Edexcel International A Level (IAL) Physics

Your notes

Circular Motion

Contents

Your notes

Radians

- A **radian** (rad) is defined as:

  The angle subtended at the centre of a circle by an arc equal in length to the radius of the circle

- Radians are used whenever describing the angular displacement of objects in circular motion
- Angular displacement can be calculated using the equation:

$$\Delta\theta = \frac{\Delta S}{r}$$

- Where:
  - $\Delta\theta$ = angular displacement, or angle of rotation (radians)
  - s = length of the arc, or the distance travelled around the circle (m)
  - r = radius of the circle (m)

- Radians are commonly written in terms of π
- The angle in radians for a complete circle (360°) is equal to:

$$\frac{\text{circumference of circle}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi$$

Radian Conversions

- If an angle of 360° = 2π radians, then 1 radian in degrees is equal to:

$$\frac{360}{2\pi} = \frac{180}{\pi} \approx 57.3^\circ$$

- Use the following equation to convert from degrees to radians:

$$\theta^\circ \times \frac{\pi}{180} = \theta \text{ rad}$$

Table of common degrees to radians conversions

Degrees (°)	Radians (rads)
360	2π
270	$\frac{3\pi}{2}$
180	π
90	$\frac{\pi}{2}$

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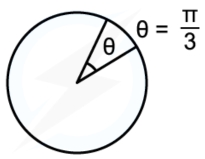
Worked Example

Convert the following angular displacement into degrees:



Answer:

STEP 1

REARRANGE DEGREES TO RADIANS CONVERSION EQUATION

DEGREES ⟶ RADIANS $\theta^\circ \times \frac{\pi}{180} = \theta \text{ RAD}$

RADIANS ⟶ DEGREES $\theta \text{ RAD} \times \frac{180}{\pi} = \theta^\circ$

STEP 2

SUBSTITUTE VALUE

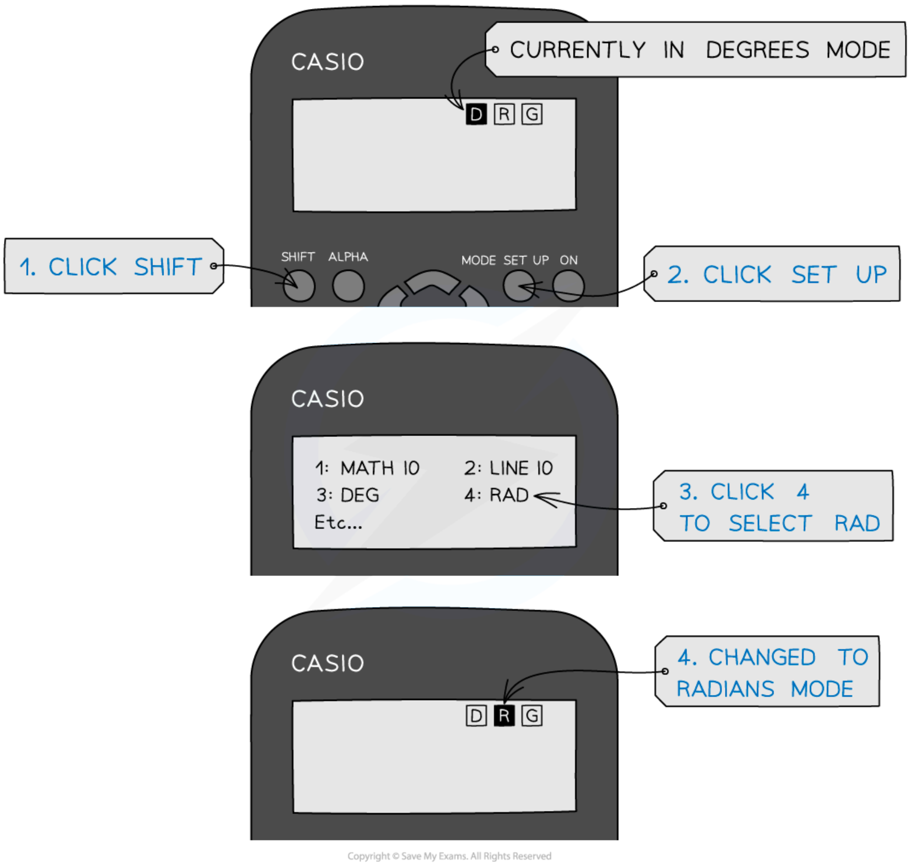
π's WILL CANCEL OUT

$\frac{\pi}{3} \text{ RAD} \times \frac{180}{\pi} = \frac{180^\circ}{3} = 60^\circ$

Examiner Tips and Tricks

- You will notice your calculator has a degree (Deg) and radians (Rad) mode
- This is shown by the "D" or "R" highlighted at the top of the screen
- Remember to make sure it's in the right mode when using **trigonometric** functions (sin, cos, tan) depending on whether the answer is required in **degrees** or **radians**

- It is extremely common for students to get the wrong answer (and lose marks) because their calculator is in the wrong mode when using trigonometric functions - make sure this doesn't happen to you!

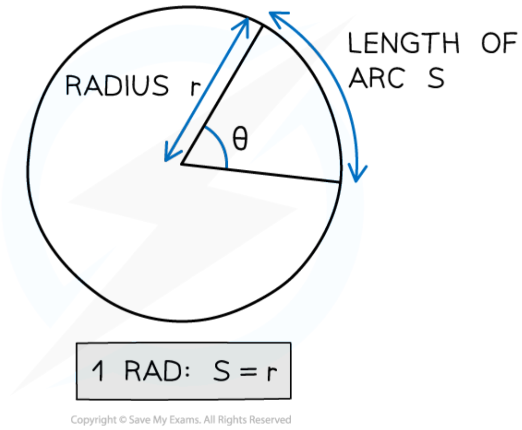


Angular Displacement

- The **angular displacement** $\Delta\theta$ is the ratio of:

$$\Delta\theta = \frac{\text{distance travelled around the circle}}{\text{radius of the circle}}$$

- Angular displacement describes the **change in angle**, in **radians**, of a body as it moves in a circle
  - This angle is measured with respect to the centre of orbit



When the angle is equal to one radian, the length of the arc (Δs) is equal to the radius (r) of the circle

Examiner Tips and Tricks

Since the equation for angular displacement gives the angle in **radians**, make sure you're comfortable with then converting to degrees if you need to for the question!

Your notes

Angular Velocity

- The **angular velocity** ω of a body in circular motion is defined as:

  The rate of change of angular displacement

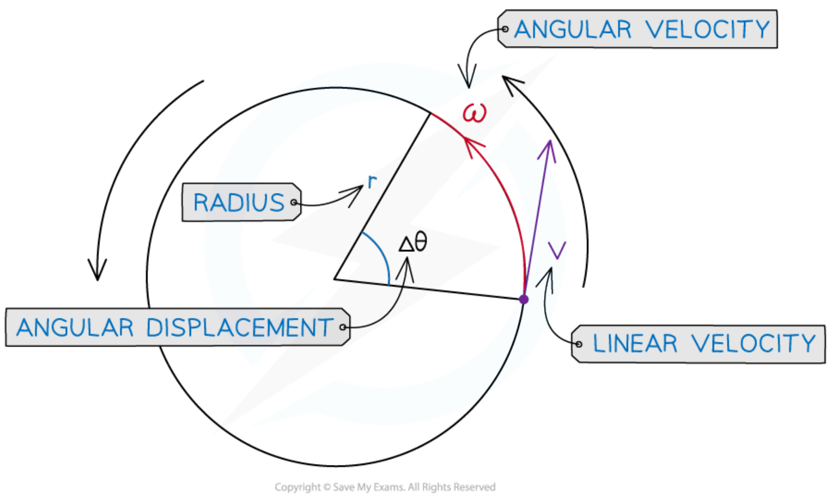
- In other words, angular velocity is the angle swept out by an object in circular motion, per second
- Angular velocity is a **vector quantity** and is measured in rad s^{-1}
  - Since it is a vector, it has a magnitude (angular **speed**) and direction
- Angular velocity is calculated using:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Where:
  - $\Delta\theta$ = change in angular displacement (radians)
  - Δt = time interval (s)
- It is related to linear speed, v by the equation

$$v = \omega r$$

- Where:
  - v = linear speed, v (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of orbit (m)



When an object is in uniform circular motion, velocity constantly changes direction, but the speed stays the same

Your notes

- Taking the angular displacement of a complete orbit or revolution as 2π radians, the angular velocity *ω* an be calculated using the equation:

$$\omega = \frac{v}{r} = 2\pi f = \frac{2\pi}{T}$$

- Where:
  - T = the time period (s)
  - f = frequency (Hz)
- This equation shows that:
  - The greater the rotation angle θ in a given amount of time T, the greater the angular velocity ω
  - An object travelling with the same linear velocity, but further from the centre of orbit (larger r) moves with a smaller angular velocity (smaller ω)

Worked Example

A bird flies in a horizontal circle with an angular speed of 5.25 rad s^{-1} of radius 650 m.

Calculate:

a) The linear speed of the bird

b) The angular frequency of the bird

Answer:

a) STEP 1 — LINEAR SPEED EQUATION

$v = r\omega$

STEP 2 — SUBSTITUTE IN VALUES

$V = 650 \times 5.25 = 3412.5 = 3410 \text{ ms}^{-1}$ (3 s.f.)

b) STEP 1 — ANGULAR SPEED WITH FREQUENCY EQUATION

$\omega = 2\pi f$

STEP 2 — REARRANGE FOR FREQUENCY

$f = \dfrac{\omega}{2\pi}$

STEP 3 — SUBSTITUTE IN VALUES

$f = \dfrac{5.25}{2\pi} = 0.83556... = 0.836 \text{ Hz}$ (3 s.f.)

Your notes

Examiner Tips and Tricks

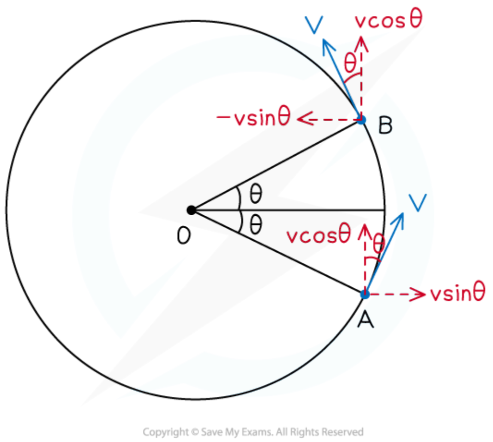
Try not to be confused by similar sounding terms like "angular velocity" and "angular speed". Just like in regular linear motion, you have linear velocity and linear speed: one is a scalar (speed) and the other is a vector (velocity).

In this worked example, the equation $v = r\omega$ is used to calculate the linear speed. This is fine, because v in this context is just the **magnitude** of the linear velocity (and similarly, ω is the magnitude of the angular velocity).

Finally, you may sometimes come across ω being labelled as 'angular frequency', because of its relationship to linear frequency f as given by the alternative equation $\omega = 2\pi f$. Remember, the units of ω are rad s^{-1}, whereas the units of f are Hz.

Deriving Equations for Centripetal Acceleration

- An object moving in **uniform circular motion** travels with a constant **angular velocity** and **angular speed**
- However, its **direction** is always changing
  - Therefore, its **linear velocity** changes, so it must be **accelerating**
  - This is called a **centripetal acceleration**
- An object in circular motion is thus always accelerating
- This acceleration is called 'centripetal' because it is directed toward the **centre of orbit**
- To derive an equation for the magnitude of centripetal acceleration, consider an object in uniform circular motion between point A and B on a circle, as shown below:



An object in uniform circular motion is accelerating toward the centre of orbit, O. Between A and B, the horizontal component of motion changes from v sinθ to −v sinθ

- At A and B, by resolving the horizontal and vertical components of linear velocity *v*, it can be seen that:
  - Initial vertical component of *v* = final vertical component of *v* which is *v* cos*θ*
  - Initial horizontal component of *v* is *v* sin*θ*
  - Final horizontal component of *v* is –*v* sin*θ*
- This means the acceleration of the object is only horizontal, given by:

$$a = \frac{\Delta v}{\Delta t} = \frac{(-v\sin\theta) - (v\sin\theta)}{t} = \frac{-2v\sin\theta}{t}$$

- Recalling the equations for angular velocity $\omega = \theta / t$ and $v = r\omega$, then:

$$t = \frac{\theta}{\omega} = \frac{\theta}{\left(\frac{v}{r}\right)} = \frac{r\theta}{v}$$

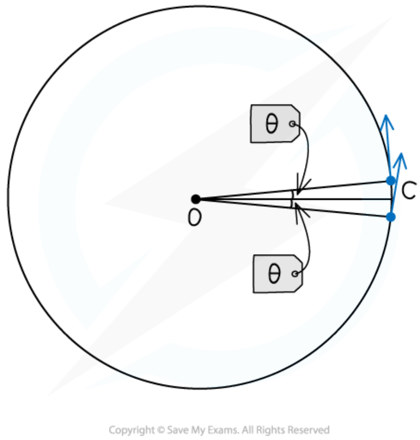
- The object's angular displacement is actually 2θ, therefore, the time t is given by

$$t = \frac{2r\theta}{v}$$

- Therefore, substituting this into the equation for acceleration gives:

$$a = \frac{-2v\sin\theta}{\left(\frac{2r\theta}{v}\right)} = \frac{-v^2\sin\theta}{r\theta}$$

- This equation is the acceleration of the object between points A and B
    - To find the **instantaneous acceleration** at an exact point on the circle, say point C, reduce the size of the angular displacement θ so it becomes infinitesimally small
    - This is shown in the image below:



By taking the limit of angular displacement as zero, we can derive an equation for the instantaneous centripetal acceleration of the object at point C

- In the limit $\theta \rightarrow 0$ radians
    - The value of $\sin\theta$ is approximately equal to θ
    - Therefore, $\frac{\sin\,\theta}{\theta} \approx 1$ (for very small angles)
    - This is known as the **small angle approximation**
- Therefore, the **instantaneous acceleration** is the centripetal acceleration:

Your notes

$$a = -\frac{v^2}{r} = -r\omega^2$$

- Where:
  - a = centripetal acceleration (m s^{-2})
  - v = linear velocity (m s^{-1})
  - r = radius of orbit (m)
  - ω = angular velocity (rad s^{-1})
- The negative sign indicates that the centripetal acceleration is directed toward the **centre of orbit**

Examiner Tips and Tricks

This seems like a complicated derivation, but there is no maths in there that you haven't been introduced to already. It is important you know how to use the **vector diagrams** to reach the final equations for angular accelerations, understanding every step along the way. Try and do it without the notes to help memorise and see how far you get!

Using Equations for Centripetal Acceleration

- Centripetal acceleration is defined as:

  The acceleration of an object towards the centre of a circle when an object is in motion (rotating) around a circle at a constant speed

- Its magnitude is calculated using the radius r and linear speed v:

$$a = \frac{v^2}{r}$$

- Using the equation relating angular speed ω and linear speed v:

$$\mathbf{v = r\omega}$$

- These equations can be combined to give another form of the centripetal acceleration equation:

$$a = \frac{(r\omega)^2}{r}$$

$$\mathbf{a = r\omega^2}$$

Your notes

- This equation shows that centripetal acceleration is equal to the radius times the square of the angular speed
- Alternatively, rearrange for *r:*

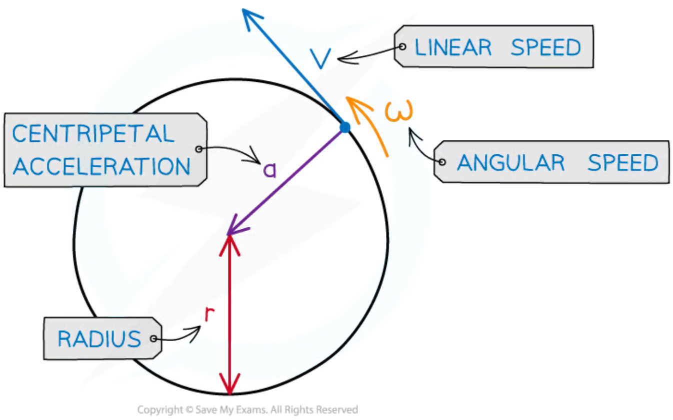
$$r = \frac{v}{\omega}$$

- This equation can be combined with the first one to give us another form of the centripetal acceleration equation:

$$a = \frac{v^2}{\left(\frac{v}{\omega}\right)}$$

$$a = v\omega$$

- This equation shows how the centripetal acceleration relates to the linear speed and the angular speed



Centripetal acceleration is always directed toward the centre of the circle, and is perpendicular to the object's velocity

- Where:
  - a = centripetal acceleration (m s^{-2})
  - v = linear speed (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)

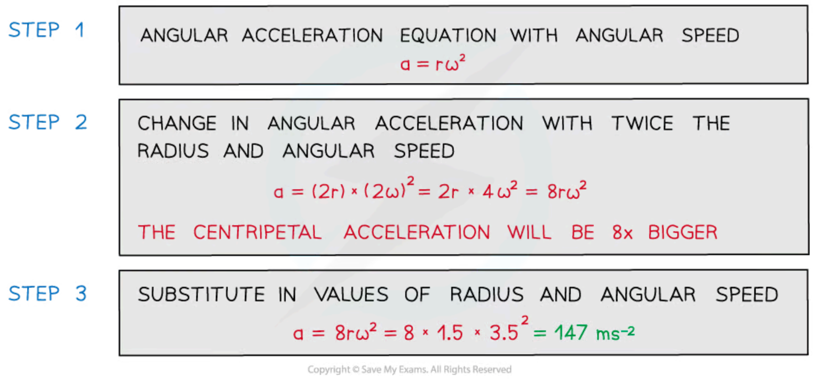
Your notes

Worked Example

A ball tied to a string is rotating in a horizontal circle with a radius of 1.5 m and an angular speed of 3.5 rad s^{-1}.

Calculate its centripetal acceleration if the radius was twice as large and angular speed was twice as fast.

Answer:



Examiner Tips and Tricks

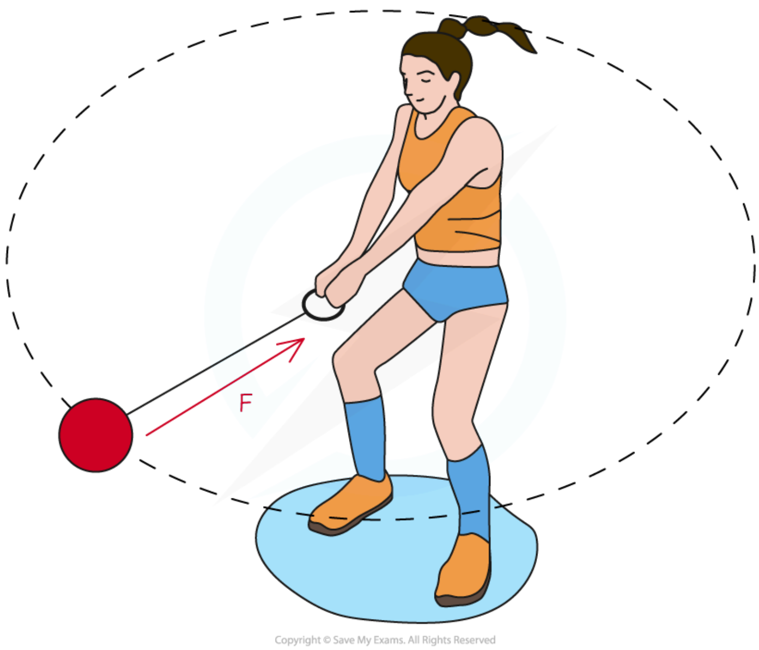
Make sure you understand both the derivation and how to use the equation for centripetal acceleration. The most crucial step is to remember the **small angle approximation,** that sin θ is approximately equal to θ when the angle is very very small. Try this in your calculator (in radians!) and see for yourself!

Your notes

Maintaining Circular Motion

- An object moving in a circle is not in equilibrium, it is constantly changing direction
  - Therefore, in order to produce circular motion, an object requires a **resultant force** to act on it
  - This resultant force is known as the **centripetal force** and is what keeps an object moving in a circle
- The centripetal force *F* is defined as:

  The resultant force towards the centre of the circle required to keep a body in uniform circular motion. It is always directed towards the centre of the body's rotation.



The tension in the string provides the centripetal force F to keep the hammer in circular orbit

- **Note:** centripetal force and centripetal acceleration act in the **same direction**
  - This is due to Newton's Second Law
- The centripetal force is **not** a separate force of its own
  - It can be any type of force, depending on the situation, which keeps an object moving in a circular path

Examples of centripetal force

Situation	Centripetal force
Car travelling around a roundabout	Friction between car tyres and the road
Ball attached to a rope moving in a circle	Tension in the rope
Earth orbiting the Sun	Gravitational force

Examiner Tips and Tricks

Make sure you are able to give examples of centripetal forces, understanding that many types of familiar forces (e.g., gravity, electric) can act as centripetal forces.

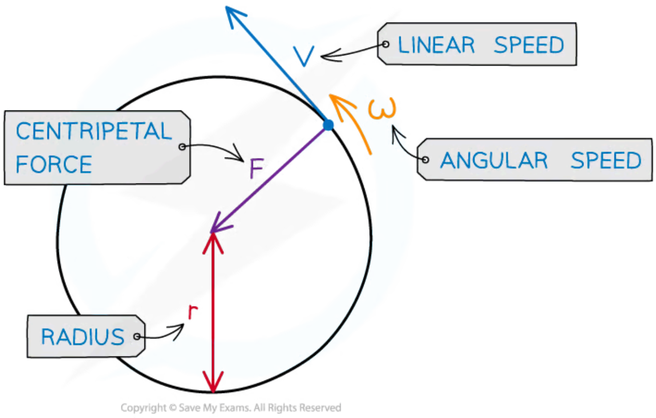
A classic example that often comes up in your magnetic fields topic is the magnetic force on a charged particle, which is always centripetal. This is because the force acts at 90° to the charged particle's velocity, causing it to move in a circle.

Centripetal Force

- Centripetal force can be calculated using any of the following equations:

$$F = \frac{mv^2}{r} = mr\omega^2 = mv\omega$$

- Where:
  - F = centripetal force (N)
  - v = linear velocity (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)



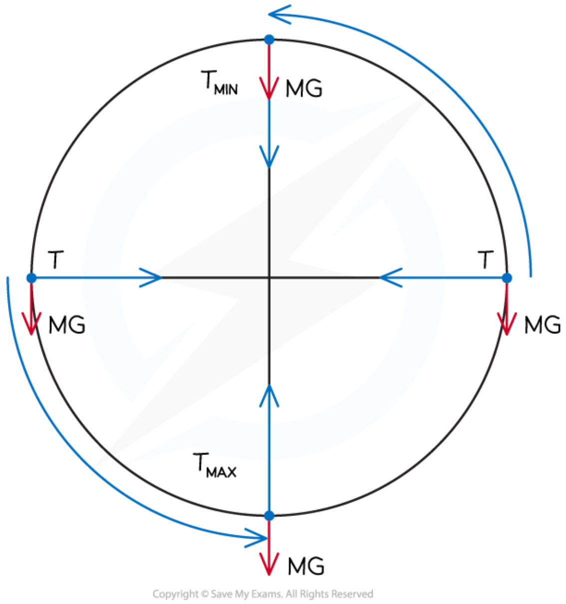
Centripetal force is always perpendicular to the direction of travel

- The centripetal force is the **resultant** force on the object moving in a circle
  - This is particularly important if there are multiple forces on the object, such as weight

Vertical Circular Motion

- An example of vertical circular motion is swinging a ball on a string in a vertical circle
- The forces acting on the ball are:
  - The **tension** in the string
  - The **weight** of the ball downwards
- As the ball moves around the circle, the **direction** of the tension will change continuously

- The **magnitude** of the tension will also vary continuously, reaching a **maximum** value at the **bottom** and a **minimum** value at the **top**
  - This is because the direction of the weight of the ball never changes, so the resultant force will vary depending on the position of the ball in the circle



- At the bottom of the circle, the tension must overcome the weight, this can be written as:

$$T_{max} = \frac{mv^2}{r} + mg$$

- As a result, the acceleration, and hence, the **speed** of the ball will be **slower** at the top
- At the top of the circle, the tension and weight act in the same direction, this can be written as:

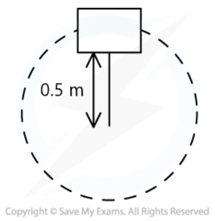
$$T_{min} = \frac{mv^2}{r} - mg$$

- As a result, the acceleration, and hence, the **speed** of the ball will be **faster** at the bottom

Worked Example

A bucket of mass 8.0 kg is filled with water and is attached to a string of length 0.5 m.

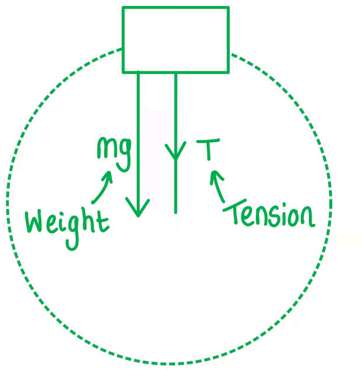
Your notes



What is the minimum speed the bucket must have at the top of the circle so no water spills out?

Answer:

Step 1: Draw the forces on the bucket at the top



Step 2: Write an expression for the centripetal force

- The weight of the bucket = mg
- At the top of the circular path, the weight and tension act in the same direction, so the centripetal force is

$$\frac{mv^2}{r} = T + mg$$

- The minimum speed *v* is when the string is taut but not stretched, so the tension here is zero (*T* = 0)

$$\frac{mv^2}{r} = mg$$

Step 3: Rearrange for velocity *v* and calculate

- *m* cancels from both sides

$$v = \sqrt{gr}$$

$$v = \sqrt{9.81 \times 0.5} = 2.2 \text{ m s}^{-1}$$

Your notes