsistent phase difference

- For example, in light, a coherent beam of light contains light waves that are **monochromatic** and have a constant phase difference
  - Monochromatic light consists of light waves of a **single frequency**
  - Laser light is an example of a coherent light source
  - Filament lamps produce incoherent light waves

Examiner Tips and Tricks

It can sometimes be tricky to identify whether constructive or destructive interference is taking place. If two waves meet at the same point on each wave e.g. two crests then the interference will be constructive, if not it will be destructive.

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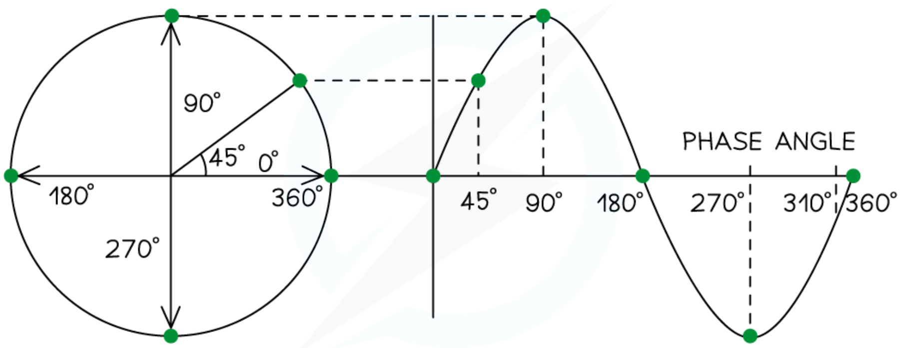
Phase & Path Difference

- Waves are said to be **coherent** if they have:
  - The same **frequency**
  - A **constant phase difference**

Phase Difference

- Two points on a wave, or on different waves, are in phase when they are the same point in their wave cycle
- The angle between their wave cycles is the phase difference

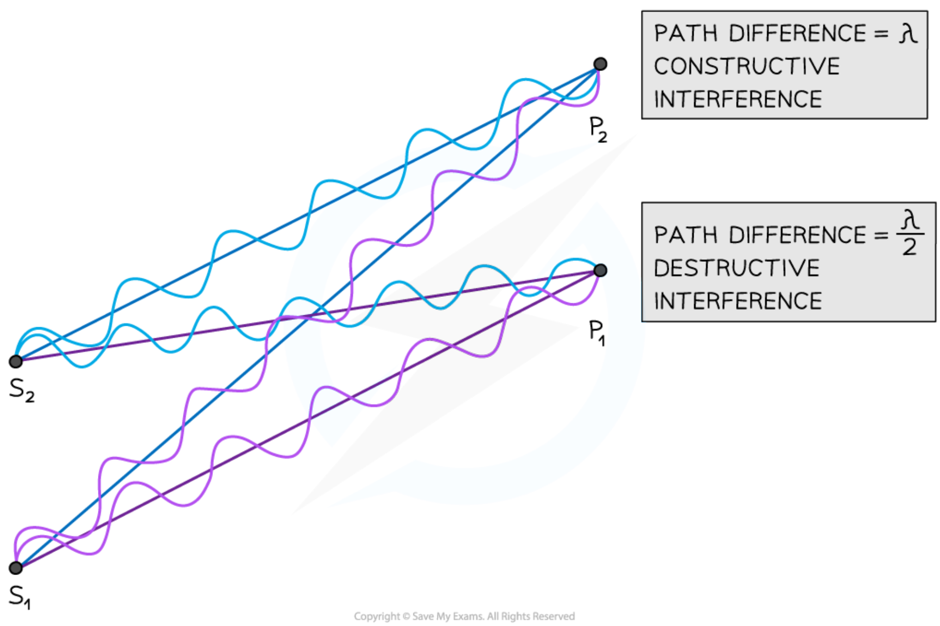


Path Difference

- The type of interference occurring at a given point (i.e. constructive or destructive) depends on the **path difference** of the overlapping waves
- Path difference is defined as:

  The difference in distance travelled by two waves from their sources to the point where they meet

- Path difference is generally expressed in multiples of wavelength



At point P_2 the waves have a path difference of a whole number of wavelengths resulting in constructive interference. At point P_1 the waves have a path difference of an odd number of half wavelengths resulting in destructive interference

- In the diagram above, the number of wavelengths between:
  - $S_1 \rightarrow P_1 = 6\lambda$
  - $S_2 \rightarrow P_1 = 6.5\lambda$
  - $S_1 \rightarrow P_2 = 7\lambda$
  - $S_2 \rightarrow P_2 = 6\lambda$
- The path difference at point P_1 is $6.5\lambda - 6\lambda =$ **$\lambda / 2$**
- The path difference at point P_2 is $7\lambda - 6\lambda =$ **λ**
- In general:
  - The condition for constructive interference is a path difference of **$n\lambda$**
  - The condition for destructive interference is a path difference of **$(n + ½)\lambda$**
  - In this case, n is an integer i.e. 0, 1, 2, 3...
- Hence:
  - **Destructive interference** occurs **at** point **P_1**
  - **Constructive interference** occurs **at** point **P_2**

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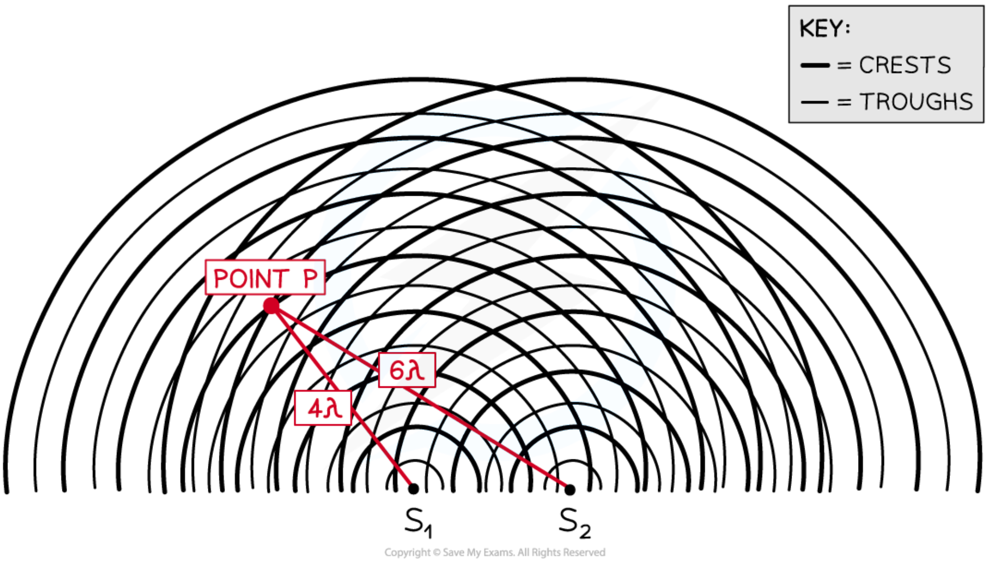

At point P the waves have a path difference of a whole number of wavelengths resulting in constructive interference

- Another way to represent waves spreading out from two sources is shown in the diagram above
- At point **P**, the number of **crests** from:
  - Source $S_1 = 4\lambda$
  - Source $S_2 = 6\lambda$
- The path difference at **P** is $6\lambda - 4\lambda = \mathbf{2\lambda}$
- This is a whole number of wavelengths, hence **constructive interference** occurs at point **P**

Worked Example

The diagram shows the interferences of coherent waves from two point sources.

Which row in the table correctly identifies the type of interference at points **X**, **Y** and **Z**.

	X	Y	Z
A	Constructive	Destructive	Constructive
B	Constructive	Constructive	Destructive
C	Destructive	Constructive	Destructive
D	Destructive	Constructive	Constructive

Answer: B

- At point **X**:
    - Both peaks of the waves are overlapping
    - Path difference = $5.5\lambda - 4.5\lambda = \lambda$
    - This is **constructive** interference and rules out options **C** and **D**
- At point **Y**:
    - Both troughs are overlapping
    - Path difference = $3.5\lambda - 3.5\lambda = 0$
    - Therefore **constructive** interference occurs
- At point **Z**:
    - A peak of one of the waves meets the trough of the other
    - Path difference = $4\lambda - 3.5\lambda = \lambda / 2$
    - This is **destructive** interference

Examiner Tips and Tricks

Phase difference and path difference are easy to confuse because the names sound similar. However they are very different concepts.

Phase difference tells us how far apart the waves are when **comparing their phases** (you can think of this as their peaks and troughs).

Path difference is how much further along one wave is than another. Think of it as how much **further along the path** it has travelled.

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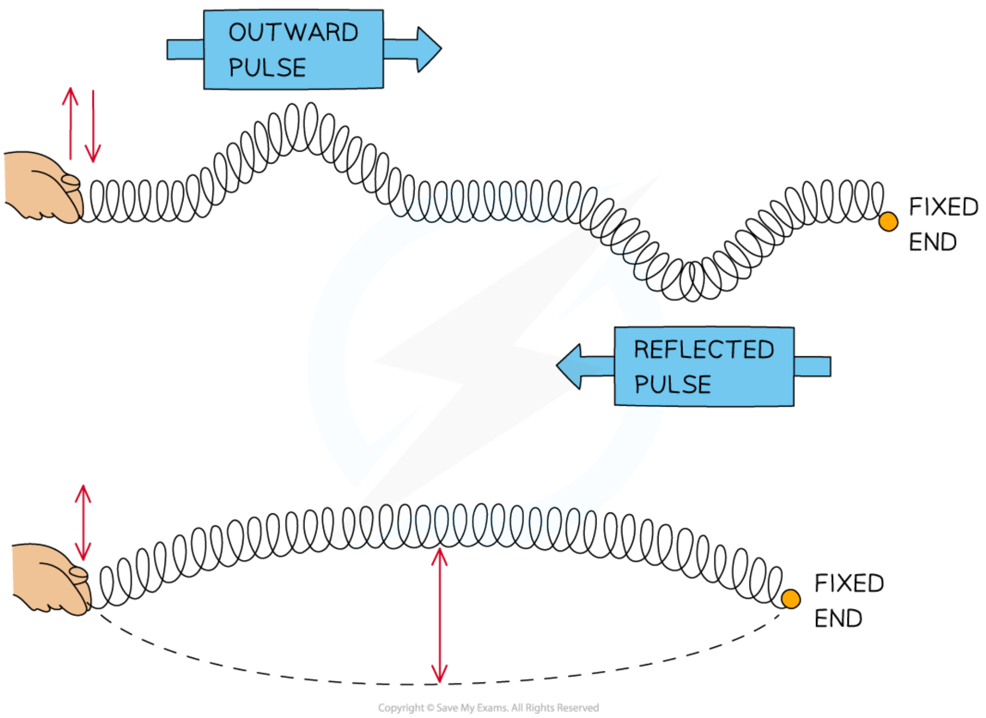
Stationary Waves

- **Stationary waves**, or standing waves, are produced by the **superposition** of two waves of the **same frequency** and **amplitude** travelling in **opposite directions**
- This is usually achieved by a travelling wave and its **reflection**. The superposition produces a wave pattern where the **peaks** and **troughs do not move**

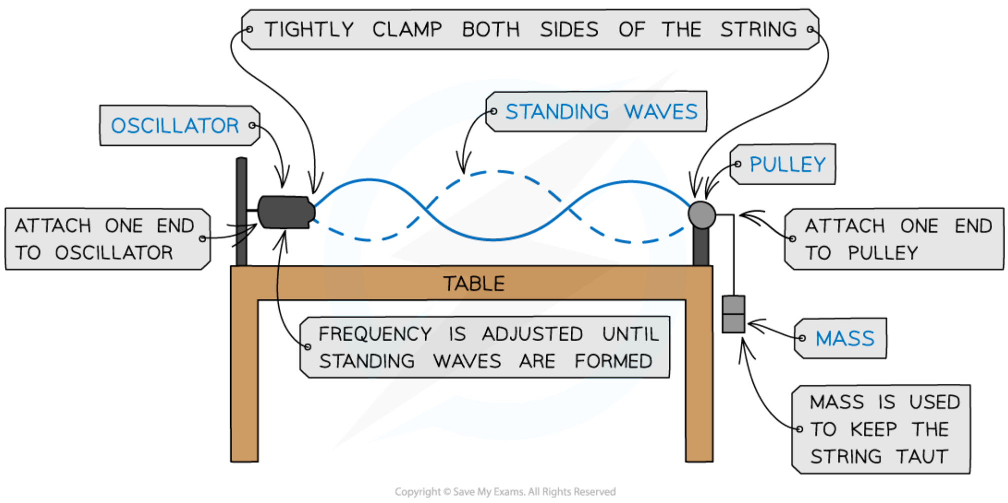


Formation of a stationary wave on a stretched spring fixed at one end

- In this section, we will look at a few experiments that demonstrate stationary waves in everyday life

Stretched Strings

- Vibrations caused by stationary waves on a stretched string produce sound
  - This is how stringed instruments, such as guitars or violins, work
- This can be demonstrated by an oscillator vibrating a length of string under tension fixed at one end:

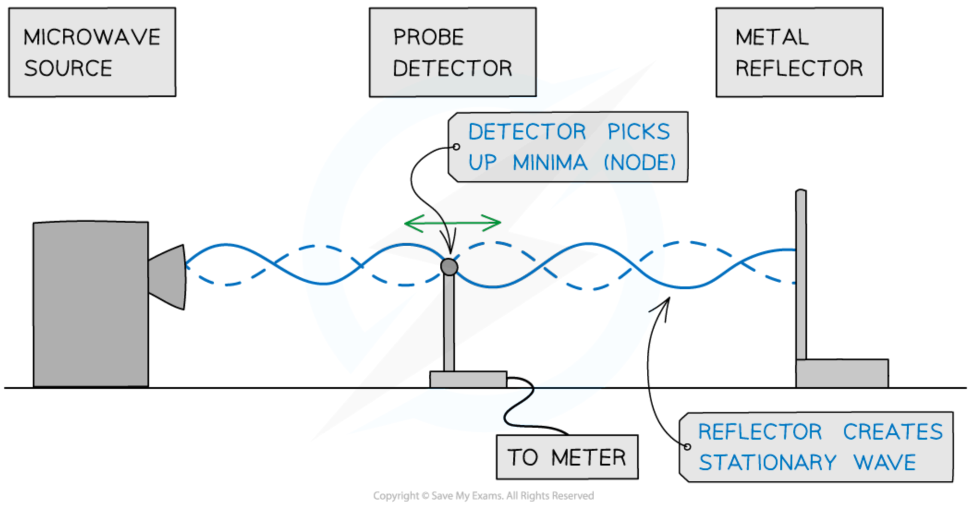


Stationary wave on a stretched string

- As the frequency of the oscillator changes, standing waves with different numbers of **minima** (**nodes**) and **maxima** (**antinodes**) form

Microwaves

- A microwave source is placed in line with a **reflecting plate** and a small **detector** between the two
- The reflector can be moved to and from the source to vary the stationary wave pattern formed
- By moving the **detector**, it can pick up the minima (nodes) and maxima (antinodes) of the stationary wave pattern



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Using microwaves to demonstrate stationary waves

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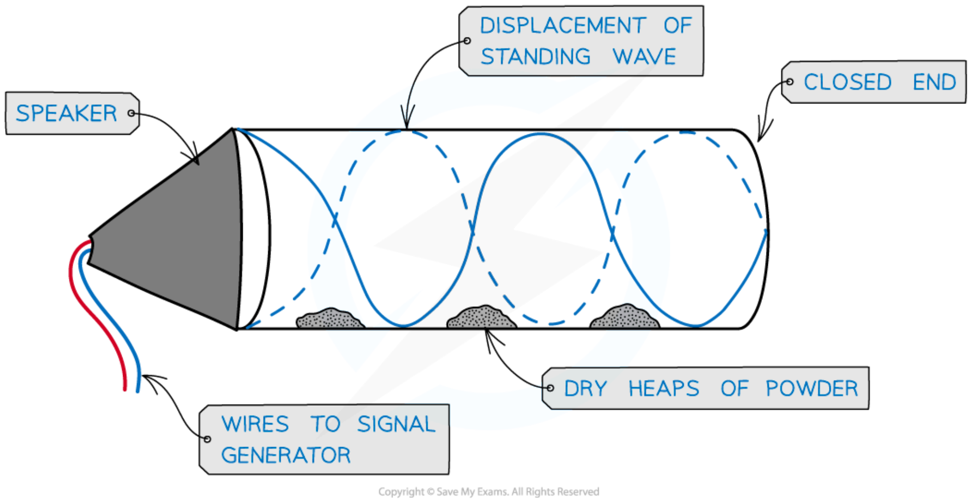
Air Columns

- The formation of stationary waves inside an air column can be produced by sound waves
  - This is how musical instruments, such as clarinets and organs, work
- This can be demonstrated by placing a fine powder inside the air column and a loudspeaker at the open end
- At certain frequencies, the powder forms evenly spaced heaps along the tube, showing where there is **zero disturbance** as a result of the nodes of the stationary wave



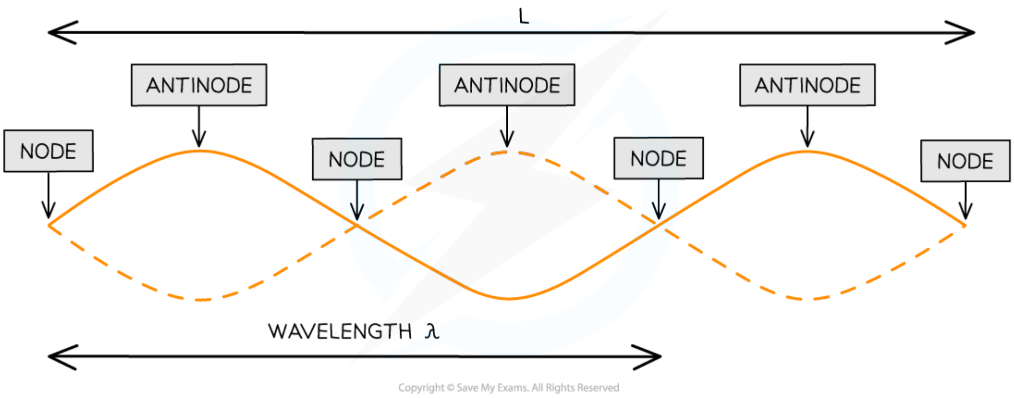
Stationary wave in an air column

- In order to produce a stationary wave, there must be a minima (node) at one end and a maxima (antinode) at the end with the loudspeaker

Nodes and Antinodes

- A stationary wave is made up **nodes** and **antinodes**
  - **Nodes** are where there is no vibration
  - **Antinodes** are where the vibrations are at their maximum amplitude
- The nodes and antinodes **do not** move along the string. Nodes are fixed and antinodes only move in the vertical direction
  - Between nodes, all points along the stationary wave are in phase

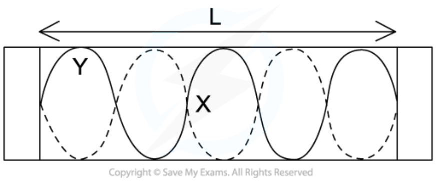
- The image below shows the nodes and antinodes on a snapshot of a stationary wave at a point in time



- L is the length of the string
- 1 wavelength λ is only a portion of the length of the string

Worked Example

A stretched string is used to demonstrate a stationary wave, as shown in the diagram.



Which row in the table correctly describes the length of L and the name of **X** and **Y**?

	Length L	Point X	Point Y
A	5 wavelengths	Node	Antinode
B	$2\frac{1}{2}$ wavelengths	Antinode	Node
C	$2\frac{1}{2}$ wavelengths	Node	Antinode
D	5 wavelengths	Antinode	Node

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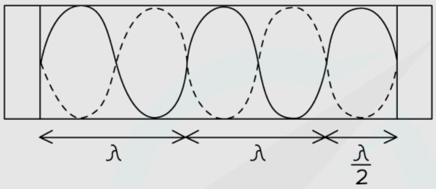
Answer: C

STEP 1

CALCULATE HOW MANY WAVELENGTHS IN THE LENGTH OF THE STRING



$= 2\frac{1}{2}$ WAVELENGTHS

THIS RULES OUT A AND D

STEP 2

X IS A POINT OF 0 DISPLACEMENT – A NODE

STEP 3

Y IS A POINT OF MAXIMUM DISPLACEMENT – AN ANTINODE

STEP 4

THE CORRECT ROW IS C

Examiner Tips and Tricks

Always refer back to the experiment or scenario in an exam question e.g. the wave produced by a **loudspeaker** reflects at the end of a **tube**. This reflected wave, with the same frequency, overlaps the initial wave to create a stationary wave.

Can't remember which is the node and which is the anti-node? **Nod**es occur at areas of **NO D**isturbance!

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Wave Speed on a Stretched String

- The speed of a wave travelling along a string with two fixed ends is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

- Where:
  - T = tension in the string (N)
  - μ = mass per unit length of the string (kg m^{-1})

- At the fundamental frequency, f_0 of a stationary wave of length L, the wavelength, $\lambda = 2L$
- Therefore, according to the wave equation, the speed of the stationary wave is:

$$v = f\lambda = f \times 2L$$

- Combining these two equations leads to the equation for the **fundamental frequency** (sometimes referred to as the first harmonic):

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

- Where:
  - f = frequency (Hz)
  - L = the length of the string (m)
  - T = the tension in the string (N)
  - μ = mass per unit length (kg m^{-1})
- Mass per unit length, μ can be calculated by dividing the mass of the string by the length of the string

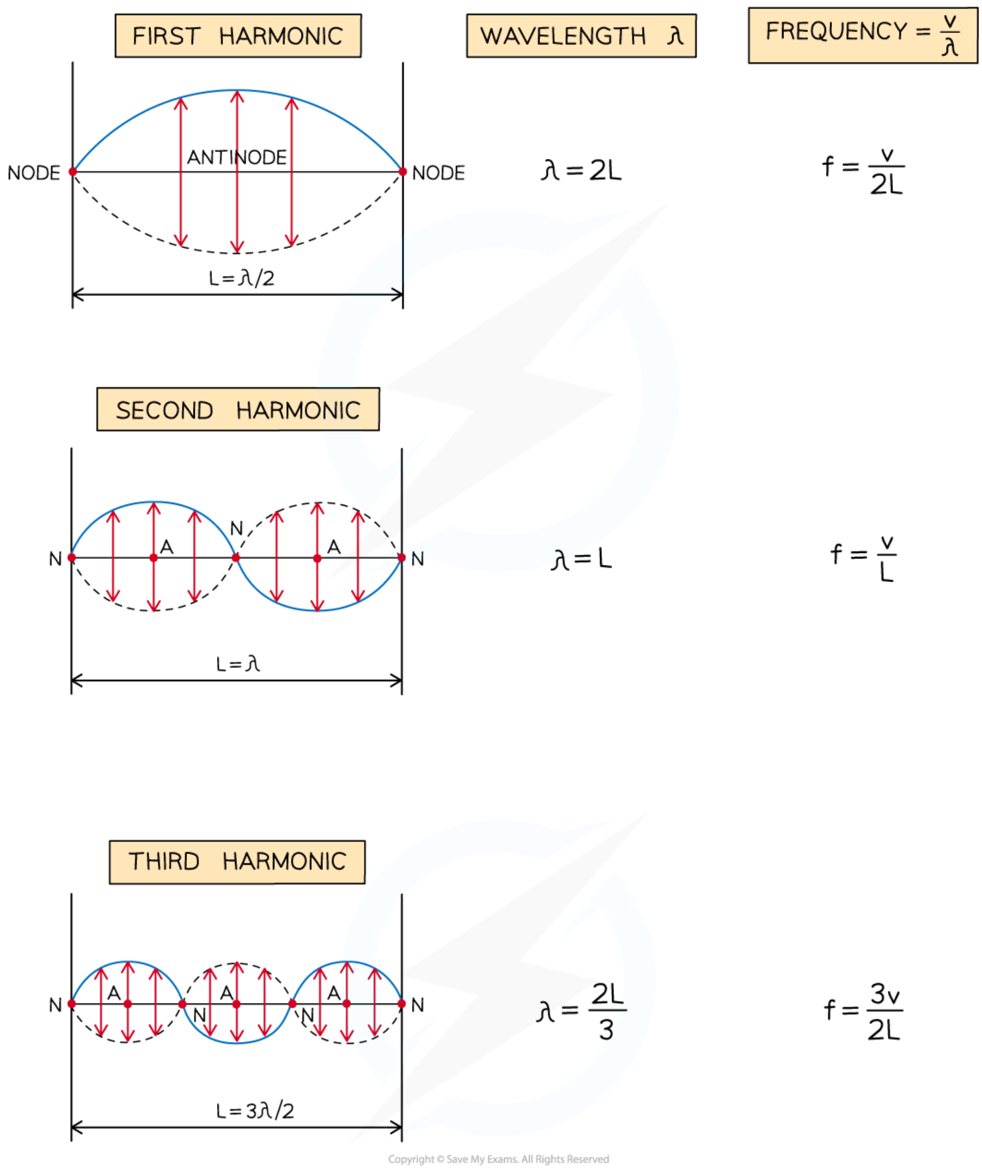

Diagram showing the first three modes of vibration of a stretched string with corresponding frequencies

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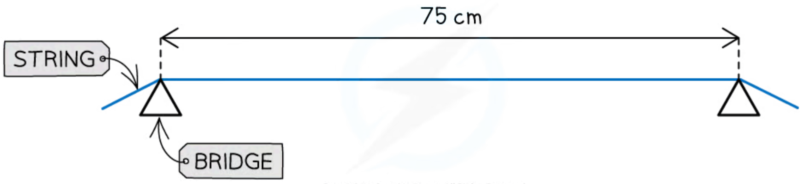
Worked Example

A guitar string of mass 3.2 g and length 90 cm is fixed onto a guitar.

The string is tightened to a tension of 65 N between two bridges at a distance of 75 cm.



Your notes

Calculate the

a) speed of the waves on the string

b) fundamental frequency of the string

Answer:

Part (a)

Step 1: Write the known quantities in S.I. units

- Tension, T = 65 N
- Mass, m = 3.2 g = 3.2×10^{-3} kg
- Length of string, L = 90 cm = 0.90 m
  - Mass per unit length, $\mu = \frac{m}{L} = \frac{3.2 \times 10^{-3}}{0.9}$ = 3.56×10^{-3} kg m^{-1}

Step 2: Write the equation for speed on a string and calculate

- $v = f\lambda = f \times 2L$ AND $\mathbf{f} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$
- So, $v = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \times 2L = \sqrt{\frac{T}{\mu}}$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{65}{3.56 \times 10^{-3}}} = 135$$

Step 3: Write the answer to the correct significant figures and include units

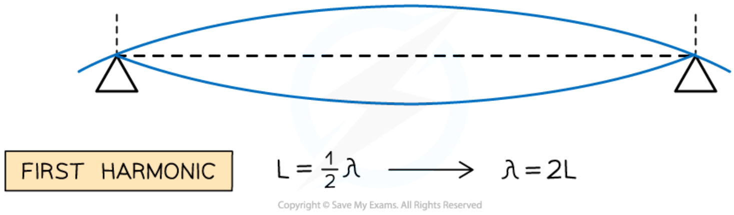
- The speed of the wave on the string, v = 140 m s^{-1}

Part (b)

Step 1: Write the known quantities in S.I. units

- Tension, T = 65 N
- Length of string under tension, L = 75 cm = 0.75 m
- Mass per unit length, μ = 3.56×10^{-3} kg m^{-1} (from part (a))

Step 2: Identify the length of one wavelength at the fundamental frequency, f_0



Step 3: Write the equation for fundamental frequency and calculate

$$f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{v}{2L} = \frac{135}{(2 \times 0.75)} = 90.1$$

Step 3: Write the answer to the correct significant figures and include units

- The fundamental frequency, f_0 = 90 Hz

Examiner Tips and Tricks

Go through solutions step by step, showing all your working. Questions like this one will be very similar so you can rely on using your tried and tested method to get the answer.

Core Practical 5: Investigating Stationary Waves

Aims of the Experiment

- The overall aim of the experiment is to measure how the frequency of the first harmonic is affected by changing one of the following variables:
  - The length of the string
  - The tension in the string
  - Strings with different values of mass per unit length

Variables

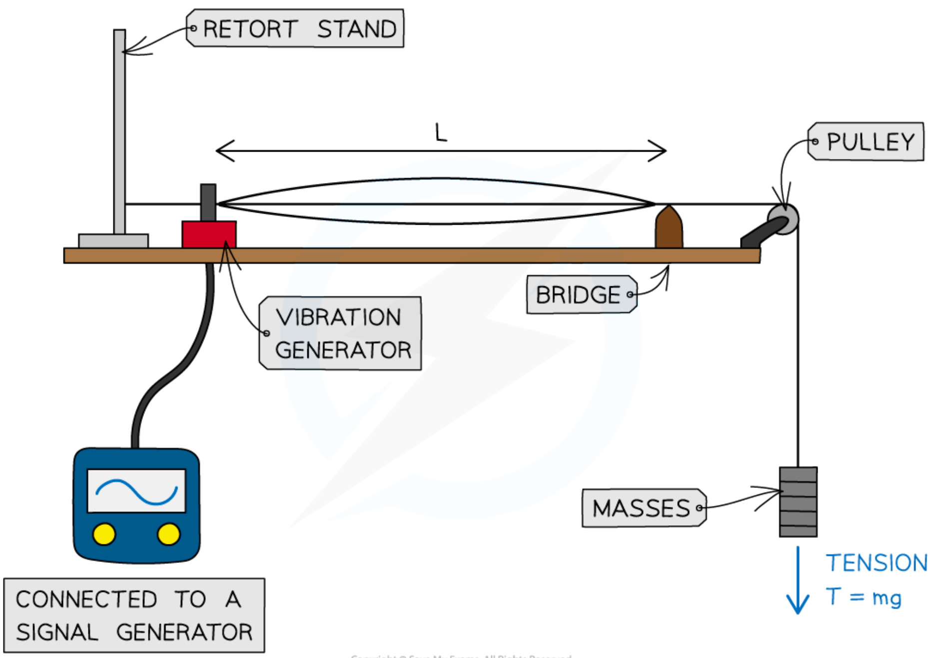
- Independent variable = either length, tension, or mass per unit length
- Dependent variable = frequency of the first harmonic
- Control variables
  - If length is varied = same masses attached (tension), same string (mass per unit length)
  - If tension is varied = same length of the string, same string (mass per unit length)
  - If mass per unit length is varied = same masses attached (tension), same length of the string

Equipment List

Apparatus	Purpose
Signal generator	Used to operate the vibration generator and measure the frequency of the first harmonic
Vibration generator	Connected to the signal generator to produce the stationary wave
Retort stand	To provide a stable fixed end on the table
G clamp or 2 kg mass	To place on the retort stand to stabilise apparatus
2.0 m of string	Used to observe the stationary wave
Pulley	To allow the masses to hang vertically, and introduces less friction than the edge of the table
Wooden bridge	To provide the other fixed end which can vary the length of the string
Mass hanger + 100 g masses	To hang from the pulley to vary the tension in the string
Metre ruler	To measure the length of the string
Top–pan balance	To measure the mass of the string

- Resolution of measuring equipment:
  - Metre ruler = 1 mm
  - Signal generator ~ 10 nHz
  - Top-pan balance = 0.005 g

Method



The setup of apparatus required to measure the frequency of the first harmonic at different values of length, tension, or mass per unit length

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This method is an example of the procedure for varying the length of the string with the frequency – this is just one possible relationship that can be tested

1. Set up the apparatus by attaching one end of the string to the **vibration generator** and pass the other end over the bench pulley and secure to the mass hanger
2. Adjust the position of the bridge so that the length L is measured from the vibration generator to the bridge using a metre ruler
3. Turn on the **signal generator** to set the string **oscillating**
4. Increase the **frequency** of the vibration generator until the **first harmonic** (**nodes** at both ends and an **antinode** in the middle) is observed and read the frequency that this occurs at
5. **Repeat** the procedure with different lengths of L
6. Repeat the frequency readings at least two more times and take the **average** of these measurements
7. Measure the **tension** in the string using $T = mg$
    - Where m is the mass attached to the string and g is the gravitational field strength on Earth (9.81 N kg^{-1})
8. Measure the **mass per unit length** of the string, μ = mass of string ÷ length of string
    - Simply take a known length of the string (1 m is ideal) and measure its mass on a balance

- An example of a table with some possible string lengths might look like this:

LENGTH OF THE STRING

FREQUENCY OF THE FIRST HARMONIC

L / m	f / Hz 1st READING	f / Hz 2nd READING	f / Hz 3rd READING	f / Hz MEAN
0.2				
0.4				
0.6				
0.8				
1.0				
1.2				
1.4				
1.6				

Before conducting an experiment, a table must be set up to detail of what measurements are to be made

Your notes

Analysing the Results

- For the **first harmonic**, wavelength, $\lambda = 2L$
- So, the **speed** of the stationary wave is:

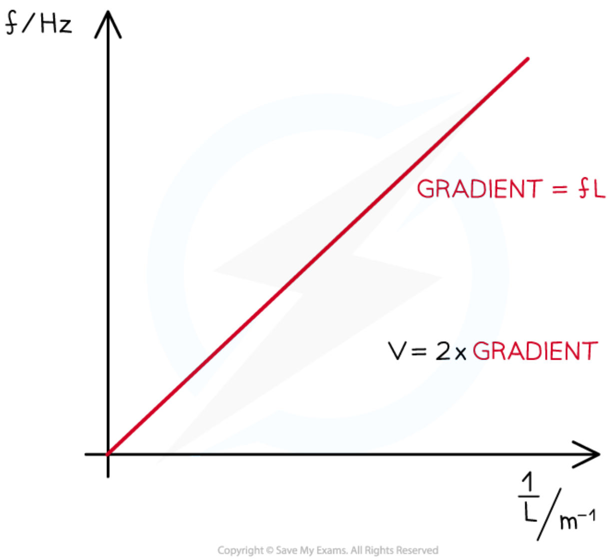
$$v = f\lambda = f \times 2L$$

- Rearranging for **frequency**, f:

$$f = \frac{v}{2L} = \frac{v}{2}\left(\frac{1}{L}\right)$$

- Comparing this to the equation of a straight line: y = mx
    - y = f (Hz)
    - x = $1/L$ (m^{-1})
    - Gradient = $v/2$ ($m\ s^{-1}$)

1. **Plot** a graph of the mean values of f against $1/L$
2. Draw a **line of best fit** and calculate the gradient
3. Work out the **wave speed**, which will be 2 × gradient



If the frequency is plotted again the inverse of the length, the velocity is twice the gradient of the graph

- Verify the **wave speed** of the travelling waves using the equation:

$$v = \sqrt{\frac{T}{\mu}}$$

- Where:
  - T = tension (N)
  - μ = mass per unit length (kg m^{-1})
- Assess the **uncertainties** in the measurements of length and frequency, and carry out calculations to determine the uncertainty in the wave speed

Evaluating the Experiment

Systematic errors:

- An **oscilloscope** can be used to verify the signal generator's readings
- The signal generator should be left for about 20 minutes to stabilise
- The measurements would have a **greater resolution** if the length used is as large as possible, or as many half-wavelengths as possible
  - This means measurements should span a **suitable range**, for example, 20 cm intervals over at least 1.0 m

Random errors:

- The **sharpness** of resonance leads to the biggest problem in deciding when the first harmonic is achieved
  - This can be **resolved** by adjusting the frequency while looking closely at a node. This is a technique to gain the largest response
  - Looking at the amplitude is likely to be less reliable since the wave will be moving very fast
- When taking repeat measurements of the frequency, the best procedure is as follows:
  - Determine the **frequency** of the **first harmonic** when the largest vibration is observed and note down the frequency at this point
  - **Increase the frequency** and then gradually reduce it until the first harmonic is observed again and note down this frequency
  - If taking three repeat readings, repeat this procedure again
  - Average the three readings and move on to the next measurement

Safety Considerations

- Use a **rubber string** instead of a metal wire, in case it snaps under tension
- If using a metal wire, **wear goggles** to protect the eyes

- **Stand well away** from the masses in case they fall onto the floor
- Place a **crash mat** or a soft surface under the masses to break their fall

Your notes

Worked Example

A student investigates the relationship between the frequency of the stationary waves on a wire and the tension in the wire. The tension is varied by adding masses to a hanger which is attached to a pulley over one end of a table.The student records the following data:

- Mass of the wire = 0.16 g
- Length of the wire weighed = 1.0 m
- Distance between the fixed ends = 0.4 m

Mass on hanger (kg)	Frequency (Hz) 1st reading	Frequency (Hz) 2nd reading	Frequency (Hz) mean
0.20	140	135	
0.40	195	200	
0.60	240	240	
0.80	280	278	
1.00	310	310	
1.20	340	340	
1.40	365	368	
1.60	392	390	

Calculate the frequency using the relation between frequency, length, tension and mass per unit length. Evaluate the percentage uncertainty in these values.

Answer:

Your notes

Step 1: Calculate the mean values of frequency

- The mean readings are: 138 Hz; 198 Hz; 240 Hz; 279 Hz; 310 Hz; 340 Hz; 367 Hz; 391 Hz

Step 2: Work out the tensions and mass per unit length

- Use the equation to calculate the frequency for a mass of 0.20 kg:

$$\mu = \frac{0.16 \times 10^{-3}}{1.0} = 0.16 \times 10^{-3} \text{ kg m}^{-1}$$

$$T = mg = 0.20 \times 9.81 = 1.96 \text{ N}$$

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 0.4}\sqrt{\frac{1.96}{0.16 \times 10^{-3}}} = 138.3 = 140 \text{ Hz (2 s.f.)}$$

- Working in the same way, the frequencies for the remaining lengths are: 200 Hz; 240 Hz; 280 Hz; 310 Hz; 340 Hz; 370 Hz; 390 Hz

Step 3: Determine the uncertainties in each quantity

Mass on hanger

- The maximum uncertainty is likely to be 0.005 kg
- Therefore, % uncertainty in measurement of 0.2 kg is $\frac{0.005}{0.20} \times 100\% = 2.5\%$
  - Since each mass has an uncertainty of 0.005 kg, these are added together as the masses increase, so each mass has the same percentage uncertainty i.e. $\frac{0.01}{0.40} \times 100\% = 2.5\%$

Length

- Resolution of a metre ruler is 1 mm, but errors of up to 1 cm are possible in this set up
- Therefore, % uncertainty in measurement of 0.4 m is $\frac{0.01}{0.4} \times 100\% = 2.5\%$

Your notes

Mass per unit length

- The resolution of a metre ruler is 1 mm and the resolution of a top-pan balance is 0.005 g
- Therefore the uncertainty in the mass per unit length:
    - $\frac{\Delta\mu}{\mu} = \frac{\Delta m}{m} + \frac{\Delta l}{l}$
    - $\Delta\mu = 0.16 \times \left(\frac{0.005}{0.16} + \frac{0.001}{1.0}\right) = 0.005$ g/m
- Therefore, % uncertainty in measurement of 0.16 g/m is $\frac{0.005}{0.16} \times 100\% = 3\%$

Step 4: Determine the uncertainties in the mean frequency values

- Since tension (proportional to the mass on the hanger) and mass per unit length are to the power of $\frac{1}{2}$, their % uncertainties are halved
- Maximum % uncertainty in calculated frequencies would be the % uncertainty in each value added together:
    - % uncertainty in frequency = 2.5 % + 1.25% + 1.5% = 5.25%