Edexcel International A Level (IAL) Physics

Your notes

Capacitance

Contents

Capacitance

- Capacitors are electrical devices used to store energy in electronic circuits, commonly for a backup release of energy if the power fails
- Capacitors do this by storing electric **charge**, which creates a build up of electric **potential energy**
- They are made in the form of two conductive **metal plates** connected to a voltage supply (parallel plate capacitor)
  - There is commonly a **dielectric** in between the plates, to ensure charge does not flow across them
- The capacitor circuit symbol is:

The capacitor circuit symbol is two parallel lines

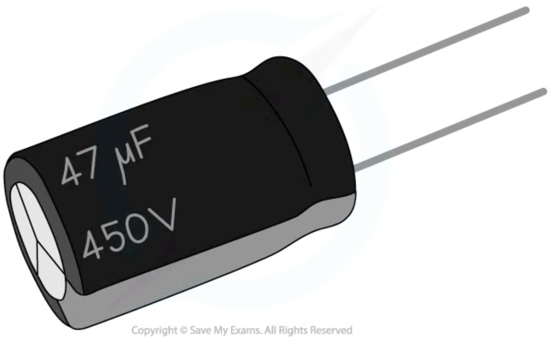
- Capacitors are marked with a value of their **capacitance**
- Capacitance is defined as:

  The charge stored per unit potential difference (between the plates)

- The greater the **capacitance**, the greater the **charge stored** in the capacitor
- The capacitance of a capacitor is defined by the equation:

$$C = \frac{Q}{V}$$

- Where:
  - C = capacitance (F)
  - Q = charge stored (C)
  - V = potential difference across the capacitor plates (V)



A capacitor used in small circuits

- Capacitance is measured in the unit **Farad (F)**
  - In practice, 1 F is a very large unit
  - Often it will be quoted in the order of micro Farads (μF), nanofarads (nF) or picofarads (pF)
- If the capacitor is made of parallel plates, *Q* is the charge on the plates and *V* is the potential difference across the capacitor
  - The charge Q is **not** the charge of the capacitor itself, it is the charge stored **on** the plates
- This capacitance equation shows that an object's capacitance is the **ratio of the charge stored by the capacitor to the potential difference between the plates**

Worked Example

A parallel plate capacitor has a capacitance of 1 nF and is connected to a voltage supply of 0.3 kV.

Calculate the charge on the plates.

Answer:

Step 1: Write down the known quantities

- Capacitance, C = 1 nF = 1×10^{-9} F
- Potential difference, *V* = 0.3 kV = 0.3×10^3 V

Step 2: Write out the equation for capacitance

$$C = \frac{Q}{V}$$

Step 3: Rearrange for charge Q

$$Q = CV$$

Your notes

Step 4: Substitute in values

$$Q = (1 \times 10^{-9}) \times (0.3 \times 10^{3}) = 3 \times 10^{-7}\ C = 300\ nC$$

Your notes

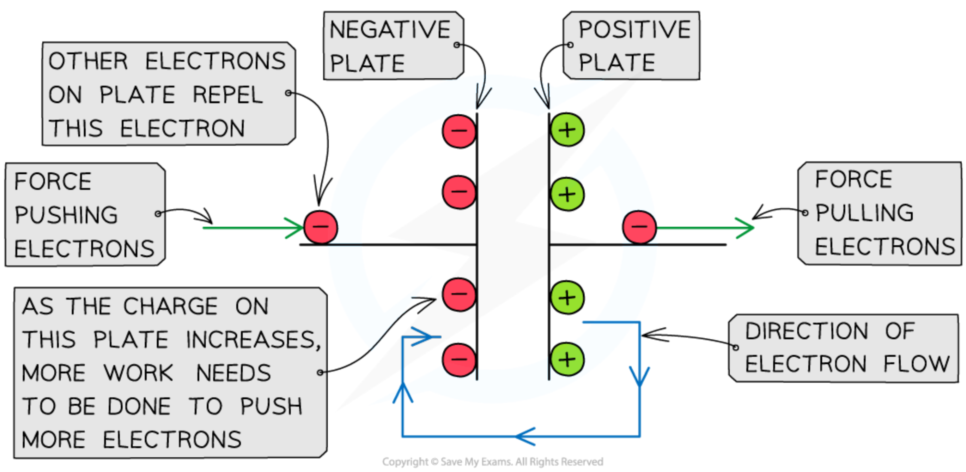
Examiner Tips and Tricks

The 'charge stored' by a capacitor refers to the magnitude of the charge stored **on** each plate in a parallel plate capacitor or **on** the surface of a spherical conductor. The letter 'C' is used both as the symbol for capacitance as well as the unit of charge (coulombs). Take care not to confuse the two!

Energy Stored by a Capacitor

- When charging a capacitor, the power supply 'pushes' electrons to one of the metal plates
  - It therefore does **work** on the electrons and **electrical energy** becomes stored on the plates
- The power supply 'pulls' electrons off of the other metal plate, attracting them to the positive terminal
  - This leaves one side positively charged, while the other side becomes negatively charged
  - Hence, in this way, charge is 'stored' by the capacitor
- Gradually, this stored charge builds up
  - Adding more electrons to the negative plate at first is relatively easy since there is little repulsion
- As the charge of the negative plate increases, i.e., becomes more negatively charged, the force of repulsion between the electrons on the plate and the new electrons being pushed onto it increases
- This means a greater amount of work must be done to increase the charge on the negative plate or in other words:

  The potential difference across the capacitor increases as the amount of charge increases



As the charge on the negative plate builds up, more work needs to be done to add more charge

Alternative Equations for Energy Stored

- The energy stored by a capacitor is given by:

$$W = \frac{1}{2} QV$$

- Substituting the charge Q with the **capacitance** equation Q = CV, the energy stored can also be calculated by the following equation:

$$W = \frac{1}{2} CV^2$$

- By substituting the potential difference *V*, the energy stored can also be defined in terms of just the charge stored Q and the capacitance, C:

$$W = \frac{Q^2}{2C}$$

Your notes

Worked Example

Calculate the change in the energy stored in a capacitor of capacitance 1500 μF when the potential difference across the capacitor changes from 10 V to 30 V.

Answer:

Step 1: Write down the equation for energy stored in terms of capacitance C and p.d V

$$W = \frac{1}{2} CV^2$$

Step 2: The change in energy stored is proportional to the change in p.d

$$\Delta W = \frac{1}{2} CV_2^{\,2} - \frac{1}{2} CV_1^{\,2} = \frac{1}{2} C\left(V_2^{\,2} - V_1^{\,2}\right)$$

Step 3: Substitute in values

$$\Delta W = \frac{1}{2} \times (1500 \times 10^{-6}) \times (30^2 - 10^2)$$

$$\Delta W = 0.6 \text{ J}$$

Examiner Tips and Tricks

Energy stored or work done are used interchangeably (and sometimes written as *E* or *W* as shown above). You should be comfortable linking the two equivalent ideas -

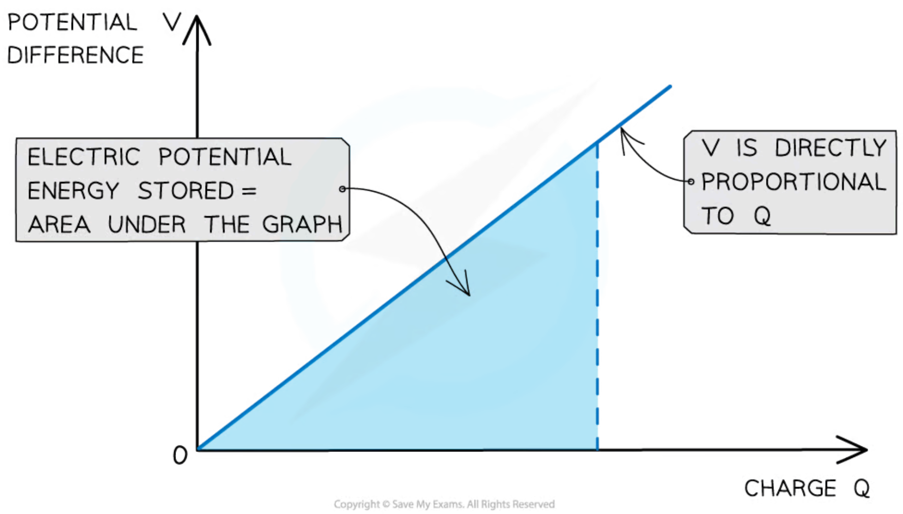
the energy stored *in* the capacitor is equal to the work done *on* it, by the power supply which charges it. Make sure you can apply each of the three equations given above!

Your notes

Area Under a Potential Difference–Charge Graph

- The charge stored Q on the capacitor is given by the equation Q = C*V*
    - Therefore, the charge stored Q is **directly proportional** to the potential difference across the plates *V*
- The graph of charge against potential difference is therefore a straight line graph through the origin
- The gradient of the graph represents $\frac{1}{capacitance\ C}$, which is a constant
- The electrical (potential) energy stored in the capacitor can be determined from the **area under the potential–charge graph** which is equal to the **area** of a right-angled triangle:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$



The area under a potential difference–charge graph represents the energy stored by a capacitor

- Therefore the work done, or **energy stored *W*** in a capacitor is defined by the equation:

$$W = \frac{1}{2}QV$$

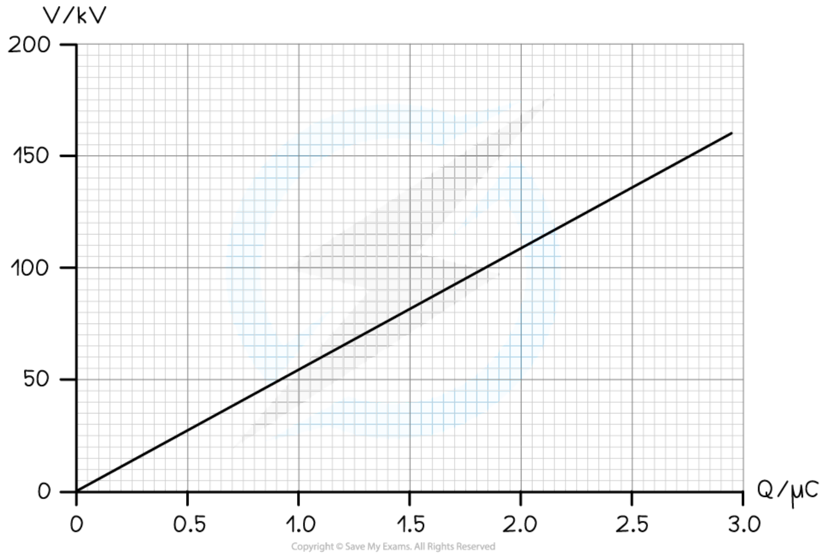
- Where:
    - *W* = energy stored (J)

- Q = charge stored (C)
- V = potential difference across the plates (V)

Your notes

Worked Example

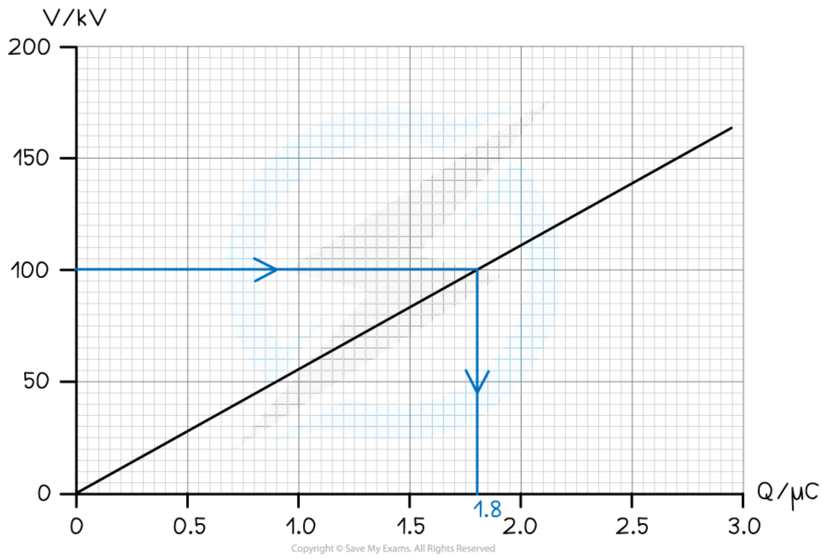
The variation of the potential V of a charged isolated metal sphere with surface charge Q is shown on the graph below.



Using the graph, determine the electric potential energy stored on the sphere when charged to a potential of 100 kV.

Answer:

Step 1: Determine the charge on the sphere at the potential of 100 kV



- From the graph, the charge on the sphere at 100 kV is **1.8 μC**

Step 2: Calculate the electric potential energy stored

- The electric potential energy stored is the area under the graph at 100 kV
- The area is equal to a right-angled triangle, so, can be calculated with the equation:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

- Substituting in the values gives:

$$\text{Area} = \frac{1}{2} \times 1.8\ \mu\text{C} \times 100\ \text{kV}$$

$$\text{E.P.E} = \frac{1}{2} \times 1.8 \times 10^{-6} \times 100 \times 10^{3} = \mathbf{0.09\ J}$$

Examiner Tips and Tricks

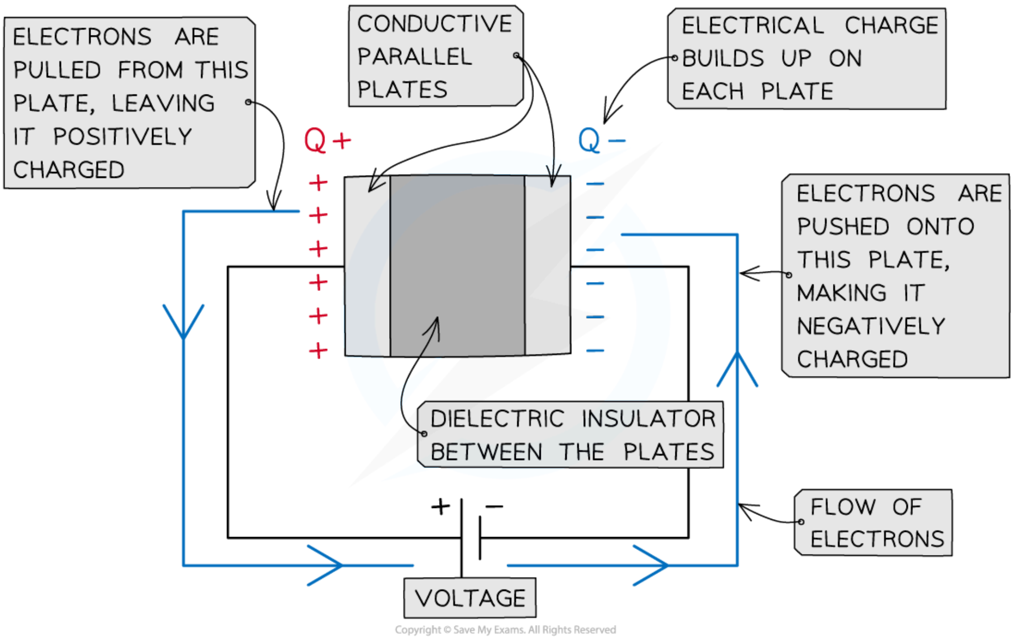
Remember to always check the **units** of the charge-potential difference graphs. The charges can often be in μC or the potential difference in kV! The units must be in C and V to get a work done in J.

Your notes

Charge & Discharge Curves

Charging Curves

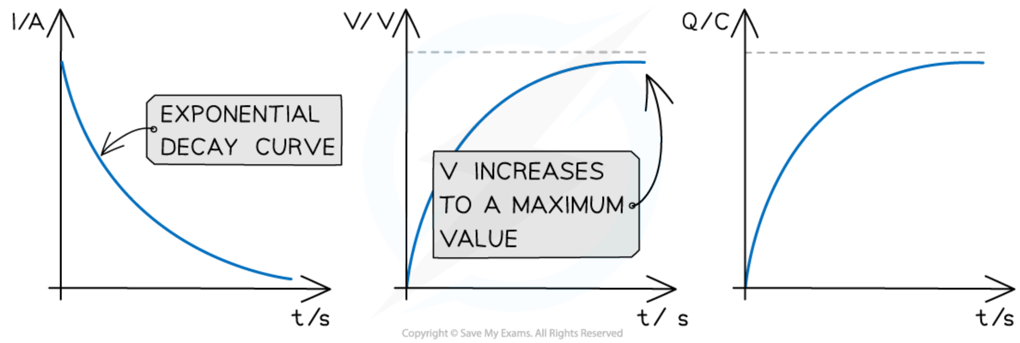
- Capacitors are charged by a **power supply** (e.g. a battery)
- When charging, electrons are 'pulled' from the plate connected to the positive terminal of the power supply
  - Hence the plate nearest the positive terminal is **positively charged**
- Oppositely, electrons are 'pushed' onto the plate connected to the negative terminal
  - Hence the plate nearest the negative terminal is **negatively charged**
- As the negative charge builds up, fewer electrons are pushed onto the plate due to electrostatic repulsion from the electrons already on the plate
- When no more electrons can be pushed onto the negative plate, the charging stops



A parallel plate capacitor is made up of two conductive plates with opposite charges building up on each plate

- At the start of charging, the current is large and gradually falls to zero as the electrons stop flowing through the circuit
  - The current decreases **exponentially**
  - This means the rate at which the charge decreases is proportional to the amount of charge it has left

- Since an equal but opposite charge builds up on each plate, the potential difference between the plates slowly increases until it is the same as that of the power supply
    - Therefore, the charge stored on the capacitor plates increases until the potential difference across the plates matches that of the power supply

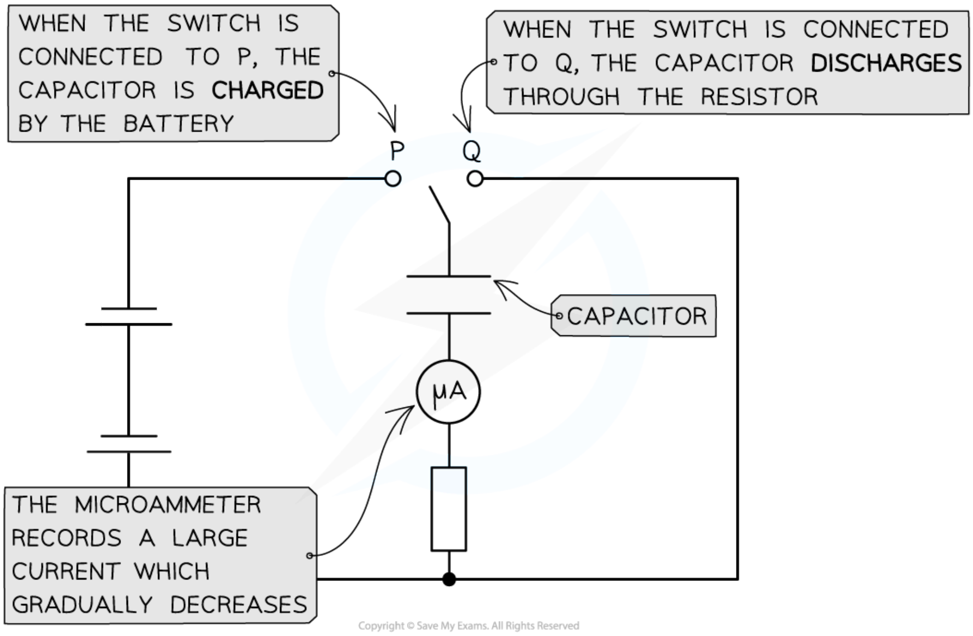


Graphs of variation of current, p.d and charge with time for a capacitor charging through a battery

- **The key features of the charging graphs are:**
    - The shapes of the p.d. and charge against time graphs are identical
    - The current against time graph is an **exponential decay** curve
    - The initial value of the current starts on the y axis and decreases exponentially
    - The initial value of the p.d and charge starts at 0 up to a maximum value

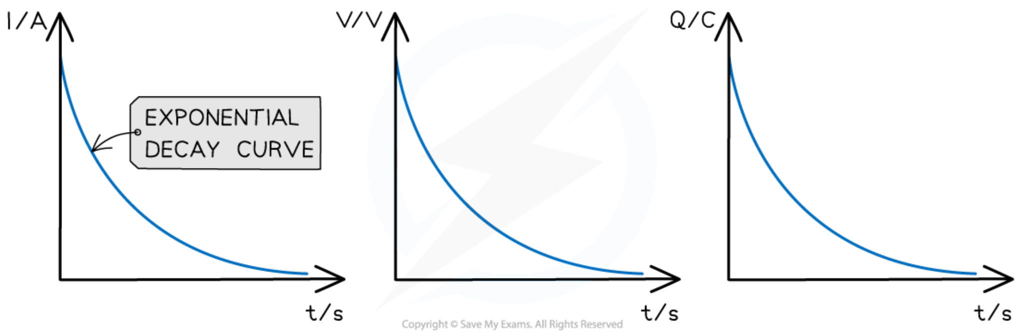
Discharging Curves

- Capacitors are **discharged** through a resistor with **no** power supply present
- The electrons now flow back from the negative plate to the positive terminal of the power supply until there is no potential difference across the capacitor plates
- Charging and discharging is commonly achieved by moving a switch that connects the capacitor between a power supply and a resistor



The capacitor charges when connected to terminal P and discharges when connected to terminal Q

- At the start of discharge, the current is **large** (but in the opposite direction to when it was charging) and gradually falls to zero
- As a capacitor discharges, the current, p.d and charge all decrease **exponentially**
  - This means the rate at which the current, p.d or charge decreases is proportional to the amount of current, p.d or charge it has left
- The graphs of the variation with time of current, p.d and charge are all identical and follow a pattern of **exponential decay**



Graphs of variation of current, p.d and charge with time for a capacitor discharging through a resistor

- **The key features of the discharge graphs are:**
  - The shape of the current, p.d. and charge against time graphs are identical
  - Each graph shows exponential decay curves with decreasing gradient

- The initial values (typically called I_0, V_0 and Q_0 respectively) start on the y axis and decrease exponentially

- The rate at which a capacitor discharges depends on the **resistance** of the circuit
  - If the resistance is **high**, the current will decrease **more slowly** and charge will flow from the capacitor plates more slowly, meaning the capacitor will take **longer** to discharge
  - If the resistance is **low**, the current will decrease **quickly** and charge will flow from the capacitor plates quickly, meaning the capacitor will discharge **faster**

Your notes

Examiner Tips and Tricks

Make sure you're comfortable with sketching and interpreting charging and discharging graphs, as these are common exam questions. A quick summary to help you remember:

- Discharging curves are all identical
- **C**urrent decreases for the **C**harging curve (but increases for potential difference and charge stored!)

The Time Constant

- The time constant of a capacitor discharging through a **resistor** is a measure of how long it takes for the capacitor to discharge
- The definition of the time constant for a **discharging** capacitor is:

  The time taken for the charge, current or potential difference of a discharging capacitor to decrease to 37% of its original value

- Alternatively, for a **charging** capacitor:

  The time taken for the charge or potential difference of a charging capacitor to rise to 63% of its maximum value

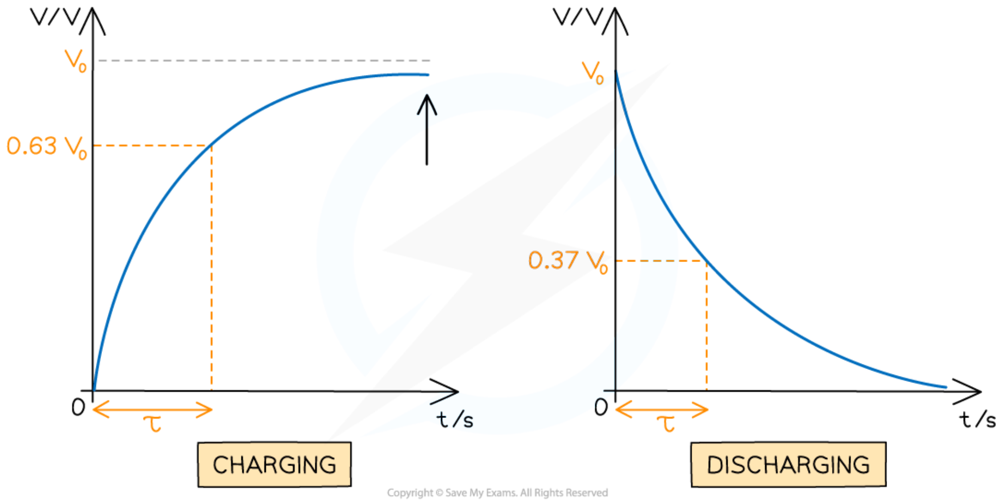
- 37% is 0.37 or 1 / e (where e is the exponential function) multiplied by the original value (I_0, Q_0 or V_0)
- This is represented by the Greek letter tau, τ, and measured in units of **seconds** (s)
- The time constant provides an easy way to compare the rate of change of similar quantities eg. charge, current and p.d.
- It is defined by the equation:

$$\tau = RC$$

- Where:
- τ = time constant (s)

- R = resistance of the resistor (Ω)
- C = capacitance of the capacitor (F)

- For example, to find the time constant for a **discharging** capacitor:
- Calculate $0.37V_0$, where V_0 is the **initial** potential difference across it
  - Determine the corresponding time taken for the potential difference to decrease to that value
- To find the time constant for a **charging** capacitor:
  - Calculate $0.63V_0$, where V_0 is the **maximum** potential difference across it
  - Determine the corresponding time taken for the potential difference to rise to that value



The time constant shown on a charging and discharging capacitor

Worked Example

A capacitor of 7 nF is discharged through a resistor of resistance R. The time constant of the discharge is 5.6×10^{-3} s.

Calculate the value of R.

Answer:

Your notes

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Step 1: Write the known quantities

Capacitance, $C = 7 \text{ nF} = 7 \times 10^{-9}$ F

Time constant, $\tau = 5.6 \times 10^{-3}$ s

Step 2: Write down the time constant equation

$$\tau = RC$$

Step 3: Rearrange for R

$$R = \frac{\tau}{C}$$

Step 4: Substitute in values and calculate

$$R = \frac{5.6 \times 10^{-3}}{7 \times 10^{-9}} = \mathbf{800\ k\Omega}$$

Examiner Tips and Tricks

Remember to check the context of an exam question, i.e., whether the capacitor is **charging** or **discharging**. The definition of the time constant depends on it!

- For a **charging** capacitor, the time constant refers to the time taken to reach 63% of its **maximum** potential difference or charge stored
- For a **discharging** capacitor, the time constant refers to the time take to discharge to 37% of its **initial** potential difference or charge stored

Required Practical: Charging & Discharging Capacitors

Aim of the Experiment

- The overall aim of this experiment is to calculate the capacitance of a capacitor. This is just one example of how this required practical might be carried out

Variables

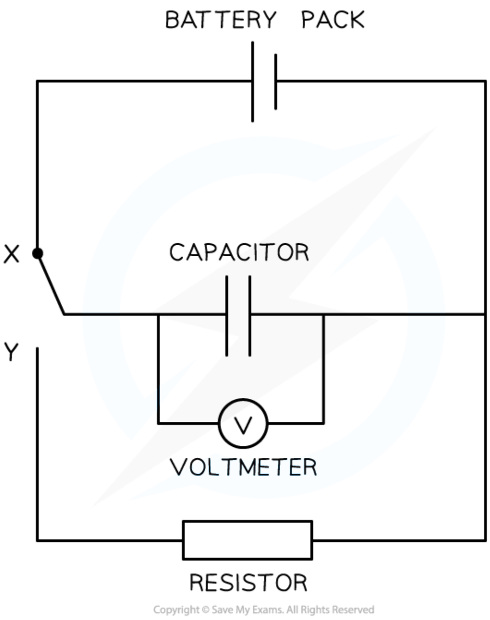
- Independent variable = time, t
- Dependent variable= potential difference, V
- Control variables:
  - Resistance of the resistor
  - Current in the circuit

Equipment List

Apparatus	Purpose
Switch	To switch between the charging and discharging circuit
Capacitor	To measure the capacitance
10 kΩ Resistor	To discharge the capacitor
Battery pack (power supply)	To provide the potential difference across the capacitor
Voltmeter	To measure the potential difference across the capacitor
Stopwatch	To measure the time taken for the capacitor to discharge

- **Resolution** of measuring equipment:
  - Voltmeter = 0.1 V
  - Stopwatch = 0.01 s

Method



1. Set up the apparatus like the circuit above, making sure the switch is not connected to **X** or **Y** (no current should be flowing through)
2. Set the battery pack to a potential difference of 10 V and use a 10 kΩ resistor. The capacitor should initially be fully discharged
3. Charge the capacitor fully by placing the switch at point **X**. The voltmeter reading should read the same voltage as the battery (10 V)
4. Move the switch to point **Y**
5. Record the voltage reading every 10 s down to a value of 0 V. A total of 8–10 readings should be taken

- An example table might look like this:

FROM STOPWATCH

FROM VOLTMETER

TIME t / s	POTENTIAL DIFFERENCE / V
0.00	
10.00	
20.00	
30.00	
40.00	
50.00	
60.00	
70.00	
80.00	

Analysing the Results

Your notes

- The potential difference (p.d) across the capacitance is defined by the equation:

$$V = V_0 e^{-\frac{t}{RC}}$$

- Where:
  - V = p.d across the capacitor (V)
  - V_0 = initial p.d across the capacitor (V)
  - t = time (s)
  - e = exponential function
  - R = resistance of the resistor (Ω)
  - C = capacitance of the capacitor (F)
- Rearranging this equation for ln(V) by taking the natural log (ln) of both sides:

$$\ln\left(\frac{V}{V_0}\right) = -\frac{t}{RC}$$

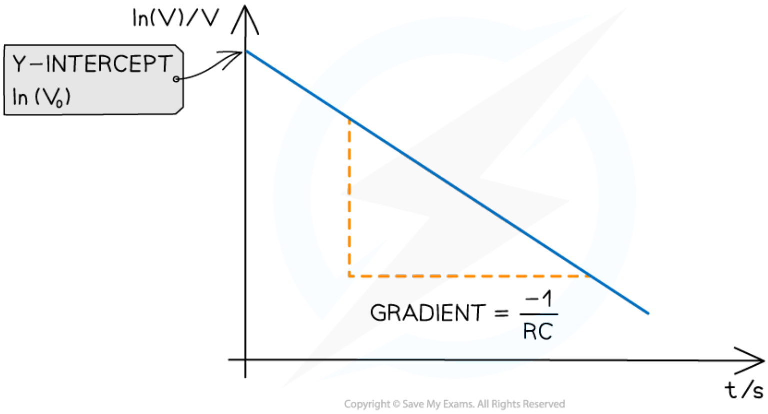
$$\ln(V) - \ln(V_0) = -\frac{t}{RC}$$

$$\ln(V) = -\frac{t}{RC} + \ln(V_0)$$

- Comparing this to the equation of a straight line: y = mx + c
  - y = ln(V)
  - x = t
  - gradient = -1/RC
  - c = ln(V_0)

1. Plot a graph of ln(V) against t and draw a line of best fit
2. Calculate the gradient (this should be negative)
3. The capacitance of the capacitor is equal to:

$$C = -\frac{1}{R \times \text{gradient}}$$



Your notes

Evaluating the Experiment

Systematic Errors:

- If a digital voltmeter is used, wait until the reading is settled on a value if it is switching between two
- If an analogue voltmeter is used, reduce parallax error by reading the p.d at eye level to the meter
- Make sure the voltmeter starts at zero to avoid a zero error

Random Errors:

- Use a resistor with a large resistance so the capacitor discharges slowly enough for the time to be taken accurately at p.d intervals
- Using a datalogger will provide more accurate results for the p.d at a certain time. This will reduce the error in the speed of the reflex needed to stop the stopwatch at a certain p.d
- The experiment could be repeated by measuring the time for the capacitor to charge instead

Safety Considerations

- Keep water or any fluids away from the electrical equipment
- Make sure no wires or connections are damaged and contain appropriate fuses to avoid a short circuit or a fire
- Using a resistor with too low a resistance will not only mean the capacitor discharges too quickly but also that the wires will become very hot due to the high current
- Capacitors can still retain charge after power is removed which could cause an electric shock. These should be fully discharged and removed after a few minutes

Worked Example

Your notes

A student investigates the relationship between the potential difference and the time it takes to discharge a capacitor. They obtain the following results:

Time t/s	Potential Difference / V
0.00	10.0
10.00	8.95
20.00	7.58
30.00	6.13
40.00	4.90
50.00	4.12
60.00	3.00
70.00	2.03

The capacitor is labelled with a capacitance of 4200 μF. Calculate:

(i) The value of the capacitance of the capacitor discharged.

(ii) The relative percentage error of the value obtained from the graph and this true value of the capacitance.

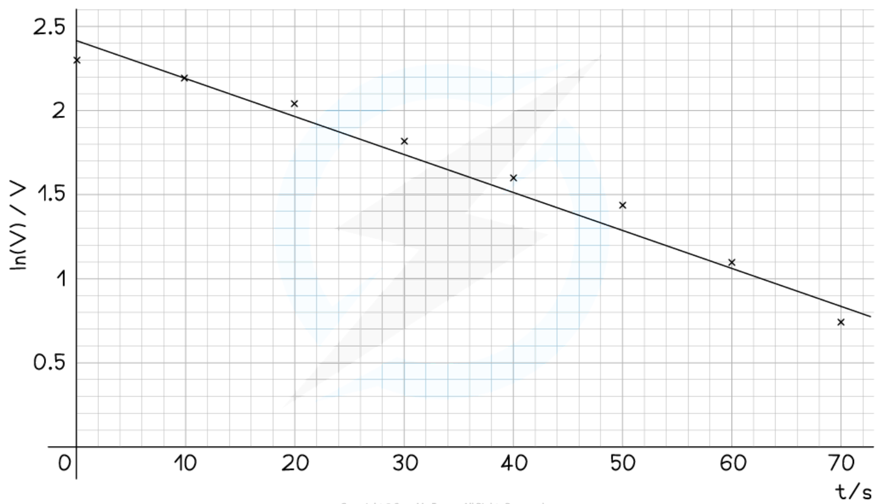
Answer:

Step 1: Complete the table

- Add an extra column ln(V) and calculate this for each p.d

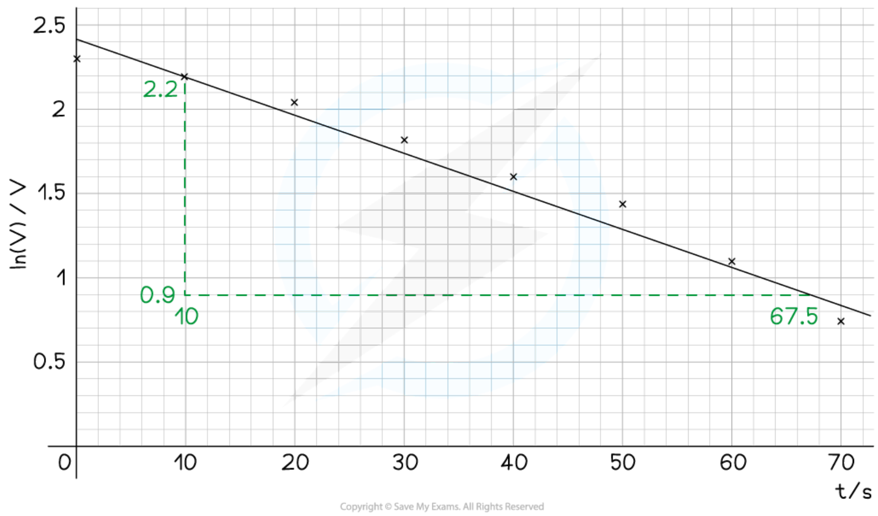
Time t/s	Potential Difference / V	ln(V) / V
0.00	10.0	2.30
10.00	8.95	2.19
20.00	7.58	2.03
30.00	6.13	1.81
40.00	4.90	1.59
50.00	4.12	1.42
60.00	3.00	1.10
70.00	2.03	0.71

Step 2: Plot the graph of ln(*V*) against average time t



- Make sure the axes are properly labelled and the line of best fit is drawn with a ruler

Step 3: Calculate the gradient of the graph



- The gradient is calculated by:

$$\text{gradient} = \frac{0.9 - 2.2}{67.5 - 10} = -0.0226$$

Step 4: Calculate the capacitance, C

$$C = -\frac{1}{R \times \text{gradient}}$$

$$C = -\frac{1}{10\ 000 \times -0.0226} = 4.42 \times 10^{-3}\ \text{F} = 4400\ \mu\text{F}$$

Step 5: Calculate the relative percentage error of the value obtained

$$\text{Relative percentage error} = \frac{\text{Measured value} - \text{Accepted value}}{\text{Accepted value}} \times 100\ \%$$

$$\text{Relative percentage error} = \frac{\left(4.42 \times 10^{-3}\right) - \left(4200 \times 10^{-6}\right)}{4200 \times 10^{-6}} \times 100\ \% = 5.2\ \%$$

Your notes

Your notes

Exponential Discharge in a Capacitor

The Discharge Equation

- When a capacitor discharges through a resistor, the charge stored on it decreases **exponentially**
- The amount of charge remaining on the capacitor Q after some elapsed time *t* is governed by the **exponential decay equation**:

$$Q = Q_0 e^{-(t/RC)}$$

- Where:
  - Q = charge remaining (C)
  - Q_0 = initial charge stored (C)
  - e = exponential function
  - *t* = elapsed time (s)
  - *R* = circuit resistance (Ω)
  - *C* = capacitance (F)

Discharge Equation for Potential Difference

- The exponential decay equation for **charge** can be used to derive a decay equation for potential difference
- Recall the equation for charge *Q* = *CV*
  - It also follows that the initial charge $Q_0 = CV_0$ (where V_0 is the initial potential difference)
- Therefore, substituting *CV* for Q into the original exponential decay equation gives:

$$CV = CV_0 e^{-(t/RC)}$$

- Cancelling C from both sides gives the exponential decay equation for **potential difference *V***:

$$V = V_0 e^{-(t/RC)}$$

- Where:
  - *V* = potential difference after some time *t* (V)
  - V_0 = initial potential difference (V)
  - *t* = elapsed time (s)

- R = resistance (Ω)
- C = capacitance (F)

- This equation shows that the potential difference also decreases **exponentially,** from some initial value V_0

Your notes

Discharge Equation for Current

- The exponential decay equation for **potential difference** can be used to derive a decay equation for current
- Recall Ohm's law ***V = IR***
  - It follows that the initial potential difference $V_0 = I_0R$ (where I_0 is the initial current)
- Therefore, substituting IR for V into the decay equation for potential difference gives:

$$IR = I_0 R e^{-(t/RC)}$$

- Cancelling R from both sides gives the exponential decay equation for **current *I*:**

$$I = I_0 e^{-(t/RC)}$$

- Where:
  - I = current after some time t (A)
  - I_0 = initial current (A)
  - t = elapsed time (s)
  - R = resistance (Ω)
  - C = capacitance (F)
- This equation shows that the current also decreases **exponentially,** from some initial value I_0

Worked Example

A 10 mF capacitor is fully charged by a 12 V power supply and then discharged through a 1 kΩ resistor.

What is the discharge current after 15 s?

Answer:

Step 1: Write the known quantities

- Initial potential difference V_0 = 12 V
- Resistance R = 1 kΩ = 1000 Ω
- Capacitance C = 10 mF = 0.01 F
- Time elapsed = 15 s

Step 2: Determine the initial current I_0

- Since the initial potential difference is 12 V and the resistance is 1000 Ω, then:

$$I_0 = \frac{V_0}{R} = \frac{12}{1000} = 0.012 \text{ A}$$

Step 3: Write the decay equation for current

- The decay equation for current is:

$$I = I_0 e^{-(t/RC)}$$

Step 4: Substitute quantities and calculate the current after 15 s

- Substituting quantities gives the following:

$$I = (0.012) \times (e^{-(15/(1000 \times 0.01)})$$

$$I = (0.012) \times (e^{-1.5})$$

$$I = (0.012) \times (0.223...)$$

$$I = 2.7 \times 10^{-3} \text{ A} = 2.7 \text{ mA}$$

Examiner Tips and Tricks

Remember you can work out initial quantities like current or potential difference or charge using the equations:

- $V_0 = I_0R$
- $Q_0 = CV_0$

You will then usually have enough information to substitute all necessary values into the decay equations!

Natural Logarithms & Discharge Equations

- The exponential decay equations are not **linear**
- They can be turned into linear equations by using the **natural logarithm** function
- Recall the exponential decay equation for charge:

$$Q = Q_0 e^{-(t/RC)}$$

- Dividing both sides by Q_0 gives:

$$\frac{Q}{Q_0} = e^{-(t/RC)}$$

- Taking the **natural logarithm** of both sides 'cancels' the exponential function e, giving:

Your notes

Your notes

$$\ln\left(\frac{Q}{Q_0}\right) = \ln\left(e^{-(t/RC)}\right) = -\frac{t}{RC}$$

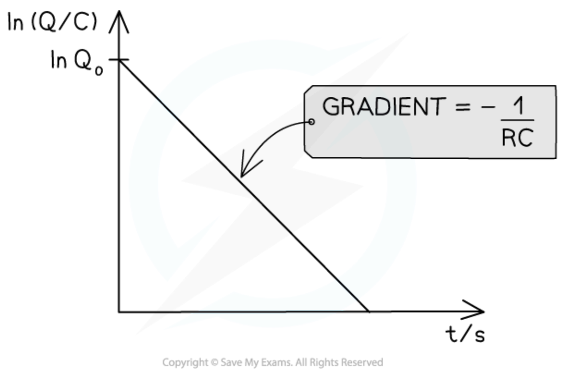
- This simplifies to:

$$\ln Q - \ln Q_0 = -\frac{t}{RC}$$

- Leaving an equation for the natural logarithm of charge Q as:

$$\ln Q = -\frac{1}{RC}t + \ln Q_0$$

- This is the equation of a **straight line graph**, where:
  - ln Q is plotted on the *y*-axis
  - *t* is plotted on the *x*-axis
  - The gradient of the line is therefore equal to –1/*RC*



The natural logarithm of the exponential decay curve line arises it to a straight-line graph with a gradient equal to –1/RC

- Following similar steps, the linearised versions of the decay equations for **potential difference** *V* is:

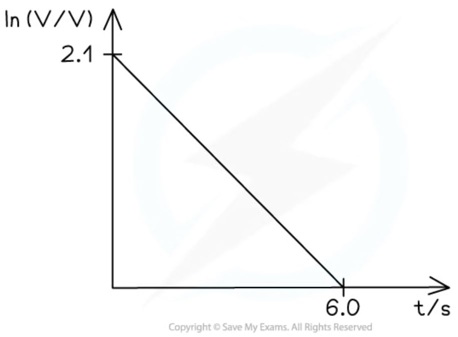
$$\ln V = -\frac{1}{RC}t + \ln V_0$$

- And for current *I* is:

$$\ln I = -\frac{1}{RC}t + \ln I_0$$

Worked Example

When a capacitor discharges, the voltage V across it varies with time t. A graph showing the variation of ln V against t is shown for a particular discharging capacitor.



Use the graph to determine the initial voltage across the capacitor.

Answer:

Step 1: Write the equation for the linearised decay equation for potential difference

- The linearised decay equation for potential difference is given by:

$$\ln V = -\frac{1}{RC}t + \ln V_0$$

Step 2: Interpret the graph given using the linearised equation

- The equation says the y-intercept of the straight line is represented by ln V_0

Step 3: Use the y-intercept to determine the initial voltage

- The y-intercept is equal to 2.1
- Therefore:

$$\ln V_0 = 2.1$$

Step 4: Cancel the natural logarithm using the exponential function:

- Raising both sides using the exponential function e 'cancels' the natural logarithm

- This gives:

$$V_0 = e^{(2.1)} = 8.2 \text{ V}$$

Examiner Tips and Tricks

You need to know how to derive decay equations for pd and for current from the decay equation for charge, as well as how to use and interpret natural logarithm equations. If you can understand that these natural log equations are **linear**, because they can plotted as a graph in the form $y = mx + c$, then you are well set for exam questions on this topic! Remember:

- The gradient of the straight line is given by -1/RC

Your notes

- The y-intercept of the line represents $\ln Q_0$ or $\ln V_0$ or $\ln I_0$

Your notes