Edexcel International A Level (IAL) Physics

Your notes

Density, Upthrust & Viscous Drag

Contents

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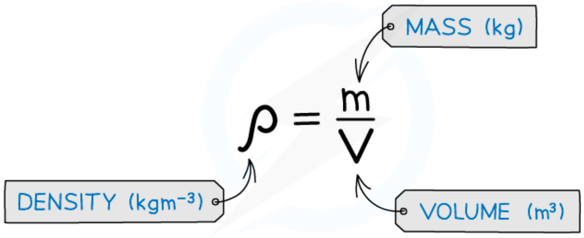
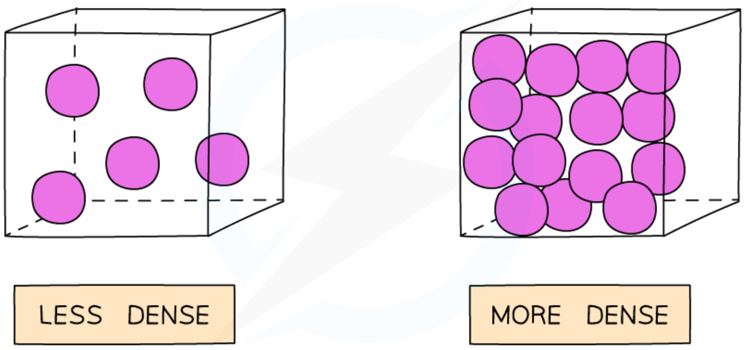
Density

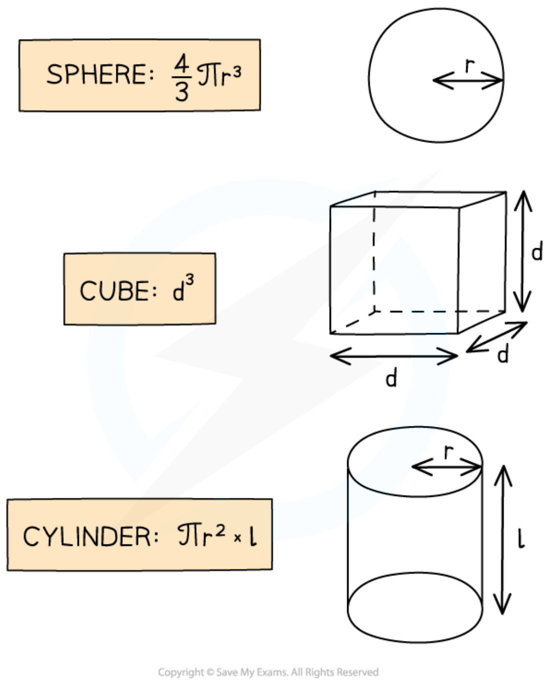
- Density is the **mass per unit volume** of an object
  - Objects made from low-density materials typically have a lower mass
  - For example, a balloon is less dense than a small bar of lead despite occupying a larger volume
- The units of density depend on the units used for mass and volume:
  - If the mass is measured in *g* and volume in **cm^3**, then the density will be in **g / cm^3**
  - If the mass is measured in *kg* and volume in **m^3**, then the density will be in **kg / m^3**





Gases are less dense than a solid

- The volume of an object may not always be given directly, but can be calculated with the appropriate equation depending on the object's shape

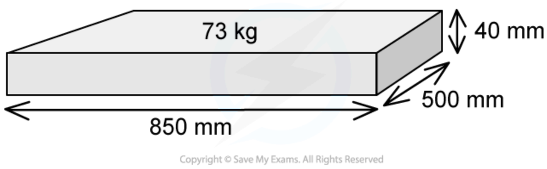


Volumes of common 3D shapes

Worked Example

A paving slab has a mass of 73 kg and dimensions 40 mm × 500 mm × 850 mm.

Calculate the density, in kg m^{-3} of the material from which the paving slab is made.



Answer:

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STEP 1

EQUATION FOR DENSITY

$$\rho = \frac{M}{V}$$

STEP 2

CALCULATE THE VOLUME

$$V = 40\ \text{mm} \times 500\ \text{mm} \times 850\ \text{mm} = 1.7 \times 10^{7}\ \text{mm}^3$$

STEP 3

CONVERT mm^3 TO m^3

$$1\ \text{mm} = 0.001\ \text{m} = 1 \times 10^{-3}\ \text{m}$$

$$1\ \text{mm}^3 = (0.001)^3\ \text{m}^3 = (1 \times 10^{-3})^3\ \text{m}^3 = 1 \times 10^{-9}\ \text{m}^3$$

$$V = 1.7 \times 10^{7} \times 1 \times 10^{-9} = 1.7 \times 10^{-2}\ \text{m}$$

STEP 4

SUBSTITUTE MASS AND VOLUME INTO DENSITY EQUATION

$$\rho = \frac{73\ \text{kg}}{1.7 \times 10^{-2}\ \text{m}^3} = 4300\ \text{kgm}^{-3}\ \ (2\ \text{s.f.})$$

Examiner Tips and Tricks

- When converting a **larger** unit to a **smaller** one, you **multiply (×)**
  - E.g. 125 m = 125 × 100 = 12 500 cm
- When you convert a **smaller** unit to a **larger** one, you **divide (÷)**
  - E.g. 5 g = 5 / 1000 = 0.005 or 5×10^{-3} kg
- When dealing with squared or cubic conversions, cube or square the conversion factor too
  - E.g. 1 $\text{mm}^3 = 1 / (1000)^3 = 1 \times 10^{-9}\ \text{m}^3$
  - E.g. 1 $\text{cm}^3 = 1 / (100)^3 = 1 \times 10^{-6}\ \text{m}^3$

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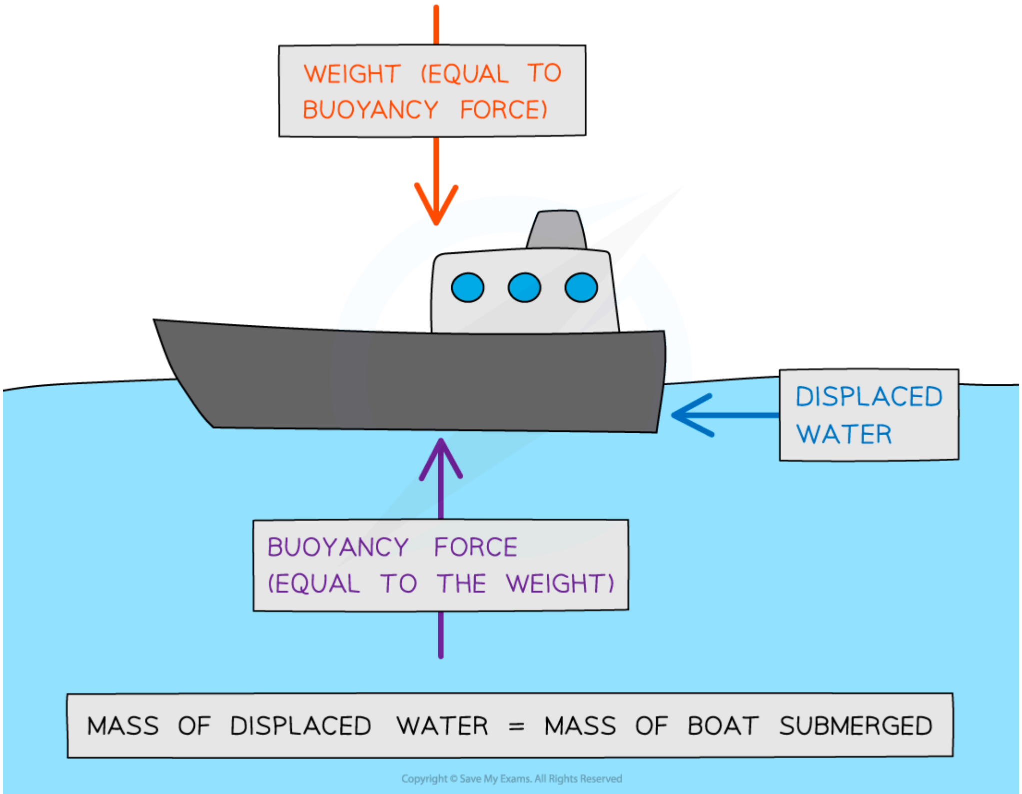
Upthrust

Archimedes' Principle

- Archimedes' principle states:

  An object submerged in a fluid at rest has an upward buoyancy force (upthrust) equal to the weight of the fluid displaced by the object

- The object sinks until the weight of the fluid displaced is equal to its own weight
  - Therefore the object floats when the magnitude of the upthrust equals the weight of the object
- The magnitude of upthrust can be calculated in steps by:
  - Find the volume of the submerged object, which is also the volume of the displaced fluid
  - Find the weight of the displaced fluid
  - Since $m = \rho V$ (density × volume), upthrust is equal to $F = mg$ which is the weight of the fluid displaced by the object
- Archimedes' Principle explains how ships float:



Boats float because they displace an amount of water that is equal to their weight

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Worked Example

Atmospheric pressure at sea level has a value of 100 kPa. The density of sea water is 1020 kg m^{-3}.

At what depth in the sea would the total pressure be 250 kPa?

A. 20 m

B. 9.5 m

C. 18 m

D. 15 m

ANSWER: D

STEP 1

THE TOTAL PRESSURE = HYDROSTATIC PRESSURE + ATMOSPHERIC PRESSURE

$$250 \times 10^3 = \rho g h + 100 \times 10^3$$

$$250 \times 10^3 = 1020 \times 9.81 \times h + 100 \times 10^3$$

STEP 2

REARRANGING FOR HEIGHT h

$$h = \frac{250 \times 10^3 - 100 \times 10^3}{1020 \times 9.81} = 15 \text{ m}$$

Worked Example

Icebergs typically float with a large volume of ice beneath the water. Ice has a density of 917 kg m^{-3} and a volume of V_i.

The density of seawater is 1020 kg m^{-3}. What fraction of the iceberg is above the water?

A. 0.10 V_i

B. 0.90 V_i

C. 0.97 V_i

D. 0.20 V_i

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ANSWER: A

STEP 1

ACCORDING TO ARCHIMEDES' PRINCIPLE, THE UPTHRUST IS EQUAL TO THE WEIGHT OF THE SEAWATER DISPLACED BY THE ICEBERG

ICEBERG WEIGHT $W_i = m_i g$

BUOYANCY FORCE IS THE WEIGHT OF THE DISPLACED WATER $W_w = m_w g$

STEP 2

SINCE THE ICEBERG IS FLOATING, ITS WEIGHT IS EXACTLY EQUAL TO THE BUOYANCY FORCE

$$W_i = W_w$$

$$m_i g = m_w g$$

STEP 3

REARRANGE DENSITY EQUATION FOR MASS

$$m = \rho V$$

$$\rho_i V_i g = \rho_w V_w g$$

STEP 4

CANCELLING g SHOWS THE FRACTION OF ICE UNDERWATER IS GIVEN BY RATIO OF DENSITIES

$$\frac{V_i}{V_w} = \frac{\rho_w}{\rho_i}$$

STEP 5

REARRANGE FOR V_w

$$V_w = \frac{\rho_i V_i}{\rho_w}$$

$$V_w = \frac{917}{1020} V_i = 0.9 V_i$$

STEP 6

THEREFORE 90% OF THE ICEBERG'S VOLUME IS SUBMERGED UNDERWATER

THIS MEANS THAT $1 - 0.9 = 0.1\ V_i$ IS ABOVE WATER

Examiner Tips and Tricks

Don't get confused by the two step process to find upthrust.

- Step 1: You need the volume of the submerged object, but **only** because you want to know **how much fluid was displaced**

- Step 2: What you **really** want to know is the **weight of the displaced fluid**.

A couple of familiar equations will help;

- $m = \rho V$ to get mass (and that's the V from step 1 out of the way),

then

- $W = mg$ to get weight

If you are feeling particularly mathematical, you can combine your equations, so that $W = \rho V g$

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Stoke's Law

Viscous drag

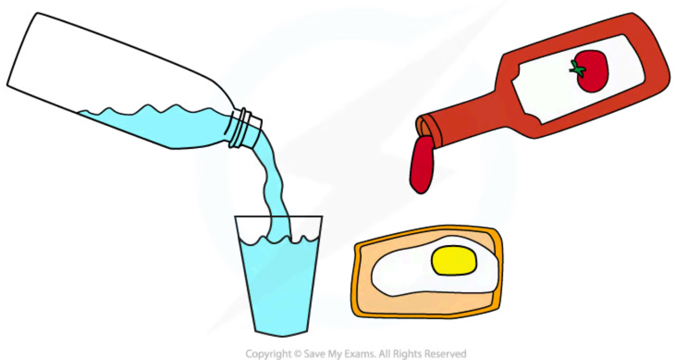
- Viscous drag is defined as:

The frictional force between an object and a fluid which opposes the motion between the object and the fluid

- The viscous drag on a small sphere can be calculated using Stokes' law:

$$F = 6\pi\eta r v$$

- Where
  - F = viscous drag force (N)
  - η = coefficient of viscosity of the fluid (N s m^{-2} *or* Pa s)
  - r = radius of the object (m)
  - v = velocity of the object (m s^{-1})
- The viscosity of a fluid can be thought of as its thickness, or how much it resists flowing
  - Fluids with **low viscosity** are **easy** to pour, while those with **high viscosity** are **difficult** to pour



- The **coefficient of viscosity** is a property of the fluid (at a given temperature) that indicates how much it will resist flow
  - **The rate of flow** of a fluid is inversely proportional to the coefficient of viscosity

Terminal velocity of a sphere in a fluid

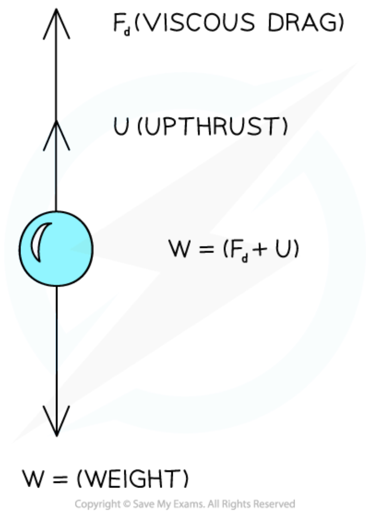
- When an object falls through a fluid (e.g. a skydiver falling through the air), it reaches **terminal velocity**

- This occurs when the **weight** of the object balances with the upthrust and the **viscous drag** force

weight = upthrust + viscous drag

$$W = U + F_d$$

Forces acting on a sphere falling through a fluid



At terminal velocity, the forces on the sphere are balanced: W (downwards) = F_d + U (upwards)

- Consider a small, solid sphere of radius r moving slowly at terminal velocity through a fluid of viscosity η
- The upthrust U on the sphere is equal to the weight of the fluid displaced

$$W_s = W_f + 6\pi\eta r v_{\text{term}}$$

$$m_s g = m_f g + 6\pi\eta r v_{\text{term}}$$

- Where
  - m_s = mass of the sphere (kg)
  - m_f = mass of the fluid displaced (kg)
  - g = acceleration due to gravity (m s^{-2})
  - v_{term} = terminal velocity of the sphere (m s^{-1})
- Using the density equation, the mass m_s of the **sphere** is given by:

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$$m_s = \rho_s V = \frac{4}{3}\pi r^3 \rho_s$$

- Where:
  - ρ_s = density of the sphere (kg m^{-3})
- The **volume of displaced fluid** is the **same** as the **volume of the sphere**
- Therefore, the mass m_f of **displaced fluid** is given by:

$$m_f = \rho_f V = \frac{4}{3}\pi r^3 \rho_f$$

- Where:
  - ρ_f = density of the fluid (kg m^{-3})
- Substituting the expressions for mass back into the original equation:

$$\left(\frac{4}{3}\pi r^3 \rho_s\right)g = \left(\frac{4}{3}\pi r^3 \rho_f\right)g + 6\pi\eta r v_{\text{term}}$$

- Rearrange to make terminal velocity the subject of the equation

$$6\pi\eta r v_{\text{term}} = \frac{4}{3}\pi r^3 g(\rho_s - \rho_f)$$

$$v_{\text{term}} = \frac{\frac{4}{3}\pi r^3 g(\rho_s - \rho_f)}{6\pi\eta r}$$

$$v_{\text{term}} = \frac{2\pi r^3 g(\rho_s - \rho_f)}{9\pi\eta r}$$

- Finally, cancel out *r* from the top and bottom to find an expression for **terminal velocity** in terms of the **radius of the sphere** and the **coefficient of viscosity**

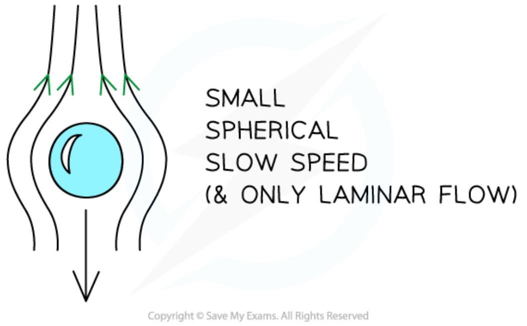
$$v_{\text{term}} = \frac{2r^2 g(\rho_s - \rho_f)}{9\eta}$$

- This final equation shows that terminal velocity is:
  - directly proportional to the square of the radius of the sphere
  - inversely proportional to the viscosity of the fluid

Understanding Viscosity & Stoke's Law

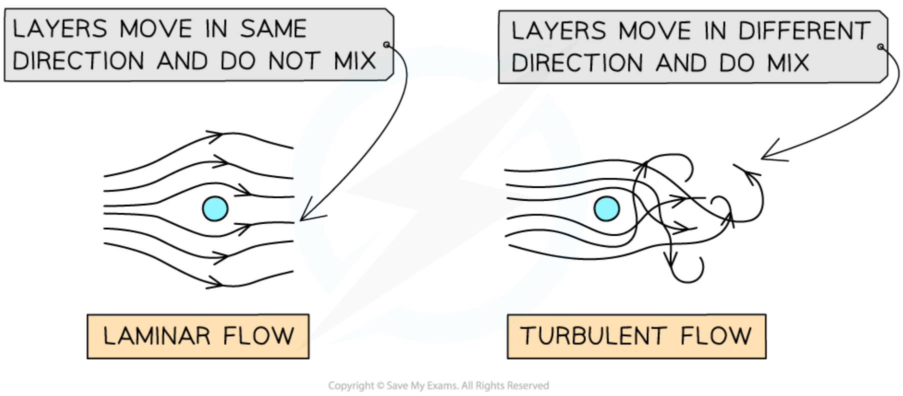
Conditions for Stoke's Law Equation

- The equation can only be used when certain conditions are met:
  - The flow is laminar
  - The object is small
  - The object is spherical
  - Motion between the sphere and the fluid is at a slow speed



Laminar flow and turbulent flow

- As an object moves through a fluid, or a fluid moves around an object, layers in the fluid are created
- In **laminar flow**, all the layers are moving in the **same** direction, and they do **not** mix
  - This tends to happen for slow-moving objects or slow-flowing liquids
  - The equation above only applies to laminar flow
- In **turbulent flow**, the layers move in **different** directions, and the layers **do** mix



Changing viscosity

- Viscosity is temperature-dependent
  - **Liquids** are **less** viscous as the temperature increases

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- **Gases** get **more** viscous as the temperature increases

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Worked Example

A ball bearing of radius 5.0 mm falls at a constant speed of 0.030 m s^{-1} through an oil which has a viscosity of 0.3 Pa s and a density of 900 kg m^{-3}.

Determine the viscous drag acting on the ball bearing.

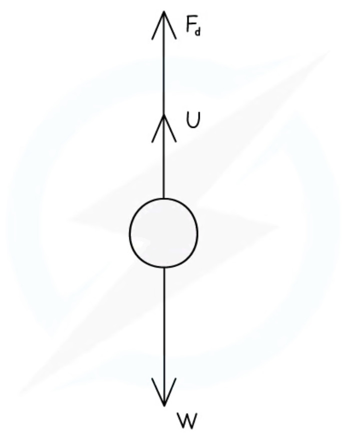
Answer:

Step 1: List the known quantities in SI units

- Radius of the sphere, r_s = 5.0 mm = 5.0×10^{-3} m
- Terminal velocity of the sphere, v = 0.03 m s^{-1}
- Viscosity of oil, η = 0.3 Pa s
- Density of oil, ρ_f = 900 kg m^{-3}

Step 2: Sketch a free-body diagram to resolve the forces at constant speed

$$W_s = F_d + U$$



Step 3: Calculate the value for viscous drag, F_d

$$F_d = 6\pi\eta rv = 6 \times \pi \times 0.3 \times 5.0 \times 10^{-3} \times 0.03 = 0.008482$$

Step 4: Write the complete answer to the correct significant figures and include units

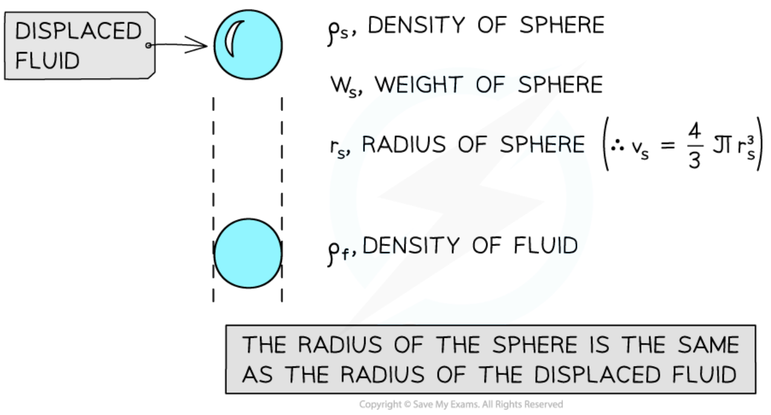
- The viscous drag, $F_d = 8.5 \times 10^{-4}$ N

Examiner Tips and Tricks

You may need to write out some or all of the derivation given in the first part above.

It is really important to keep clear whether you are talking about the density of the sphere or the fluid, and the mass of the sphere or the fluid.

Practice using subscripts, and do try this at home. It isn't one to do for the first time in an exam!



Core Practical 2: Investigating Viscosity of a Liquid

Aim of the experiment

- By allowing small spherical objects of known weight to fall through a fluid until they reach terminal velocity, the **viscosity of the fluid** can be calculated

Variables

- Independent variable: weight of ball bearing, W_s
- Dependent variable: terminal velocity, v_{term}
- Control variables: fluid being tested, temperature

Equipment List

- Long measuring cylinder
- Viscous liquid to be tested (thin oil of known density or washing up liquid)
- Stand and clamp
- Metre rule
- Rubber bands
- Steel ball bearings of different weights
- Digital scales
- Vernier calipers
- Digital stopwatch
- Magnet

Method

1. Weigh the balls, measure their radius using Vernier callipers and calculate their density
2. Place three rubber bands around the tube. The highest should be far enough below the surface of the liquid to ensure the ball is travelling at terminal velocity when it reaches this band. The remaining two bands should be 10 – 15 cm apart so that time can be measured accurately
3. Release the ball and wait until it reaches the first rubber band. Start the timer at the first band, then use the lap timer to find the time to fall d_1 and also d_2
    1. If lap timing is not available, two stopwatches operated by different people should be used

2. If the ball is still accelerating as it passes the markers, they need to be moved downwards until the ball has reached terminal velocity before passing the first mark

4. Measure and record the distances d_1 (between the highest and middle rubber band) and d_2 between the highest and lowest bands.

5. Repeat at least three times for balls of this diameter and three times for each different diameter

6. Ball bearings are removed from the bottom of the tube using the magnet against the outside wall of the measuring cylinder

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Analysis

- Terminal velocity is used in this investigation since at terminal velocity the forces in each direction are balanced

$$W_s = F_d + U \text{ (equation 1)}$$

- Where;
  - W_s = weight of the sphere
  - F_d = the drag force (N)
  - U = upthrust (N)
- The weight of the sphere is found using volume, density and gravitational force

$$W_s = v_s \rho_s g$$

$$W_s = \frac{4}{3}\pi r^3 \rho_s g \text{ (equation 2)}$$

- Where
  - v_s = volume of the sphere (m^3)
  - ρ_s = density of the sphere (kg m^3)
  - g = gravitational force (N kg^{-1})
- Recall Stoke's Law

$$F_d = 6\pi\eta r v_{term} \text{ (equation 3)}$$

- Upthrust equals the weight of the displaced fluid
  - The **volume of displaced fluid is the same as the volume of the sphere**
  - The weight of the fluid is found from volume, density and gravitational force as above

$$U = \frac{4}{3}\pi r^3 \rho_f g \text{ (equation 4)}$$

- Substitute equations 2, 3 and 4 into equation 1

$$\frac{4}{3}\pi r^3 \rho_s g = 6\pi\eta r v_{term} + \frac{4}{3}\pi r^3 \rho_f g$$

- Rearrange to make viscosity the subject of the equation

$$\frac{4}{3}\pi r^3 \rho_s g - \frac{4}{3}\pi r^3 \rho_f g = 6\pi\eta r v_{term}$$

$$\frac{4\pi r^3 g(\rho_s - \rho_f)}{3 \times (6\pi r v_{term})} = \eta$$

$$\eta = \frac{2r^2 g(\rho_s - \rho_f)}{9v_{term}}$$

Evaluating the Experiment

Systematic Errors:

- Ruler must be clamped vertically and close to the tube to avoid parallax errors in measurement
- Ball bearing must reach terminal velocity before the first marker

Random errors:

- Cylinder must have a large diameter compared to the ball bearing to avoid the possibility of turbulent flow
- Ball must fall in the centre of the tube to avoid pressure differences caused by being too close to the wall which will affect the velocity

Safety Considerations

- Measuring cylinders are not stable and should be clamped into position at the top and bottom
- Spillages will be slippery and must be cleaned up immediately
- Avoid getting fluids in the eyes

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